Electric and Magnetic Fields in Matter

Static Fields

Time-Dependent Fields

Stationary Media

Moving Media

Basic Idea

The applied fields induce internal alignment

This alignment produces an additional field

Formalism

Introduce fields that do not include the fields due to alignment

free charges	free currents
bound charges	bound currents

Introduce polarization due to the bound charges and currents

Table 30–2

THREE ELECTRIC VECTORS

Name	Symbol	Associated with	Boundary Condition
Electric field strength	**E**	All charges	Tangential component continuous
Electric displacement	**D**	Free charges only	Normal component continuous
Polarization (electric dipole moment per unit volume)	**P**	Polarization charges only	Vanishes in a vacuum

Defining equation for **E**	$\mathbf{F} = q\mathbf{E}$	Eq. 27–2
General relation among the three vectors	$\mathbf{D} = \epsilon_0\mathbf{E} + \mathbf{P}$	Eq. 30–21
Gauss's law when dielectric media are present	$\oint \mathbf{D}\cdot d\mathbf{S} = q$ (q = free charge only)	Eq. 30–24
Empirical relations for certain dielectric materials *	$\mathbf{D} = \kappa\epsilon_0\mathbf{E}$ $\mathbf{P} = (\kappa - 1)\epsilon_0\mathbf{E}$	Eq. 30–22 Eq. 30–23

* Generally true, with κ independent of **E**, except for certain materials called *ferroelectrics*; see footnote on page 758.

Table 37–1

THREE MAGNETIC VECTORS

Name	Symbol	Associated with	Boundary Condition
Magnetic induction	$\mathbf{B}$	All currents	Normal component continuous
Magnetic field strength	$\mathbf{H}$	True currents only	Tangential component continuous †
Magnetization (magnetic dipole moment per unit volume)	$\mathbf{M}$	Magnetization currents only	Vanishes in a vacuum

Defining equations for **B**	$\mathbf{F} = q\mathbf{v} \times \mathbf{B}$ or $= i\mathbf{l} \times \mathbf{B}$	Eq. 33–3*a* Eq. 33–6*a*
General relation among the three vectors	$\mathbf{B} = \mu_0\mathbf{H} + \mu_0\mathbf{M}$	Eq. 37–26
Ampère's law when magnetic materials are present	$\oint \mathbf{H}\cdot d\mathbf{l} = i$ (i = true current only)	Eq. 37–27
Empirical relations for certain magnetic materials *	$\mathbf{B} = \kappa_m\mu_0\mathbf{H}$ $\mathbf{M} = (\kappa_m - 1)\mathbf{H}$	Eq. 37–29 Eq. 37–30

* For paramagnetic and diamagnetic materials only, if κ_m is to be independent of **H**.
† Assuming no true currents exist at the boundary.

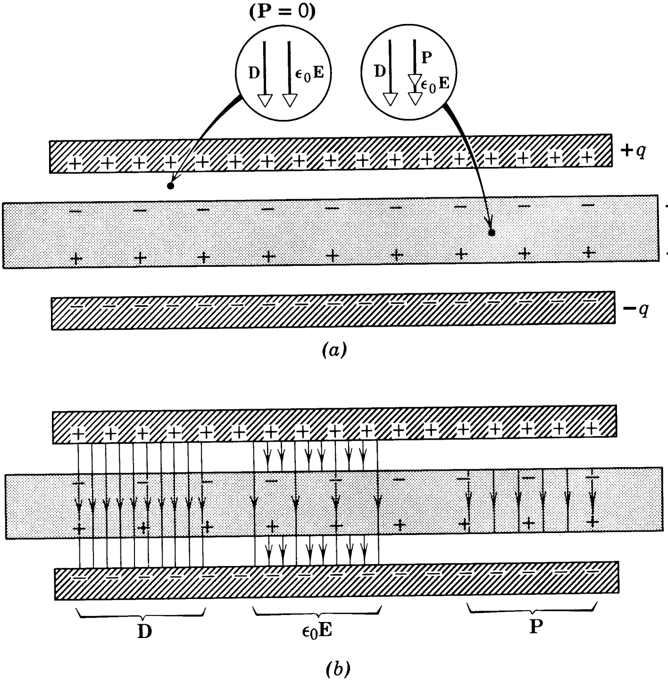


Fig. 30–13 (*a*) Showing **D**, ϵ_0**E**, and **P** in the dielectric (*upper right*) and in the gap (*upper left*) for a parallel-plate capacitor. (*b*) Showing samples of the lines associated with **D** (free charge), ϵ_0**E** (all charges), and **P** (polarization charge).

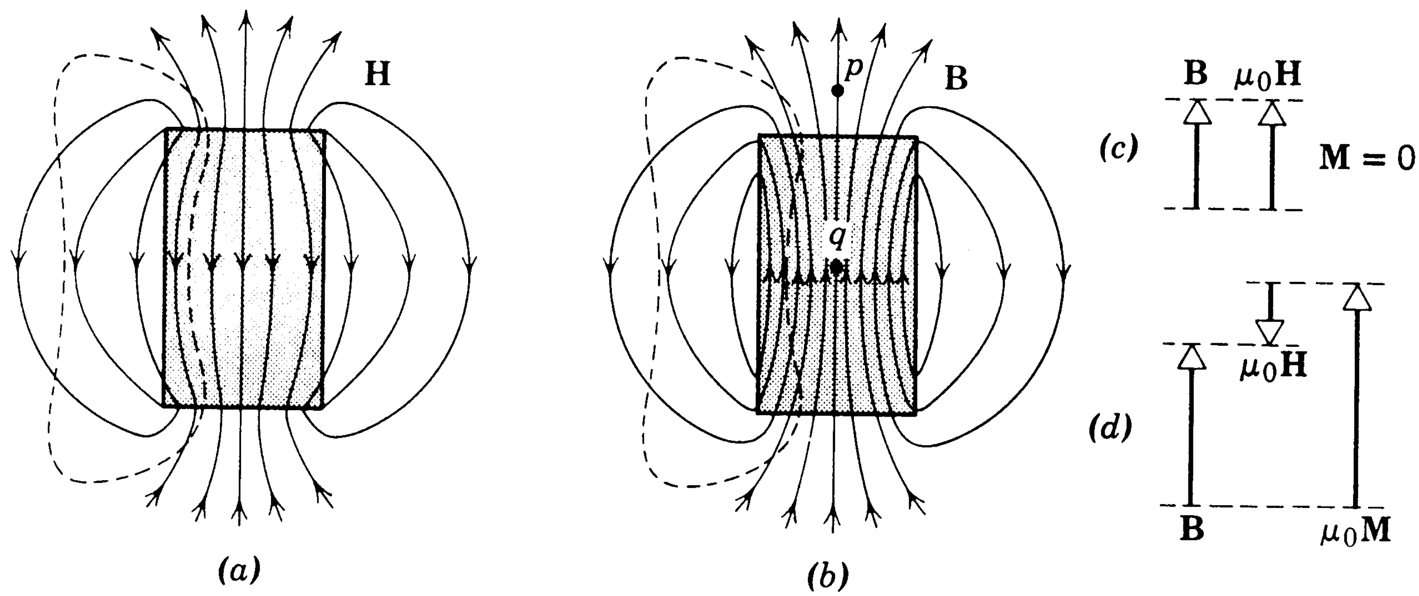


Fig. 37–22 (*a*) The lines of **H** and (*b*) the lines of **B** for a permanent magnet. Note that the lines of **H** change direction at the boundary. The closed dashed curves are paths of integration around which Ampère's law may be applied. The relation $\mathbf{B} = \mu_0\mathbf{H} + \mu_0\mathbf{M}$ is shown to be satisfied for (*c*) a particular outside point *p* and (*d*) a particular inside point *q*.

BASIC EQUATIONS OF ELECTRODYNAMICS

Maxwell's Equations

In general :

$$\begin{cases} \nabla \cdot \mathbf{E} = \frac{1}{\epsilon_0}\rho \\ \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \\ \nabla \cdot \mathbf{B} = 0 \\ \nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} \end{cases}$$

In matter :

$$\begin{cases} \nabla \cdot \mathbf{D} = \rho_f \\ \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \\ \nabla \cdot \mathbf{B} = 0 \\ \nabla \times \mathbf{H} = \mathbf{J}_f + \frac{\partial \mathbf{D}}{\partial t} \end{cases}$$

Auxiliary Fields

Definitions :

$$\begin{cases} \mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P} \\ \mathbf{H} = \frac{1}{\mu_0}\mathbf{B} - \mathbf{M} \end{cases}$$

Linear media :

$$\begin{cases} \mathbf{P} = \epsilon_0 \chi_e \mathbf{E}, \quad \mathbf{D} = \epsilon \mathbf{E} \\ \mathbf{M} = \chi_m \mathbf{H}, \quad \mathbf{H} = \frac{1}{\mu}\mathbf{B} \end{cases}$$

SO FAR: $\vec{E}$ and $\vec{B}$ in vacuum

quasi-static limit

THIS WEEK: $\vec{E}$ in matter

DIELECTRICS

NEXT WEEK: $\vec{B}$ in matter

DIAMAGNETS

FINALLY: COMPLETE MAXWELL EQUATIONS

AUXILARY FIELDS

$$\vec{D} = \epsilon_0 \vec{E} + \vec{P}$$

$$\vec{H} = \frac{1}{\mu_0} \vec{B} - \vec{M}$$

LINEAR MEDIA

$$\vec{P} = \chi_e (\epsilon_0 \vec{E})$$

$$\vec{M} = \chi_m \vec{H}$$

$$\vec{D} = \epsilon_0 \vec{E}$$

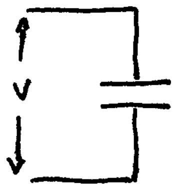
$$\vec{H} = \frac{1}{\mu} \vec{B}$$

WHICH PAIR IS FUNDAMENTAL? $\vec{E}$ and $\vec{B}$?

$\vec{D}$ and $\vec{H}$?

FEYNMAN'S VIEW

FARADAY FOUND



$$C = \epsilon_0 \frac{A}{d}$$ ϵ_0 OF VACUUM

$$C = \epsilon \frac{A}{d}$$ ϵ OF MEDIUM

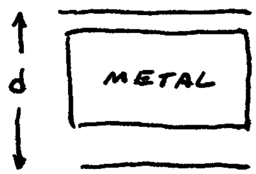
$$\epsilon > \epsilon_0$$

MORE CAPACITANCE!

MORE STORED ENERGY!

FARADAY ASKED WHY?

KNEW DECREASE GAP => MORE CAPACITANCE



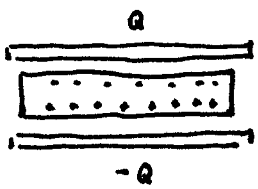
$$C = \epsilon_0 \frac{A}{d}$$

$$C' = \epsilon_0 \frac{A}{d'}$$

F => WANT INSULATOR WITH CONDUCTORS INSIDE

HOW CAN THAT BE?

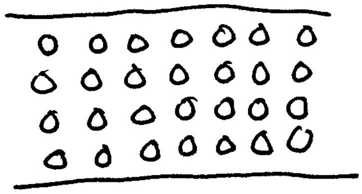
SUPPOSE WE PUT A SLAB OF METAL



$$C' = \frac{\epsilon_0 A}{d'}$$

$$d' < d \Rightarrow C' > C$$

SO FARADAY HYPOTHESIZED



conducting spheres inside an insulator

~~induced charge in the~~

Induced charge separation in the spheres

Spheres separated by insulator

$\Rightarrow$ Atoms are perfect conductors

Atoms are separated by an insulator

We now know 2 EFFECTS

Charge separation → polarization FARADAY'S IDEA

→ reorientation of dipoles

First principles

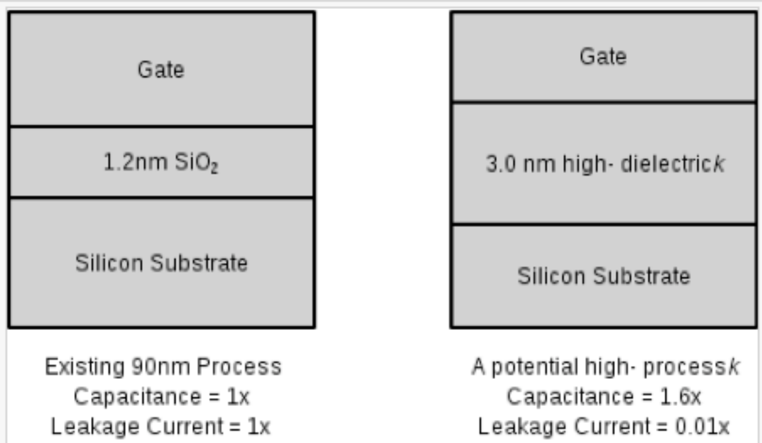
[edit]

The gate oxide in a MOSFET can be modeled as a parallel plate capacitor. Ignoring quantum mechanical and depletion effects from the Si substrate and gate, the capacitance C of this parallel plate capacitor is given by

$$C = \frac{\kappa \varepsilon_0 A}{t}$$

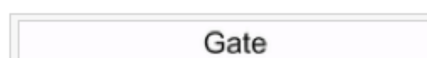
Where

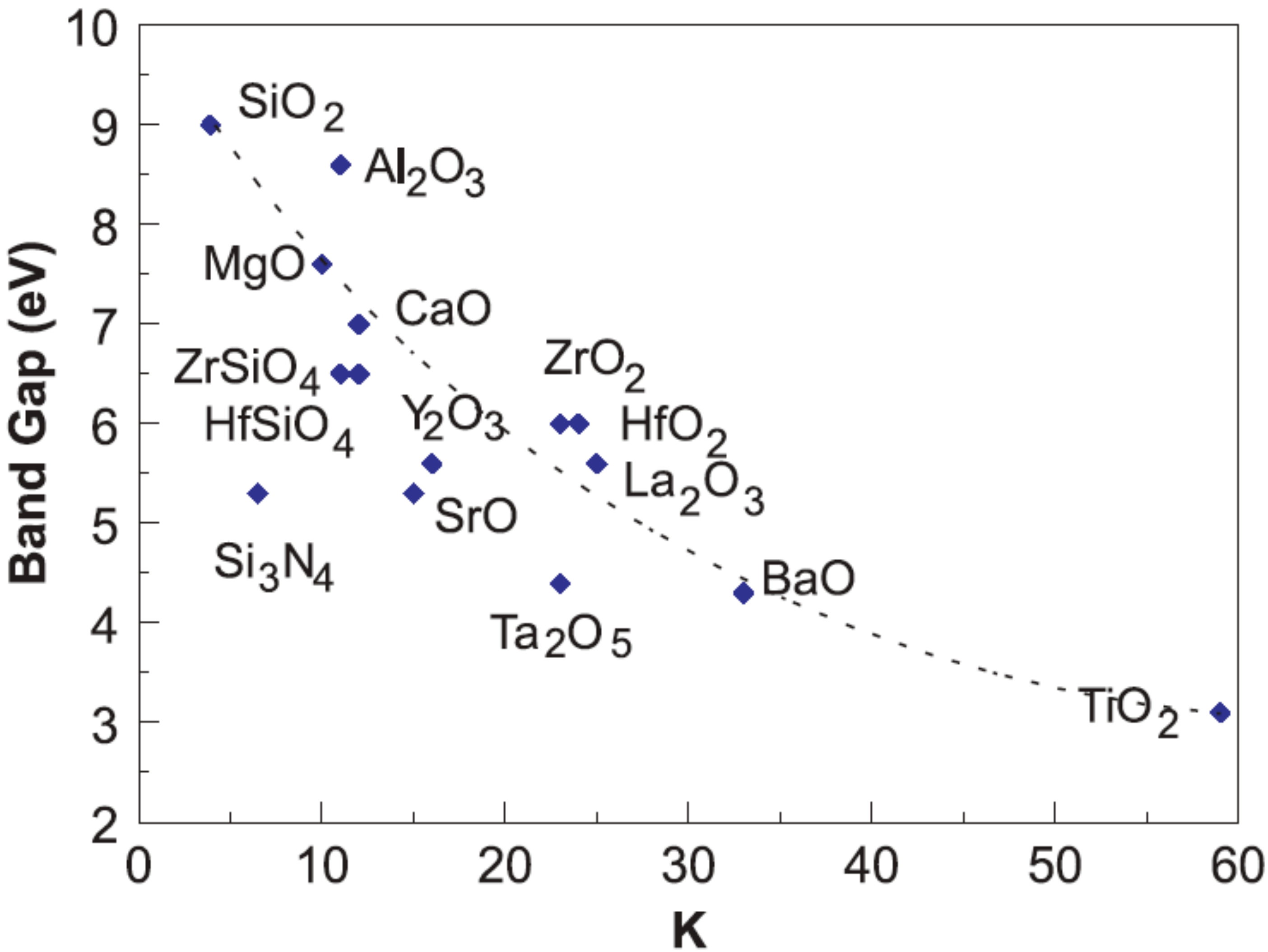
- A is the capacitor area
- κ is the relative dielectric constant of the material (3.9 for silicon dioxide)
- ε_0 is the permittivity of free space
- t is the thickness of the capacitor oxide insulator



Conventional silicon dioxide gate dielectric structure compared to a potential high-k dielectric structure

Since leakage limitation constrains further reduction of t, an alternative method to increase gate capacitance is alter κ by replacing silicon dioxide with a high-κ material. In such a scenario, a thicker gate layer might be used which can reduce the leakage current flowing through the structure as well as improving the gate dielectric reliability.

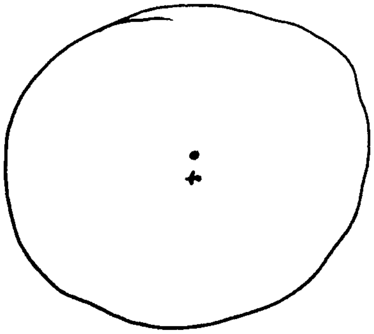


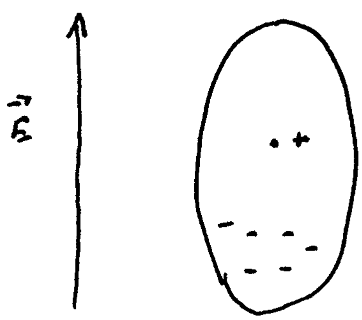


Band Gap (eV)
K
SiO2
Al2O3
MgO
CaO
ZrSiO4
HfSiO4
ZrO2
Y2O3
HfO2
La2O3
SrO
Si3N4
BaO
Ta2O5
TiO2

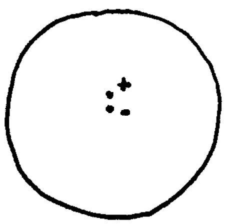
POLARIZABILITY OF ATOMS

INDUCED DIPOLES ~ induced dipole in Faraday's metal spheres.





ACTUALLY THE DISTORTION IS EXTREMELY SMALL



SUPERPOSITION : POSITIVE POINT CHARGE

NEGATIVE CHARGED SPHERE

FIELD OUTSIDE

• +

• −

Our old friend the dipole!

INDUCED DIPOLE MOMENT

$$\vec{p} = q\vec{r}$$

INDUCED POLARIZATION VECTOR

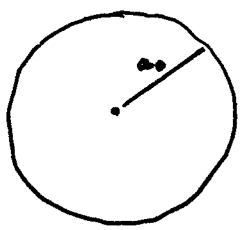
$$\vec{P} = n\vec{p} = nq\vec{r}$$

atoms per unit volume

FOR SMALL FIELDS (linear dielectrics)

$$\vec{p} = \alpha \vec{E}$$

atomic polarizability



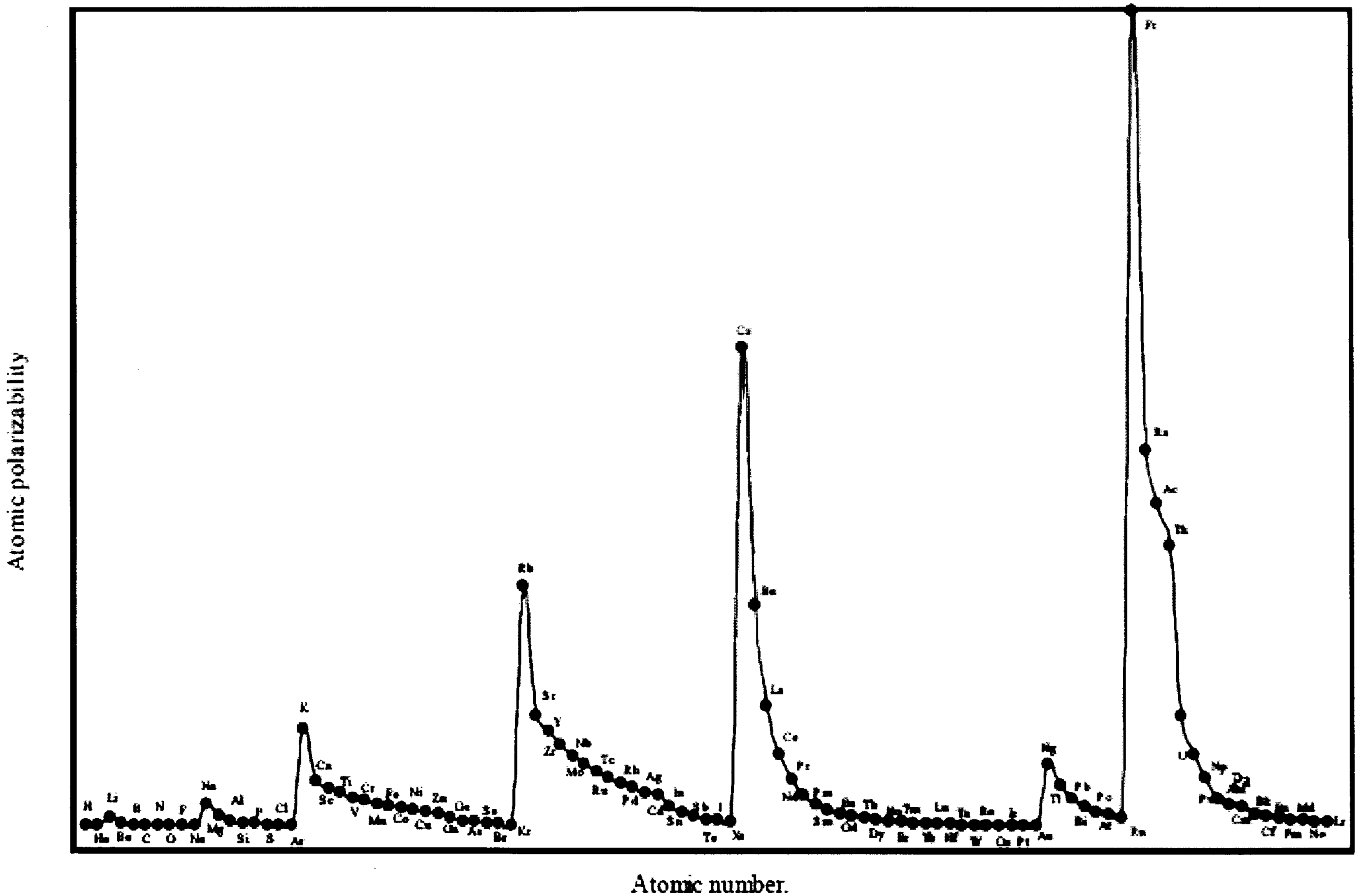
$$E_a = \frac{e}{a_0^2}$$

$$\text{DISTORTION} \propto \frac{E_{ext}}{E_a} = \frac{E}{(e/a_0^2)}$$

ATOMIC FIELDS: 3×10^{11} V/m

EXTERNAL FIELDS: 3×10^{6} V/m

=> DISTORTIONS ARE SMALL


Atomic polarizability
Atomic number.

SECOND EFFECT

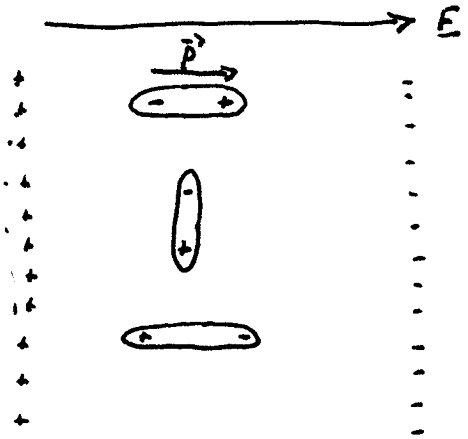
PERMANENT DIPOLE MOMENTS

$NaCl$ ionic linear $Na^+ Cl^-$

HCl

H_2O covalent not linear

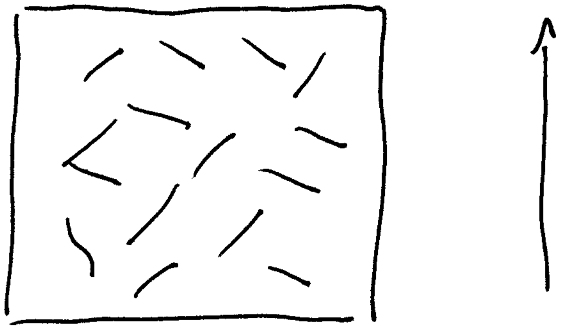
PUT A DIPOLE IN A FIELD



$$U = -\vec{p} \cdot \vec{E}$$

$$\tau = \vec{p} \times \vec{E}$$

MOLECULAR DIELECTRICS



RANDOM DIPOLES

APPLY FIELD

DIPOLES WANT TO ALIGN WITH THE APPLIED FIELD

IF THEY DID $\vec{E} = 3 \times 10^9$ V/m

SO WHY DON'T THEY?

ENERGY GAIN pE

THERMAL ENERGY $k_B T$

PROB $\sim e^{-pE/k_B T}$

FOR WATER

APPLY 30,000 V/m $\Rightarrow \dfrac{pE}{kT} \sim 10^{-4}$

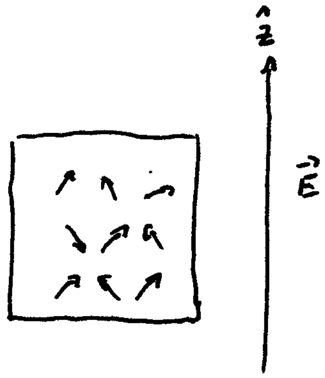
FOR WATER

$\epsilon = 80$ $\Rightarrow$ SMALL ALIGNMENT

MORE THAN ATOMS

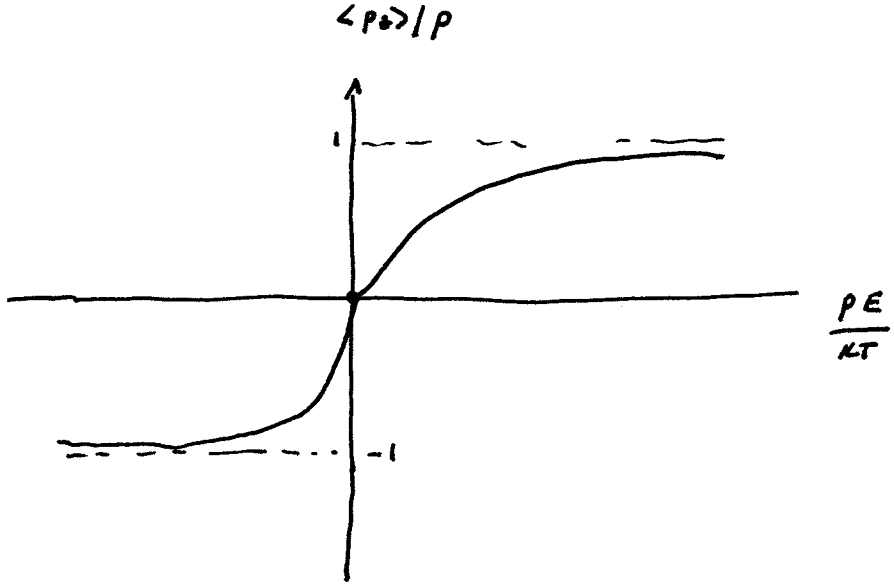
OR NONPOLAR DIELECTRICS

THE LANGEVIN EQUATION



some dipoles align

$$\frac{\langle p_z \rangle}{p} = \left[\coth\left(\frac{pE}{kT}\right) - \frac{kT}{pE}\right]$$



CONDUCTORS : free charges

DIELECTRICS : no free charges

free dipoles

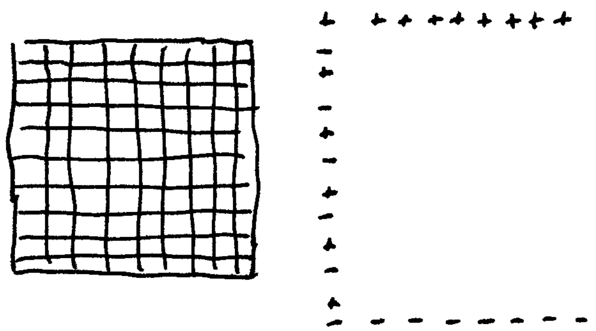
WHAT DO THEY DO?

POLARIZATION

$\vec{p}$ dipole moment of atom or molecule

N number of molecules per unit volume

$\vec{P} = N\vec{p}$ dipole moment per unit volume



surface charge density $\sigma_p = \vec{P}\cdot\hat{n}$

volume charge density $\rho_p = \vec{\nabla}\cdot\vec{P}$

if $\vec{P}$ is uniform $\rho_p = 0$

$\sigma_p = \vec{P}\cdot\hat{n}$

σ_p and ρ_p determine $\vec{E}$ outside

ALIASES:

σ_p polarization

σ_b bound

σ_i induced

$$\vec{\nabla} \cdot \vec{E} = \rho_t / \epsilon_0$$

$$\rho_t = \rho_f + \rho_p$$

$$\nabla^2 V = - \rho_t / \epsilon_0 \qquad \text{POISSON'S EQUATION}$$

$$\vec{\nabla} \cdot \vec{E} = \frac{1}{\epsilon_0} (\rho_f + \rho_p)$$

$$\rho_p = - \vec{\nabla} \cdot \vec{P}$$

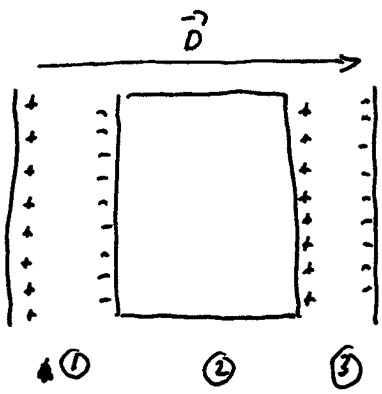
$$\vec{\nabla} \cdot \vec{E} = \frac{1}{\epsilon_0} (\rho_f - \vec{\nabla} \cdot \vec{P})$$

$$\vec{\nabla} \cdot (\epsilon_0 \vec{E} + \vec{P}) = \rho_f$$

$$\vec{\nabla} \cdot \vec{D} = \rho_f$$

lines of $\vec{D}$ start and end on free charges

lines of $\vec{E}$ on all charges



① $E = \frac{\sigma_f}{\epsilon_0}$ $\qquad D = \sigma_f$

② $\epsilon = \sigma_f / \epsilon$ $\qquad D = \sigma_f$

③ $E = \sigma_f / \epsilon_0$ $\qquad D = \sigma_f$

$D = \sigma_f$ in ①, ②, and ③

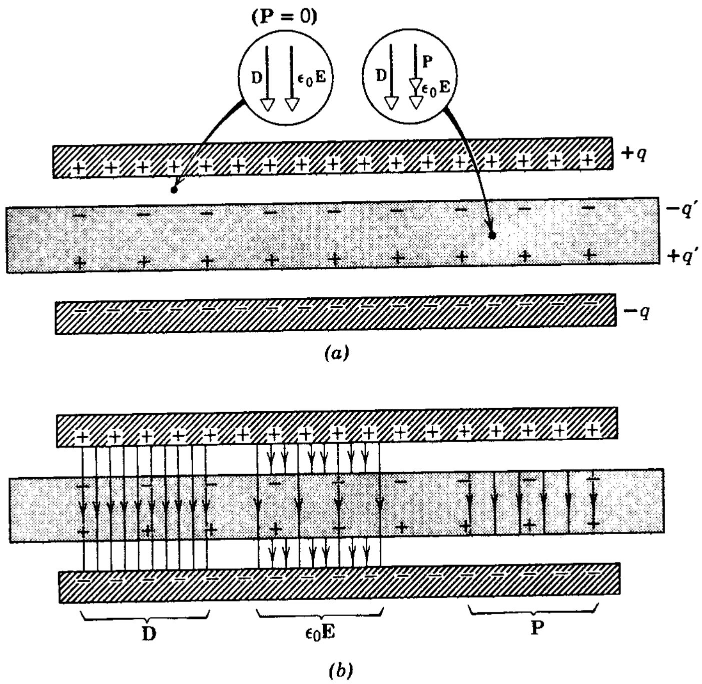
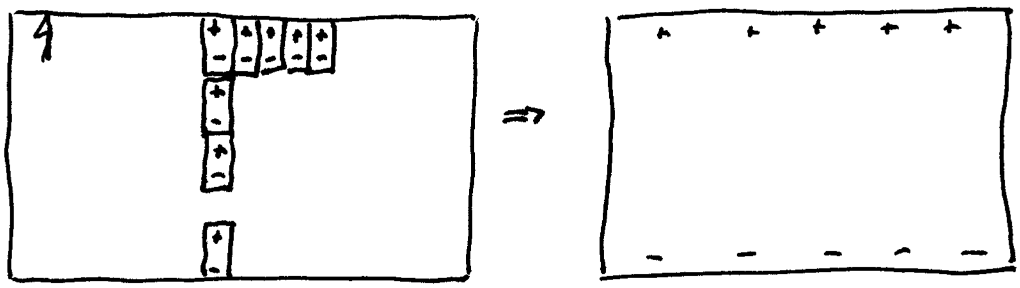


Fig. 30–13 (*a*) Showing **D**, ϵ_0**E**, and **P** in the dielectric (*upper right*) and in the gap (*upper left*) for a parallel-plate capacitor. (*b*) Showing samples of the lines associated with **D** (free charge), ϵ_0**E** (all charges), and **P** (polarization charge).

POLARIZATION CHARGES



if dielectric is uniform and isotropic

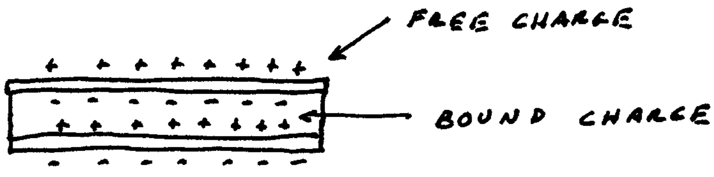
⇒ polarization charge on the surface

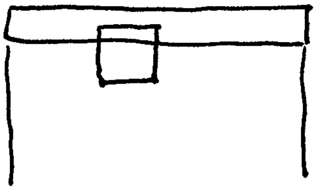
surface charge density

$$\sigma_{pol} = n\,\vec{P}$$

$$\sigma_{pol} = P$$

surface charge density = polarization inside the dielectric





$$\int \vec{E}'\cdot d\vec{S} = Q_T/\epsilon_0$$

$$E'A = \epsilon_0(Q_F - Q_b)/\epsilon_0$$

$$E' = \frac{\sigma_F - \sigma_b}{\epsilon_0} = \frac{\sigma_F - P}{\epsilon_0}$$

for low fields
and typical dielectrics

$$\vec{P} = \chi_e \epsilon_0 \vec{E}$$

electric
susceptibility

$$E = \frac{\sigma_F}{\epsilon_0}\left(\frac{1}{\chi_e + 1}\right)$$

the reduction factor

PARALLEL PLATE CAPACITOR

$$V = E d = \frac{\sigma_F}{\epsilon_0}\left(\frac{1}{\chi_e + 1}\right) d$$

$$Q = \sigma_F A$$

$$C = \frac{\epsilon_0 A}{d}(1 + \chi)$$

$$\epsilon = (1 + \chi_e)\epsilon_0$$

capacitance
increased
by this factor

$$= \frac{\epsilon_0 A}{d}(K)$$

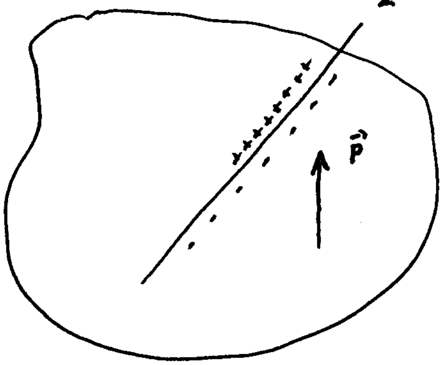
$$\epsilon = K \epsilon_0$$

FOR ANY SURFACE

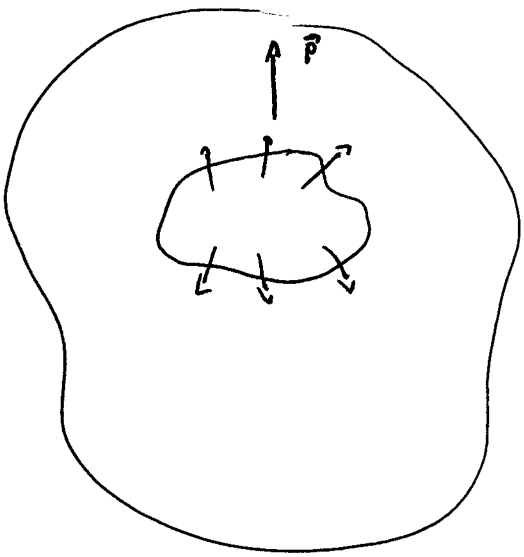
inside or out

$$\sigma_{pol} = \vec{P} \cdot \hat{n}$$

interface is neutral



FOR ANY VOLUME



$$\Delta Q_{pol} = -\int_S \vec{P} \cdot d\vec{S}$$

$$\Delta Q_{pol} = \int_V \rho_{pol}\, dV$$

$$\int \rho_{pol}\, dV = -\int_S \vec{P} \cdot d\vec{S}$$

~~a kind of~~ an integral Gauss's Law

for $\vec{P}$ and ρ_{pol}

$$\rho_{pol} = -\vec{\nabla} \cdot \vec{P}$$

$$\vec{\nabla} \cdot \vec{E} = \rho_T / \epsilon_0$$

$$\vec{\nabla} \cdot \vec{P} = -\rho_{pol}$$

if $\vec{P}$ is not uniform, then there will be ~~no~~ a net polarization charge density

Perfectly Real Charge

ρ_{pol} ~~is not~~ JUST REMINDS US WHERE IT CAME FROM

ELECTROSTATICS WITH DIELECTRICS

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho_T}{\epsilon_0}$$ FUNDAMENTAL EQUATION

$$\rho_T = \rho_{FREE} + \rho_{POL} = \rho_{FREE} - \vec{\nabla} \cdot \vec{P}$$

$$\vec{\nabla} \cdot \vec{E} = \frac{1}{\epsilon_0} \left(\rho_{FREE} - \vec{\nabla} \cdot \vec{P} \right)$$

$$\vec{\nabla} \cdot \left(\vec{E} + \frac{\vec{P}}{\epsilon_0} \right) = \frac{\rho_{FREE}}{\epsilon_0}$$

$\vec{\nabla} \times \vec{E} = 0$ is still equal to zero

$$\vec{P} = \chi \vec{E}$$

$$\vec{\nabla} \cdot \left((1+\chi) \vec{E} \right) = \vec{\nabla} \cdot (\kappa \vec{E}) = \frac{\rho_{free}}{\epsilon_0}$$

NOTHING NEW

CONVENIENT WHEN WE KNOW ρ_{FREE} AND $\vec{P}$

if κ is constant, we can take it outside

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho_{FREE}}{\epsilon}$$

if it varys in space, interesting effects occur

MAXWELL INTRODUCED $\vec{D}$

$$\vec{D} = \epsilon_0 \vec{E} + \vec{P}$$

$$\vec{\nabla} \cdot \vec{D} = \rho_{FREE}$$

$$\vec{\nabla} \times \vec{E} = 0$$

in general $\vec{\nabla} \times \vec{D} \neq 0$
no potential for $\vec{D}$
in high symmetry cases,
$\vec{D}$ works well

NOT ENOUGH TO SOLVE!

MUST ADD SOMETHING

$$\vec{D} = \epsilon \vec{E} = \epsilon_0 (1 + \chi) \vec{E} = \kappa \epsilon_0 \vec{E}$$

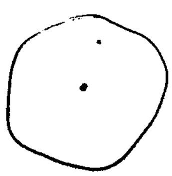
ϵ is called the permittivity

χ electric susceptibility

κ dielectric constant

ϵ_0 permittivity of free space

$\vec{D}$ is not fundamental!
it just includes the effects of matter



mostly empty
space

WHY DOES A COMB PICK UP LITTLE

PIECES OF PAPER?

FOR A POINT DIPOLE $\vec{p}$ IN AN INHOMOGENEOUS FIELD

$$\vec{F} = \vec{p} \cdot \left(\vec{\nabla}\,\vec{E}\right)$$

$$F_x = \vec{p} \cdot \vec{\nabla} E_x$$

$$F_y = \vec{p} \cdot \vec{\nabla} E_y$$

$$F_z = \vec{p} \cdot \vec{\nabla} E_z$$

HOW DOES THE FORCE SCALE WITH $\vec{E}$?

$\vec{p} = \alpha \vec{E}$ $\qquad p \sim E$

$$\vec{\nabla} E \sim E$$

$$F \sim E^2$$

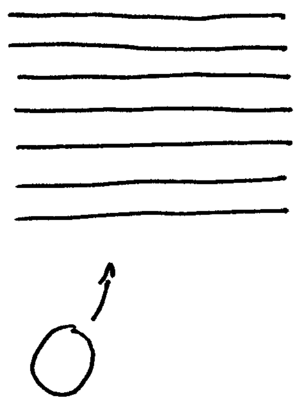
NO FORCE IN A UNIFORM FIELD

THE CANONICAL PROBLEMS

	OBJECTS	SOURCES
dielectric	sphere	uniform field
	cylinder	point charge
	plane	line charge

CANONICAL PROBLEMS

POINT
CHARGE
semi-infinite
dielectric



sphere
cylinder
plate

SHOW FALSTAD.COM

solve Maxwell's equations

images

Laplace's eqn KNOWN VOLTAGES

Poisson's eqn

http://www.falstad.com/vector3de/

This applet is an electrostatics demonstration which displays the electric field in a number of situations. You can select from a number of fields and see how particles move in the field if it is treated as either a velocity field (where the particles move along the field lines) or an actual force field (where the particles move as if they were charged particles). This helps you visualize the field When you start the applet, you will see 500 particles moving in a point charge field. By default the particles are treating the field as a *velocity* field, which means that the field vectors determine how fast the particles are moving and in what direction. In this case, the particles just move toward the center. The velocity of all the particles at a certain point on the grid is always the same. If the field a *force* field, then the field vectors determine the acceleration of the particles, but their velocity ma vary depending on where they started.

The **Field Selection** popup will allow you to select a vector field.

The choices that we looked at in class are:

conducting sphere + pt: A conducting sphere near a point charge. The size of the sphere, the separation between it and the point charge, and the potential of the sphere are all adjustable. By default the sphere is grounded.

charged sphere + pt: A charged sphere near a point charge. This is provided to show the difference between a charged sphere and a conducting sphere. (The main difference is that the electric field lines are always perpendicular to the surface of the conducting sphere, whereas this is not true with a charged sphere. This is easier to see with a Y Slice.) By default the sphere has no charge, but this can be adjusted to a positive or negative value.

cyl + line charge: A conducting cylinder near a line charge.

conducting sphere in field: A grounded conducting sphere in a uniform external field.

dielec sphere in field E: This is the electric field of a dielectric sphere in a uniform external field. The size of the sphere and the dielectric strength are adjustable. A dielectric is an insulating materi whose atoms are polarized in response to an external field; this causes the field to be weaker inside the dielectric.

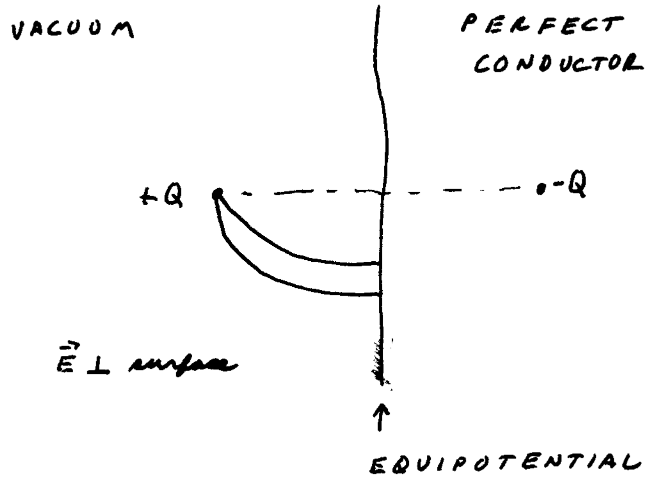
cylinder in field: A grounded conducting cylinder in a uniform external field.

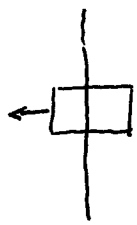
dielec cyl in field E: This is the electric field of a dielectric cylinder in a uniform external field. The size of the cylinder and the dielectric strength are adjustable.

dielec boundary E: This is the electric field of a point charge near a dielectric boundary. The poin charge is located outside of the dielectric by default; so the dielectric is the area below the bounda plane. The location of the point charge and the dielectric strength are adjustable.

conducting plane + pt: This is the electric field of a point charge near a conducting boundary.

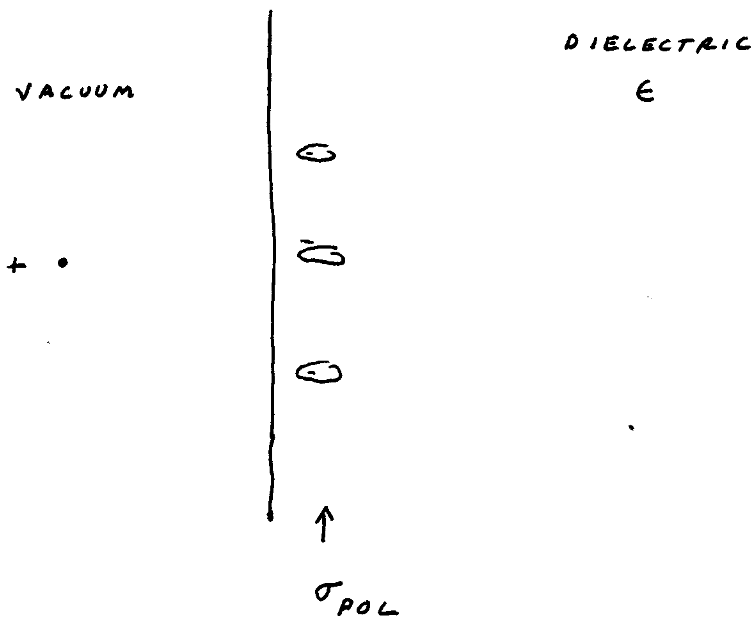
IMAGES IN DIELECTRICS





$$E \cdot A = \sigma A / \epsilon_0$$

$$E = \frac{\sigma}{\epsilon_0}$$



OUTSIDE

Q $\bullet$ $\qquad\qquad$ $\bullet$ $Q' = \dfrac{\kappa_1 - \kappa_2}{\kappa_1 + \kappa_2} Q$

INSIDE

$\bullet$

$\nearrow$

$Q + Q''$

$$Q'' = \frac{2\kappa_2}{\kappa_1 + \kappa_2}$$

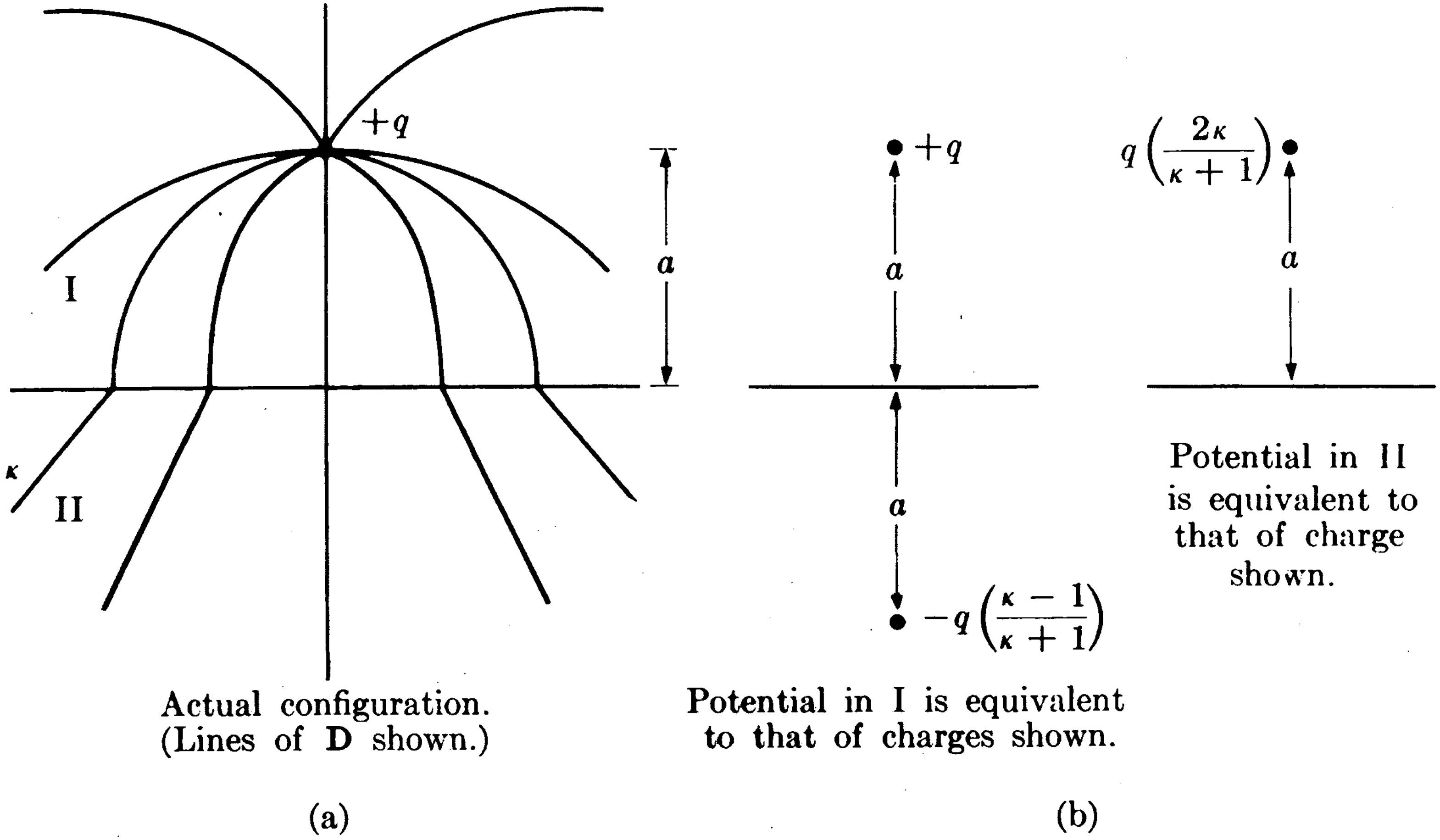
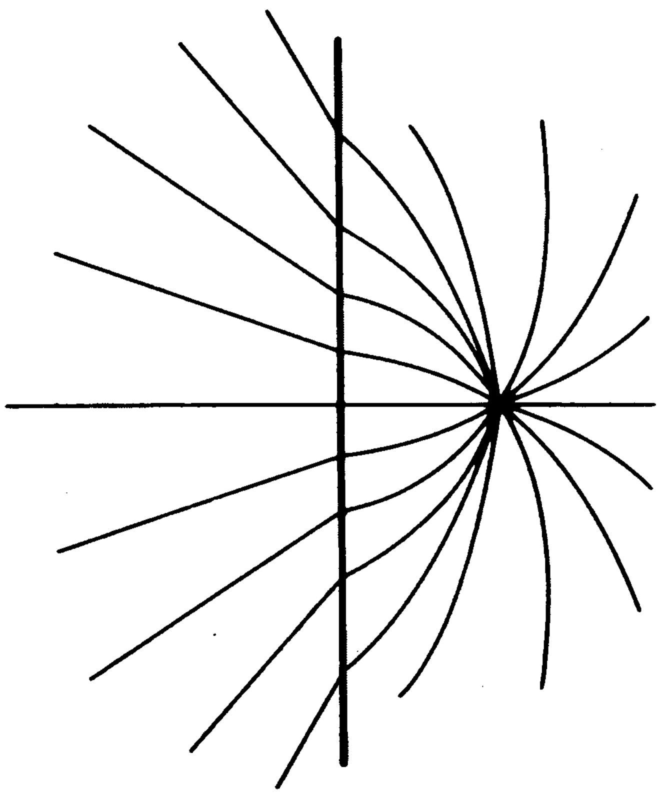
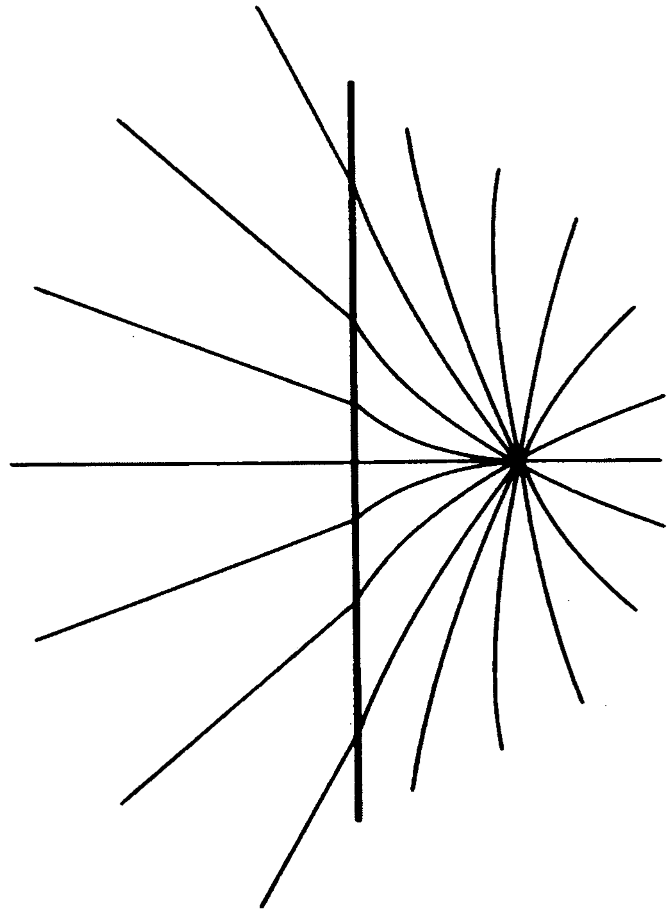


FIG. 3–3. Point charge and "dielectric half-space." (a) represents the actual physical system of a charge $+q$ at a distance a from a dielectric half-space of specific inductive capacity κ. (b) is a system of images representing the configuration: the left-hand side of the distribution (b) is a system of charges which gives the correct field distribution in region I; the right-hand part gives the correct field distribution in region II.



$\epsilon_2 > \epsilon_1$



$\epsilon_2 < \epsilon_1$

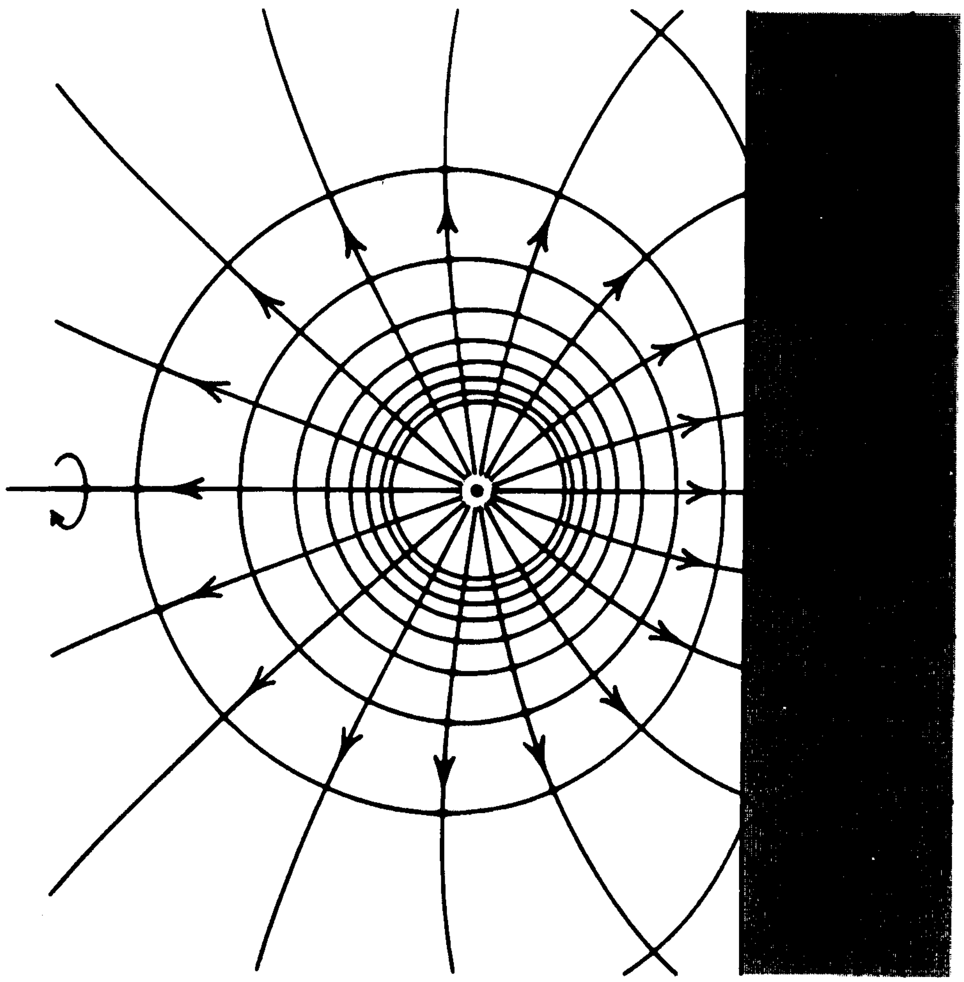


Figure 4-15. Lines of $\boldsymbol{D}$ (identified by arrows) and equipotentials for a point charge near a dielectric. As previously, equipotential surfaces are generated by rotating the figure about the axis indicated by the curved arrow. Equipotentials and lines of $\boldsymbol{D}$ near the point charge are not shown.

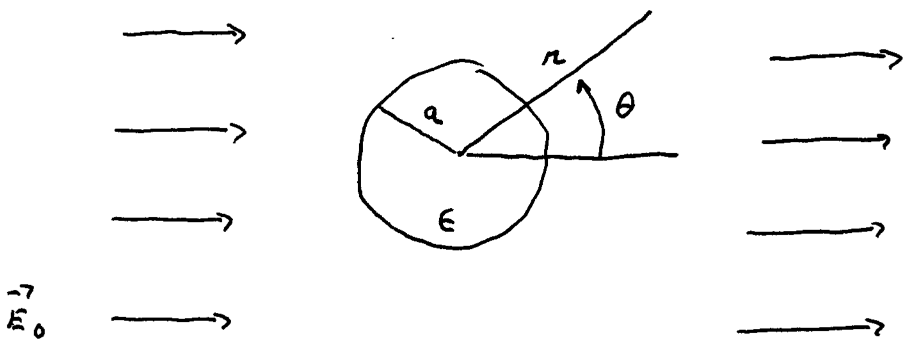
the same medium 1 on both sides of the interface, the original charge Q, and a charge

$$Q' = \frac{\epsilon_{r1} - \epsilon_{r2}}{\epsilon_{r1} + \epsilon_{r2}} Q \tag{4-55}$$

at the image position. The field in medium 2 is the same as if medium 2 extended on both sides of the interface and the original charge Q were replaced by

$$Q'' = \frac{2\epsilon_{r2}}{\epsilon_{r1} + \epsilon_{r2}} Q \tag{4-56}$$

DIELECTRIC SPHERE IN AN APPLIED FIELD



$$\nabla^2 V = 0$$

$$\nabla^2 V_{in} = 0$$

$$\nabla^2 V_{out} = 0$$

$$V_{in} = \sum_{\ell=0}^{\infty} a_\ell r^\ell P_\ell(\cos\theta)$$

$$V_{out} = \sum_{\ell=0}^{\infty} \left[b_\ell r^\ell + c_\ell r^{-(\ell+1)} \right] P_\ell(\cos\theta)$$

BCs

COMPONENT OF $\vec{E}$

TANGENTIAL CONTINUOUS ACROSS BOUNDARY

$$-\frac{1}{a} \frac{\partial V_{in}}{\partial \theta}\bigg|_{r=a} = \frac{1}{a} \frac{\partial V_{out}}{\partial \theta}\bigg|_{r=a}$$

NORMAL COMPONENT OF $\vec{D}$ CONTINUOUS

$$-\epsilon \left.\frac{\partial V_{in}}{\partial r}\right|_{r=a} = - \left.\frac{\partial V_{out}}{\partial r}\right|_{r=a}$$

E_{TAN} BC :

$$A_1 = -E_0 + \frac{C_1}{a^3}$$

$$A_\ell = \frac{C_\ell}{a^{2\ell+1}} \qquad \text{for } \ell \neq 1$$

$\vec{D}_\perp$ BC :

$$\epsilon A_1 = -E_0 - 2\frac{C_1}{a_3}$$

$$\epsilon \ell A_\ell = -(\ell+1)\frac{C_\ell}{a^{2\ell+1}}$$

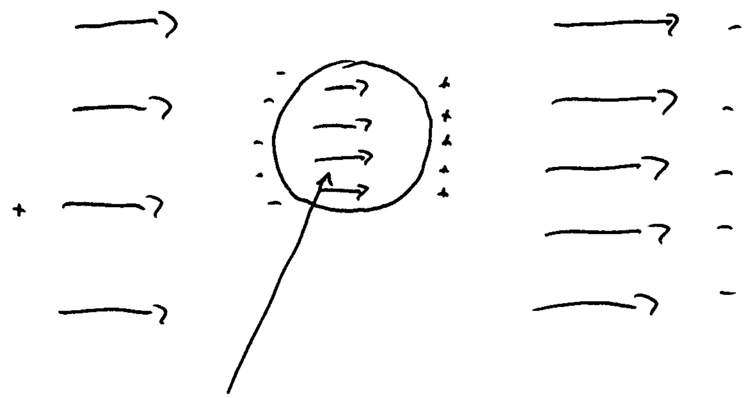
$\Rightarrow \quad A_\ell = C_\ell = 0$ for all $\ell \neq 1$

$$A_1 = -\left(\frac{3}{2+\epsilon}\right) E_0$$

$$C_1 = \left(\frac{\epsilon - 1}{\epsilon + 2}\right) a^3 E_0$$

$$V_{in} = -\left(\frac{3}{2+\epsilon}\right) E_0 \, r \cos\theta$$

$$V_{out} = -E_0 \, r \cos\theta + \left(\frac{\epsilon - 1}{\epsilon + 2}\right) E_0 \left(\frac{a^3}{r^2}\right) \cos\theta$$



UNIFORM $\vec{P}$ INSIDE

$$\vec{E}_{\text{inside due to } \vec{p}} = \left(\frac{3}{2+\epsilon}\right) E_0$$

OUTSIDE UNIFORM $\vec{E}$

PLUS

DIPOLE $p = \left(\frac{\epsilon - 1}{\epsilon + 2}\right) a^3 E_0$

$$\vec{P}_{in} = (\epsilon - 1)\, \vec{E} = (\epsilon - 1)\left(\frac{3}{2+\epsilon}\right) E_0$$

$$\vec{\nabla} \cdot \vec{P}_{in} = 0$$

$$\vec{P} \cdot \hat{n} = \sigma_{pol} = \left(\frac{3}{2+\epsilon}\right)(\epsilon - 1)\, E_0 \cos\theta$$

SPHERICAL CAVITY

OPPOSITE

FOR σ_{pol}

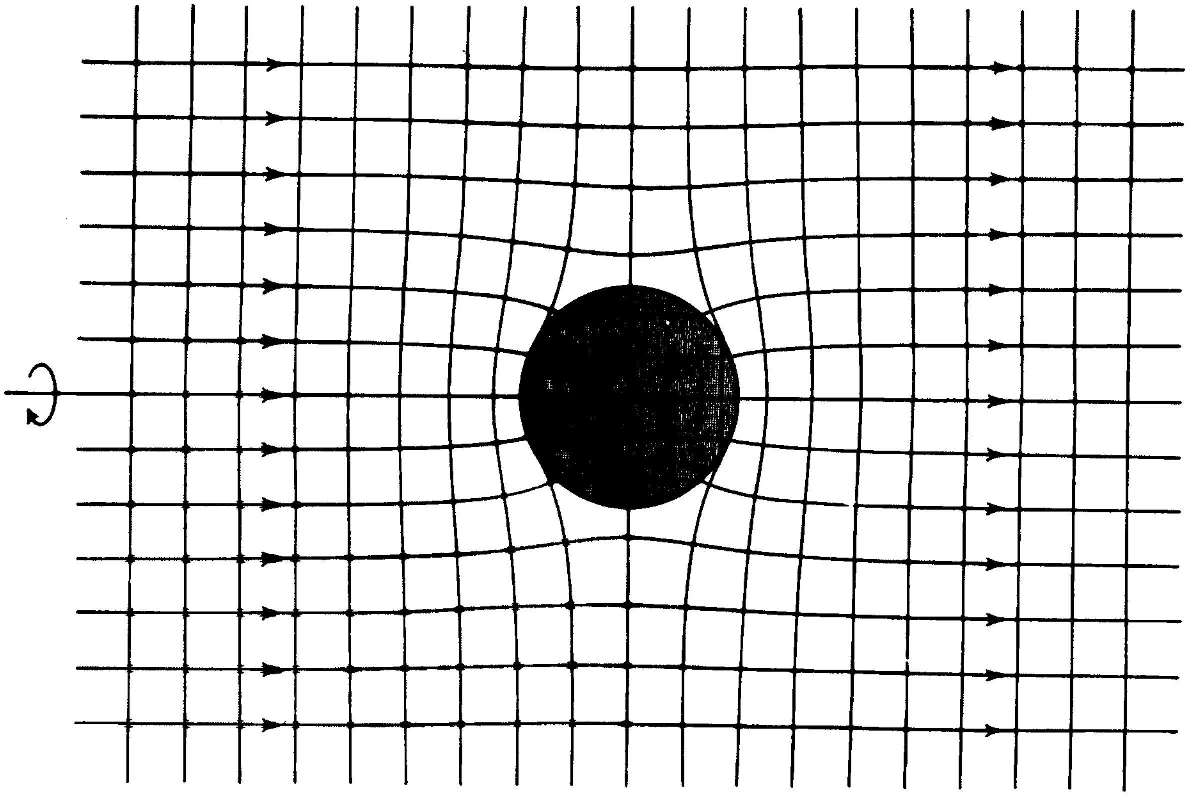


Figure 4-24. The field near a *dielectric sphere* ($\epsilon_r = 3$) in a uniform electric field. The lines of D (indicated by arrows) crowd into the sphere as shown, with the result that D is larger inside than outside. Since there is no free charge at the surface of the sphere, the lines of D neither originate nor terminate there, and they are continuous across the boundary. The equipotentials spread out inside, corresponding to a *lower* electric field intensity E. The electric field intensity E is discontinuous at the surface, and the density of lines of force is lower inside than outside. As in the conducting sphere, the field is hardly disturbed at distances larger than one radius from the surface. The field inside is uniform. The origin is chosen at the center of the sphere and the polar axis used in the calculation points to the right.

CONDUCTING SPHERE

$r = a \qquad V = 0$

$r = \infty \qquad V = -E_0 z = -E_0 r \cos\theta$

only A_1 and C_1

$A_1 = -E_0$

$C_1 = -A_1 a^3 = E_0 a^3$

$$V = -E_0 r \cos\theta + \underset{E_0 a^3}{\underset{\uparrow}{B}} \frac{\cos\theta}{r}$$

$$V = \underset{\text{APPLIED FIELD}}{\underbrace{-E_0 r \cos\theta}} + \underset{\text{OPPOSING DIPOLE}}{\underbrace{E_0 a^3 \frac{\cos\theta}{r}}}$$

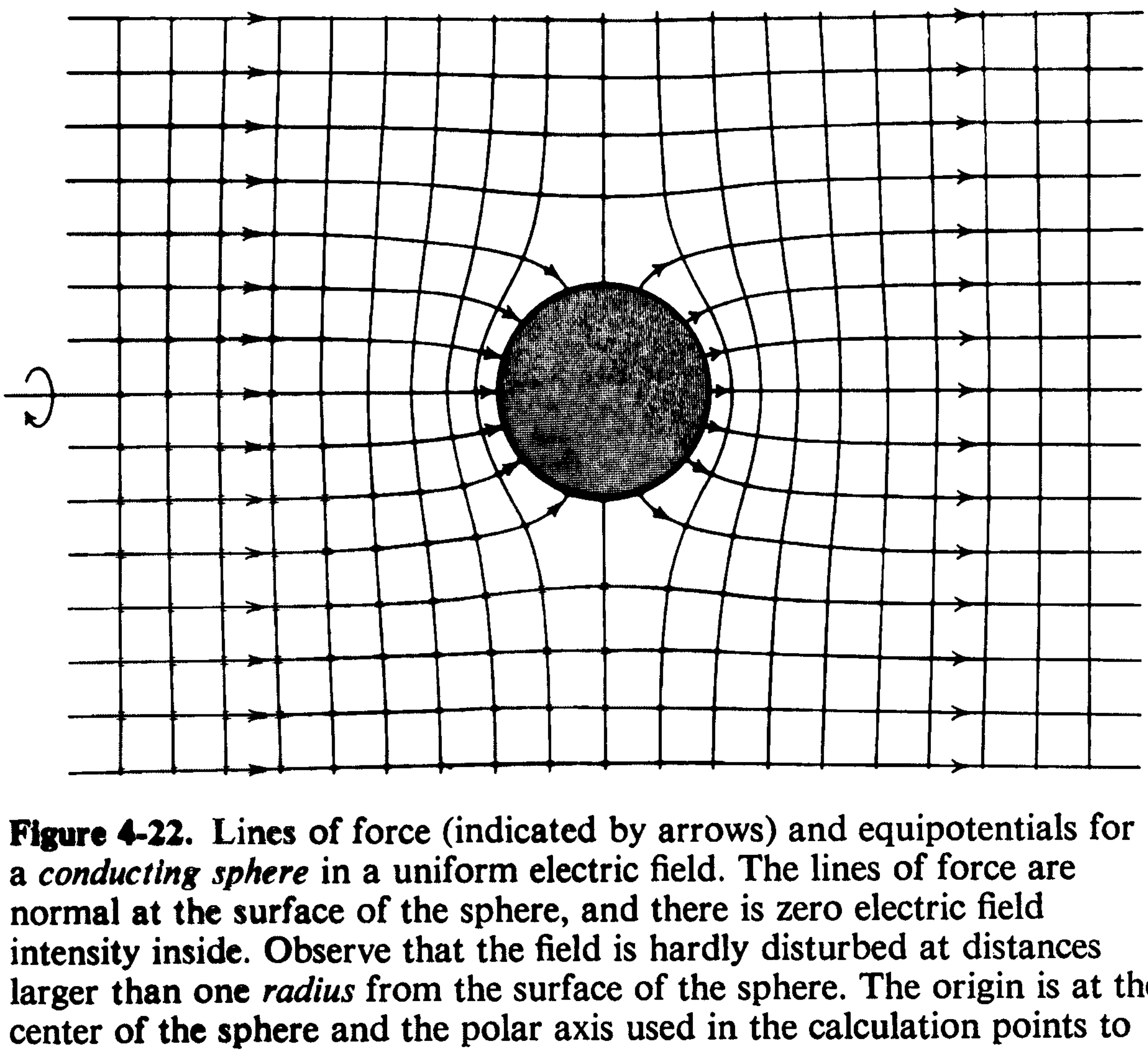
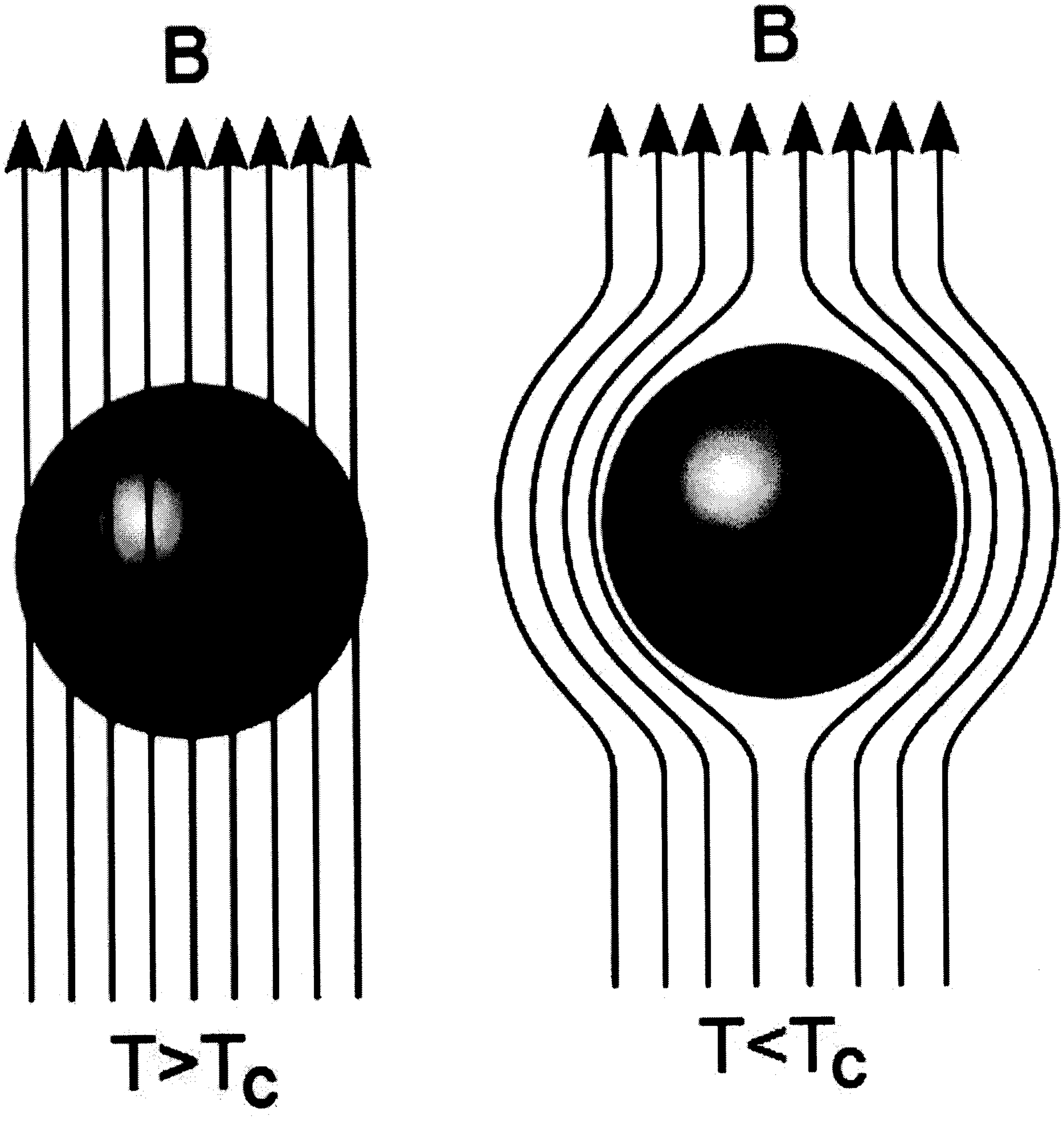


Figure 4-22. Lines of force (indicated by arrows) and equipotentials for a *conducting sphere* in a uniform electric field. The lines of force are normal at the surface of the sphere, and there is zero electric field intensity inside. Observe that the field is hardly disturbed at distances larger than one *radius* from the surface of the sphere. The origin is at the center of the sphere and the polar axis used in the calculation points to the right.


B
B
T>T$_C$
T<T$_C$