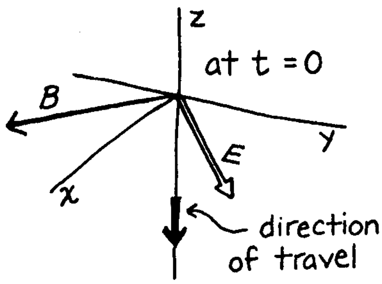
9.1



The wave is travelling in the $-\hat{z}$ direction, as shown by the sign in $(z + ct)$. Hence that is the direction of $\underset{\sim}{E} \times \underset{\sim}{B}$. $\underset{\sim}{B}$ is perpendicular to $\underset{\sim}{E}$ and equal in magnitude:

$$\underset{\sim}{B} = 2(\hat{x} - \hat{y}) \sin\left(\frac{2\pi}{\lambda}\right)(z + ct) \text{ gauss.}$$

9.2

The power density in electromagnetic waves is c times the energy density U. The power density given, 10^3 joule $m^{-2}\,sec^{-1}$, is equivalent to 10^6 erg $cm^{-2}\,sec^{-1}$. The energy density U is $10^6/3 \times 10^{10}$ erg cm^{-3}. Half of this is in magnetic field, with density $B^2_{rms}/8\pi$. Hence

$$\frac{B^2_{rms}}{8\pi} = \frac{1}{2} \times \frac{10^6}{3 \times 10^{10}}, \text{ or } B_{rms} = \left(\frac{4\pi \times 10^6}{3 \times 10^{10}}\right)^{\frac{1}{2}} = 0.02 \text{ gauss}$$

In SI, the energy density in $J\,m^{-3}$ is $\dfrac{B^2_{rms}}{2\mu_0}$, with B_{rms} in tesla. Hence:

$$B^2_{rms} = 2\mu_0 \times \frac{1}{2} \times \left(\frac{10^3}{3 \times 10^8}\right) \text{ or } B_{rms} = \left(\frac{4\pi \times 10^{-7} \times 10^3}{3 \times 10^8}\right)$$

$$= 2 \times 10^{-6} \text{ tesla}$$

9.3 The proton at the origin in Fig. 9.8 will experience the maximum electric field of 5 statvolt/cm at $t = 0$. The field falls to half value in the time, 1 nanosecond, it takes the wave to travel 30 cm (1 foot). The time variation of the field E at the origin is:

$E_y = \frac{5}{1 + (10^9 t)^2}$. The momentum acquired by the proton during the passage of the pulse will be

$$p_y = \int_{-\infty}^{\infty} e E_y \, dt = e \int_{-\infty}^{\infty} \frac{5\,dt}{1 + (10^9 t)^2} = e \times 5 \times 10^{-9} \pi$$

$p_y = 7.5 \times 10^{-18}$ gm cm sec^{-1} The proton's final speed is p_y/m or 4.7×10^6 cm/sec. Its displacement during the few nanoseconds of acceleration is negligible. One microsecond later it will be close to $y = 4.7$ cm.

9.4 Before the pulse has completely passed, the proton has acquired some velocity in the y direction, and will therefore experience a force $e\mathbf{v} \times \mathbf{B}$ in the magnetic field of the wave. This force will be in the direction $-\hat{\mathbf{x}}$, which is the direction in which the wave is travelling. And it would be in that direction for a negative particle also. The wave tends to knock the particle along. In order of magnitude, if τ is the duration of the pulse of amplitude E:

$$p_y = E e \tau \qquad v_y = \frac{eE\tau}{m}$$

$$p_x \approx e \frac{v_y}{c} B \tau = e \frac{v_y}{c} E \tau \quad , \text{ since } B = E.$$

Then $p_x / p_y \approx v_y / c$ The "knock-on" is a second order effect.

$\boxed{9.5}$ $\mathbf{E} = \hat{y}\, E_0 \sin(kx + \omega t) \qquad \mathbf{B} = -\hat{z}\, E_0 \sin(kx + \omega t)$

$\nabla \cdot \mathbf{E} = 0\ ;\ \nabla \times \mathbf{E} = \hat{z}\, k E_0 \cos(kx + \omega t)\ ;\ \frac{\partial \mathbf{E}}{\partial t} = \hat{y}\, E_0 \omega \cos(kx + \omega t)$

$\nabla \cdot \mathbf{B} = 0\ ;\ \nabla \times \mathbf{B} = \hat{y}\, k E_0 \cos(kx + \omega t)\ ;\ \frac{\partial \mathbf{B}}{\partial t} = -\hat{z}\, \omega E_0 \cos(kx + \omega t)$

$\nabla \times \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}$ requires $k = \omega/c$

$\nabla \times \mathbf{B} = \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t}$ also requires $k = \omega/c$

For $\omega = 10^{10}\ \text{sec}^{-1}$ $\qquad \lambda = 2\pi c/\omega = 18.84$ cm

$$\text{Energy density} = \Big(\underbrace{\frac{E_0^2}{8\pi}}_{\text{electric field}} + \underbrace{\frac{E_0^2}{8\pi}}_{\text{magnetic field}}\Big)\underbrace{\frac{1}{2}}_{\text{average of } \sin^2(kx+\omega t)} = \frac{E_0^2}{8\pi}$$

For $E_0 = .05$ statvolt/cm $\qquad E_0^2/8\pi = 0.99 \times 10^{-4}\ \text{erg cm}^{-3}$

Power density $= (E_0^2/8\pi)c = 3.0 \times 10^6\ \text{erg cm}^2\ \text{sec}^{-1}$

$\boxed{9.6}$ (I) $\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \qquad$ (II) $\nabla \times \mathbf{B} = \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}$

Let $\mathbf{E} = \hat{z}\, E_0 \sin(y - vt)\ ;\ \mathbf{B} = \hat{x}\, B_0 \sin(y - vt)$

$\nabla \times \mathbf{E} = \hat{x}\, \frac{\partial E_z}{\partial y} = \hat{x}\, E_0 \cos(y - vt)\ ;\ \frac{\partial \mathbf{E}}{\partial t} = -\hat{z}\, v E_0 \cos(y - vt)$

$\nabla \times \mathbf{B} = -\hat{z}\, \frac{\partial B_x}{\partial y} = -\hat{z}\, B_0 \cos(y - vt)\ ;\ \frac{\partial \mathbf{B}}{\partial t} = -\hat{x}\, v B_0 \cos(y - vt)$

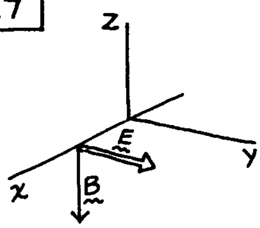
$$\left.\begin{array}{l}\text{substituting in (I) we get } E_0 = v B_0 \\ \text{substituting in (II) we get } B_0 = \epsilon_0 \mu_0 v E_0\end{array}\right\}\ \epsilon_0 \mu_0 v^2 = 1$$

$$\underbrace{v}_{\text{m sec}^{-1}} = \frac{1}{\sqrt{\mu_0 \epsilon_0}} \qquad \underbrace{E_0}_{\text{volt m}^{-1}} = \frac{\overbrace{B_0}^{\text{tesla}}}{\sqrt{\mu_0 \epsilon_0}}$$

If the magnetic field amplitude is expressed as

$$H_0 = \frac{B_0}{\mu_0}\ , \quad \text{then :} \quad \underbrace{E_0}_{\text{volt m}^{-1}} = \underbrace{H_0}_{\text{amp m}^{-1}} \underbrace{\sqrt{\frac{\mu_0}{\epsilon_0}}}_{377\ \text{ohms}}$$

9.7

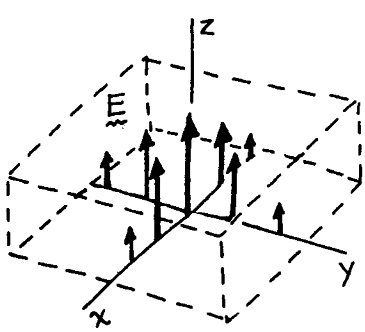


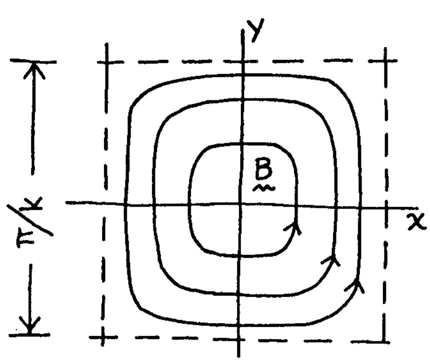
$$\omega = 2\pi f = 6.28 \times 10^8 \text{ sec}^{-1}$$
$$k = \omega/c = .0209$$
$$\mathbf{E} = \hat{\mathbf{y}} E_0 \cos(.0209x + 6.28 \times 10^8 t)$$
$$\mathbf{B} = -\hat{\mathbf{z}} E_0 \cos(.0209x + 6.28 \times 10^8 t)$$

9.8

$$E_x = E_y = 0 \;;\; E_z = E_0 \cos kx \cos ky \cos \omega t$$
$$\nabla \times \mathbf{E} = kE_0(-\hat{\mathbf{x}} \cos kx \sin ky + \hat{\mathbf{y}} \sin kx \cos ky) \cos \omega t$$
$$\frac{\partial \mathbf{E}}{\partial t} = -\omega \hat{\mathbf{z}} E_0 \cos kx \cos ky \sin \omega t$$
$$B_x = B_0 \cos kx \sin ky \sin \omega t \;;\; B_y = -\sin kx \cos ky \sin \omega t \;;\; B_z = 0$$
$$\nabla \times \mathbf{B} = \hat{z}\left(\frac{\partial B_y}{\partial x} - \frac{\partial B_x}{\partial y}\right) = -2k\hat{\mathbf{z}} B_0 \cos kx \cos ky \sin \omega t$$
$$\frac{\partial \mathbf{B}}{\partial t} = \omega B_0 (\hat{\mathbf{x}} \cos kx \sin ky - \hat{\mathbf{y}} \sin kx \cos ky) \cos \omega t$$
$$\left.\begin{aligned} \nabla \times \mathbf{E} &= -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t} \quad \text{gives : } B_0 = \frac{kc}{\omega} E_0 \\ \nabla \times \mathbf{B} &= \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t} \quad \text{gives : } B_0 = \frac{\omega}{2kc} E_0 \end{aligned}\right\} 2k^2c^2 = \omega^2$$

$$\omega = \sqrt{2}\, ck \qquad B_0 = E_0/\sqrt{2}$$





$\boxed{9.9}$ The mean energy density in a sinusoidal electromagnetic wave of amplitude E_0 is $E_0^2/8\pi$. (See Prob. 9.5 solution). $E_{rms} = E_0/\sqrt{2}$. If $E_{rms}^2/4\pi = 4\times10^{-13}$ erg, $E_{rms} = (4\pi\times4\times10^{-13})^{\frac{1}{2}}$
$= 2.2 \times 10^{-6}$ statvolt/cm
$= 2.2 \times 10^{-6} \times 3\times10^{4}$ or 6.6×10^{-2} volt/meter.

A wave in which the energy density is 4×10^{-13} erg cm^{-3} is transporting energy with power density $4\times10^{-13}\times3\times10^{10}$ or 1.2×10^{-2} erg cm^{-2}sec^{-1}, equivalent to 1.2×10^{-5} watt/m^2. If the kilowatt radiated by the transmitter is spread over a hemisphere of R meters radius, the power density there, in watt/m^2, is $10^3/2\pi R^2$. Setting this equal to 1.2×10^{-5} gives $R \approx 3000$ m, or 3 km.

If you want to do the whole calculation in SI, start with the given energy density 4×10^{-14} J m^{-3}. This times c, 3×10^{8} m sec^{-1}, gives us the power density 1.2×10^{-5} watt/m^2. To find E_{rms}, use Eq. 29:
$E_{rms} = (377\times1.2\times10^{-5})^{\frac{1}{2}} = 6.6\times10^{-2}$ volt m^{-1}.

$\boxed{9.10}$

If we neglect the edge fields, an approximation which is not very good unless $s \ll b$, the displacement current will be uniformly distributed in the gap, and the total displacement current in the gap will equal the conduction current I in the wire. The fraction of the current enclosed by a circle through P, centered on the axis, will be $\pi r^2/\pi b^2$.

Hence $2\pi r B = \frac{4\pi}{c}\frac{r^2}{b^2} I$ or $B = \frac{2Ir}{cb^2}$.

9.11 The area covered is $\pi/4 \times (1000\ \text{km})^2$, or $7 \times 10^{11}\ \text{m}^2$. The power density is therefore about 10^4 watts$/10^{12}\ \text{m}^2$, or 10^{-8} watts/m². Using the relation given by Eq. 29 we can calculate the rms electric field strength:

$$E_{rms} = (377 \times 10^{-8})^{\frac{1}{2}} = .002 \text{ or } 2 \text{ millivolt/m.}$$

9.12 Let E_i be the amplitude of the oscillating electric field of the incident wave, E_r that in the reflected wave. If half the incident energy is reflected, $E_r = E_i/\sqrt{2}$. At certain locations the two oscillating electric fields are, and remain at all times, in phase, with total amplitude $E_r + E_i$. (In Fig. 9.10 such locations are $\lambda/4$, $3\lambda/4$, $5\lambda/4$ ... from the mirror. In that case the mirror was a perfect conductor; the reflection was total, with $E_r = E_i$.) At other locations the two oscillating electric fields are, and remain, exactly 180° out of phase. The total electric field oscillates with amplitude $E_i - E_r$. (In Fig. 9.10 such a location is $\lambda/2$ from the reflector, where, because $E_r = E_i$ in that case, $\mathbf{E}$ is zero at all times.) In our case, with $E_r = E_i/\sqrt{2}$, the ratio of maximum amplitude observed to minimum amplitude observed is

$$\left(1 + \frac{1}{\sqrt{2}}\right)\Big/\left(1 - \frac{1}{\sqrt{2}}\right) = 5.83$$

9.13 $\underset{\sim}{E}' \cdot \underset{\sim}{E}' - \underset{\sim}{B}' \cdot \underset{\sim}{B}' = (\underset{\sim}{E}'_{\parallel} + \underset{\sim}{E}'_{\perp}) \cdot (\underset{\sim}{E}'_{\parallel} + \underset{\sim}{E}'_{\perp}) - (\underset{\sim}{B}'_{\parallel} + \underset{\sim}{B}'_{\perp}) \cdot (\underset{\sim}{B}'_{\parallel} + \underset{\sim}{B}'_{\perp})$

$$= \underset{\sim}{E}'_{\parallel} \cdot \underset{\sim}{E}'_{\parallel} - \underset{\sim}{B}'_{\parallel} \cdot \underset{\sim}{B}'_{\parallel} + \underset{\sim}{E}'_{\perp} \cdot \underset{\sim}{E}'_{\perp} - \underset{\sim}{B}'_{\perp} \cdot \underset{\sim}{B}'_{\perp}$$

$$= \underset{\sim}{E}_{\parallel} \cdot \underset{\sim}{E}_{\parallel} - \underset{\sim}{B}_{\parallel} \cdot \underset{\sim}{B}_{\parallel} + \left(\underset{\sim}{E}'_{\perp} \cdot \underset{\sim}{E}'_{\perp} - \underset{\sim}{B}'_{\perp} \cdot \underset{\sim}{B}'_{\perp} \right) \quad (\text{since } \underset{\sim}{E}'_{\parallel} = \underset{\sim}{E}_{\parallel};\ \underset{\sim}{B}'_{\parallel} = \underset{\sim}{B}_{\parallel})$$

$$= \gamma^2 (\underset{\sim}{E}_{\perp} + \underset{\sim}{\beta} \times \underset{\sim}{B}_{\perp}) \cdot (\underset{\sim}{E}_{\perp} + \underset{\sim}{\beta} \times B_{\perp}) - \gamma^2 (\underset{\sim}{B}_{\perp} - \underset{\sim}{\beta} \times \underset{\sim}{E}_{\perp}) \cdot (\underset{\sim}{B}_{\perp} - \underset{\sim}{\beta} \times \underset{\sim}{E}_{\perp})$$

$$= \gamma^2 \Big[\underset{\sim}{E}_{\perp}^2 + 2 \underset{\sim}{E}_{\perp} \cdot (\underset{\sim}{\beta} \times \underset{\sim}{B}_{\perp}) + (\underset{\sim}{\beta} \times \underset{\sim}{B}_{\perp})^2$$
$$- \underset{\sim}{B}_{\perp}^2 + 2 \underset{\sim}{B}_{\perp} \cdot (\underset{\sim}{\beta} \times \underset{\sim}{E}_{\perp}) - (\underset{\sim}{\beta} \times \underset{\sim}{E}_{\perp})^2 \Big]$$

Since $\underset{\sim}{E}_{\perp} \cdot \underset{\sim}{\beta} = 0$, $(\underset{\sim}{\beta} \times \underset{\sim}{E}_{\perp})^2 = \beta^2 E_{\perp}^2$; same goes for $(\underset{\sim}{\beta} \times B_{\perp})^2$.

$\underset{\sim}{E}_{\perp} \cdot (\underset{\sim}{\beta} \times B_{\perp}) = -\underset{\sim}{B}_{\perp} \cdot (\underset{\sim}{\beta} \times \underset{\sim}{E}_{\perp})$ ("box product" rule)

Thus [ ] = $\left[\underset{\sim}{E}_{\perp}^2 (1 - \beta^2) - \underset{\sim}{B}_{\perp}^2 (1 - \beta^2) \right]$ and since

$\gamma^2 (1 - \beta^2) = 1$, when we collect all that is left:

$$\underset{\sim}{E}'^2 - \underset{\sim}{B}'^2 = (\underset{\sim}{E}_{\parallel}^2 + \underset{\sim}{E}_{\perp}^2) - (\underset{\sim}{B}_{\parallel}^2 + \underset{\sim}{B}_{\perp}^2) = \underset{\sim}{E}^2 - \underset{\sim}{B}^2$$