7.1

$$\underset{\text{statvolt}}{\mathcal{E}_{max}} = \frac{1}{c}\left(\frac{d\Phi}{dt}\right)_{max} = \frac{\omega}{c} \times (\text{turns} \times \underset{\text{cm}^2}{\text{area}}) \times \underset{\text{gauss}}{B}$$

$$= \frac{2\pi \times 30}{3 \times 10^{10}} \times (4000 \times 144\pi) \times .5$$

$$= .0057 \text{ statvolt}$$

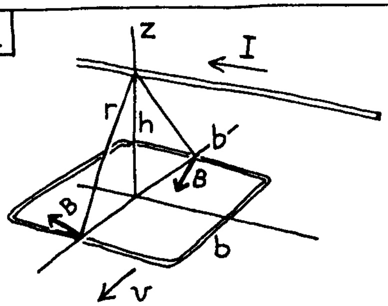
---

in S.I. : $$\underset{\text{volt}}{\mathcal{E}_{max}} = \left(\frac{d\Phi}{dt}\right)_{max} = \omega \times (\text{turns} \times \underset{\text{m}^2}{\text{area}}) \times \underset{\text{tesla}}{B}$$

$$= 2\pi \times 30 \times (4000 \times .0144\pi) \times .5 \times 10^{-4}$$

$$= 1.71 \text{ volt}$$

7.2



At the leading edge of the square loop

$$B_z = \frac{2I}{cr}\,\frac{(b/2)}{r} = \frac{Ib}{c\left(h^2 + \frac{b^2}{4}\right)}$$

At the trailing edge B_z has the opposite sign.

$$\mathcal{E} = \frac{1}{c}\,\frac{d\Phi}{dt} = \frac{2bvB_z}{c} = \frac{2I}{c^2}\,\frac{v\,b^2}{h^2 + \frac{b^2}{4}}$$

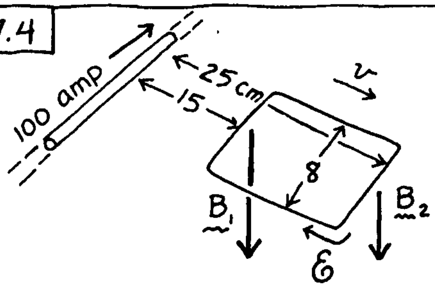
7.3 Within a circle of radius r the flux is $\pi r^2 B$.

$$\mathcal{E}_{max} = \frac{1}{c}\left(\frac{d\Phi}{dt}\right)_{max} = \frac{\omega \pi r^2 B}{c}$$

$$E = \frac{\mathcal{E}}{2\pi r} = \frac{\omega r B}{2c} = \frac{2\pi \times 2.5 \times 10^6 \times 3 \times 4}{2 \times 3 \times 10^{10}}$$

$$= 0.0031 \text{ statvolt}$$

7.4



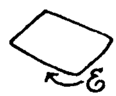
$$B = \frac{2}{10}\frac{I}{r} = \frac{20}{r} \text{ gauss}$$

$$B_1 = \frac{20}{15} = 1.33 \text{ gauss}$$

$$B_2 = \frac{20}{25} = 0.8 \text{ gauss}$$

$$v = 500 \text{ cm/sec}$$

$$\mathcal{E}(\text{volts}) = 10^{-8}\frac{d\Phi}{dt} = 10^{-8} \times 8 \times 500(1.33 - 0.8) = 2.13 \times 10^{-5} \text{ volts}$$

The flux is downward and is decreasing. $\mathcal{E}$ will be in the direction to drive a current which would make more flux downward, that is :  .

Assume the current in the loop, at any instant, is $\mathcal{E}/R$. This current I causes a field B' and a flux Φ' linking the loop. Because $\mathcal{E}$ is changing with time as the loop moves away from the wire, Φ' is changing too, resulting in an extra induced emf $\mathcal{E}'$, which we have ignored. The question is, how large must R be so that $\mathcal{E}'$ is indeed negligible compared to $\mathcal{E}$? As a very rough estimate, $B' \approx \frac{I'}{10}\cdot\frac{1}{5}$ (taking 5 cm as a typical dimension of the loop) and $\Phi' \approx B' \times$ loop area $\approx \frac{I'}{50} \times 8 \times 10 \approx 2I'$. (Strictly, we should expect a factor like $\ln\left(\frac{\text{loop diameter}}{\text{wire diameter}}\right)$ to come in when the flux of B' linking the loop is calculated – see the comments in the second paragraph on p. 282 – but unless the wire is extremely thin the logarithm won't be a very large number.) The time characteristic of these changes is of the order of magnitude of $\frac{\text{mean distance to wire}}{\text{velocity of loop}} \approx \frac{20}{500}$ sec or 0.04 sec. Thus

$$\mathcal{E}' = 10^{-8}\frac{d\Phi'}{dt} \approx 10^{-8}\frac{2I'}{.04} = 5 \times 10^{-7}\frac{\mathcal{E}}{R}$$

Thus if $R \gg 10^{-6}$ ohms, we will have $\mathcal{E}' \ll \mathcal{E}$. To state it more generally, the inductive time constant of the loop itself, L/R, should be short compared to the time of change of the externally induced emf.

7.5 $\mathcal{E} = \frac{vw}{c}(B_1 - B_2)$ $\quad I = \frac{\mathcal{E}}{R} = \frac{vw}{cR}(B_1 - B_2)$

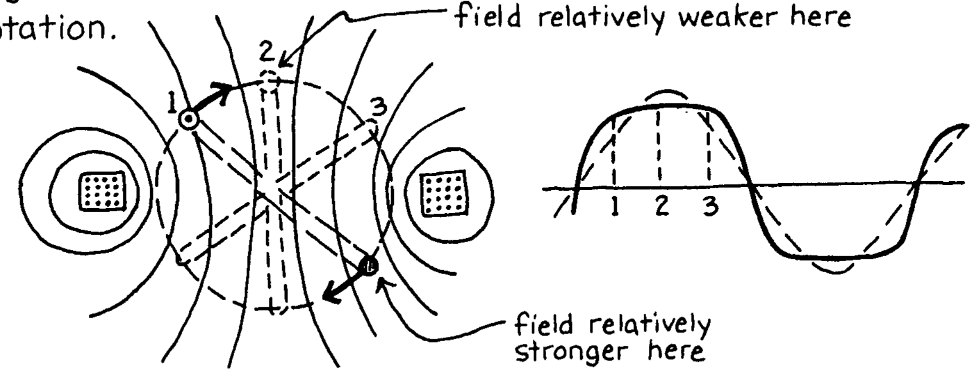
The force on the loop is : $F = \frac{IB_1 w}{c} - \frac{IB_2 w}{c} = \frac{Iw}{c}(B_1 - B_2)$

Rate at which work must be done to move the loop is

$$Fv = \frac{Iwv}{c}(B_1 - B_2) = I^2 R$$

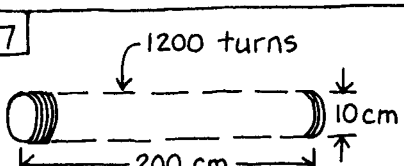
In Fig. 7.14 the energy which is dissipated in the stationary loop has to be supplied by whatever agency is moving the coil. A force is required to move the coil because of the magnetic field arising from the induced current in the loop.

7.6 If the field is uniform, $\mathcal{E}$ will be sinusoidal, regardless of the shape of the loop, at constant rate of rotation.



A loop rotating in this field will generate an emf more like a "square-wave".

7.7



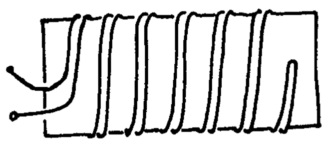
1 ampere in the coil causes a field inside the coil of magnitude $B = \frac{4\pi}{10} \times 1 \times \frac{1200}{200} = 7.55$ gauss

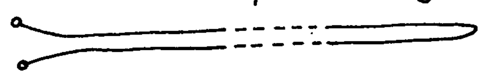
flux × turns $= \Phi N = \pi \times 5^2 \times 7.55 \times 1200 = 7.1 \times 10^5$

L (henrys) $= 10^{-8} \times 7.1 \times 10^5 = 7.1 \times 10^{-3}$ henrys

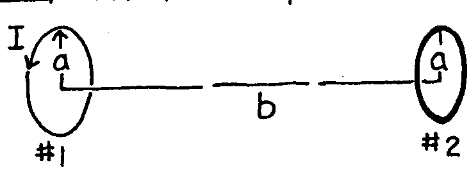
We have neglected the fact that the field inside the solenoid is not constant. It decreases near each end, so that the flux through the last turn is only about half that through a turn in the middle. This means that we have over- estimated the inductance. We might expect the error to be roughly $\frac{\text{diameter}}{\text{length}}$, or $\sim 5\%$ in this example. [In fact, the error is only 2% in this case, as one can discover by referring to tables which give exact values for the inductance of cylindrical coils.]

$\boxed{7.8}$ One way to wind resistance wire into a "non-inductive" coil is indicated below. Of course the inductance is not zero. The residual inductance is approximately that of the long, narrow "hair-pin" configuration:





$\boxed{7.9}$ With I amperes in ring #1, the field a distance b down the axis is:



$$B = \frac{2\pi a^2 I}{10(a^2+b^2)^{3/2}}$$

This is an application of Eq. 6.41, p. 227, with a replacing b, b replacing z, and $\frac{1}{10}$ replacing $\frac{1}{c}$ since I is in amperes. For $b \gg a$ this can be approximated:

$$B = \frac{2\pi a^2 I}{10 b^3}$$

and also, for $b \gg a$, we can neglect the variation of B over the interior of ring #2. Then:

$$\Phi_{12} = B \cdot \pi a^2 = \frac{2\pi^2 a^4 I}{10 b^3}$$

The mutual inductance in henrys is $10^{-8}\frac{\Phi}{I}$, or

$$M = \frac{2\pi^2 \times 10^{-9} a^4}{b^3} \text{ henrys}$$

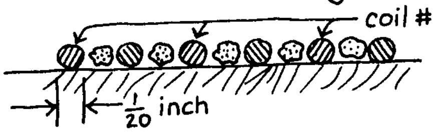
$\boxed{7.10}$ We have enough information to calculate the resistance of each coil, if we know the resistivity of copper. From the graph on page 140 we can read off an accurate enough value – say 2×10^{-6} ohm-cm, at room temperature. wire length $= 203 \times 12 \times 2.54 = 6200$ cm

$$\text{cross-section} = \frac{\pi}{4}\left(\frac{2.54}{20}\right)^2 = 0.0126 \text{ cm}^2$$

$$R = \frac{6200 \times 2 \times 10^{-6}}{1.26 \times 10^{-2}} = 1 \text{ ohm}$$

We don't know the coil size or the number of turns. Suppose the coil was cylindrical, with length b and radius a. Then if the length of a wire is ℓ and the number of turns is N, $\ell = 2\pi a N$.

We have another clue: the two coils were wound closely together separated only by twine. The winding must have looked something like this:



If we assume the twine was about as thick as the wire, the turns in one coil were spaced at intervals of $\frac{4}{20}$ inch or about 0.5 cm. Then $N = \frac{b}{0.5} = 2b$

It seems likely that Faraday's "block of wood" would have been roughly "squarish" in proportions. Let's assume $b = 2a$. Then from the three relations, we find

$$\left.\begin{aligned} \ell &= 2\pi a N \\ N &= 2b \\ b &= 2a \end{aligned}\right\}$$

at once: $\ell = 2\pi b^2$
with $\ell = 6200$ this gives $b = 30$ cm,
$2a = 30$ cm and $N = 60$ turns.

The coil is 1 ft long and 1 ft in diameter – very reasonable.

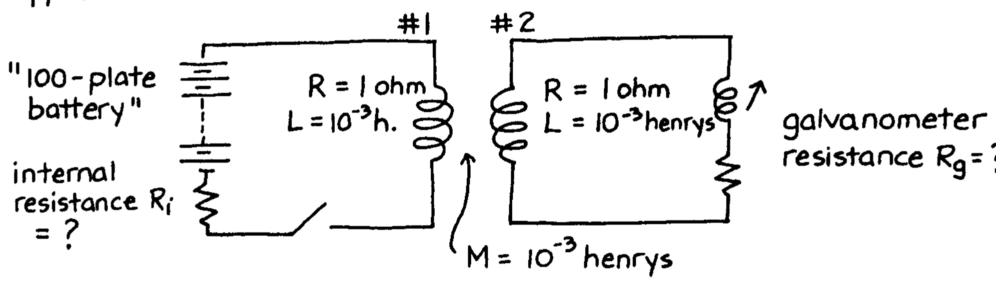
An approximate formula for the inductance L of a coil of N turns is easily derived. Taking

$$B \approx \frac{4\pi}{10} \, \frac{N I \text{(amp)}}{b}, \quad \text{as for a long solenoid,}$$

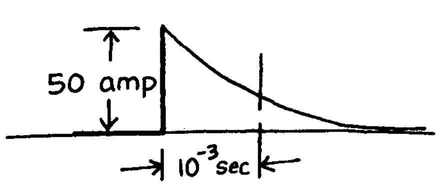
we have $\Phi = \pi a^2 B = \dfrac{4\pi^2 N I a^2}{10 b}$ and $L = 10^{-8}\dfrac{N\Phi}{I}$ henrys

$$L = \frac{4\pi^2 \times 10^{-9} N^2 a^2}{b} = \frac{4\pi^2 \times 10^{-9} \times 60^2 \times 15^2}{30} = 1.1 \times 10^{-3} \text{henrys}$$

Because end effects were neglected, this somewhat <u>over</u>-estimates the inductance, so we might as well round it off to $L = 10^{-3}$ henrys. This is also the mutual inductance of the two coils, for they link the same flux, in this approximation. The reconstructed circuit looks like this:



A 100-plate battery would have an emf of the order of 100 volts. Nothing would have been gained by using so large a battery if its internal resistance were much greater than 1 ohm, but it probably wasn't much less. So let's assume R_i = 1 ohm. Then with the switch closed, the steady current through coil #1 was 50 amperes. When the switch is opened the current must rise instantaneously to 50 amperes in coil #2, for the flux cannot decrease discontinously. Thereafter the current decays in circuit #2 with the time constant $\dfrac{L}{R + R_g}$. Assuming $R_g \ll R$ (maybe it wasn't!) we have $\dfrac{L}{R} = \dfrac{10^{-3}}{1} = 10^{-3}$ seconds. Then the current pulse looked something like this:



$\boxed{7.11}$ Referring to (a) in the Figure, if I_2 is increasing we have increasing upward flux through circuit #1. This will induce an $\mathcal{E}_1$ in a direction to drive current to make downward flux, which is opposite to the positive direction assigned to $\mathcal{E}_1$. Hence the equation must be written:

$$\mathcal{E}_1 = -L_1\frac{dI_1}{dt} - M\frac{dI_2}{dt}$$

A similar argument shows that the second equation must be written:

$$\mathcal{E}_2 = -L_2\frac{dI_2}{dt} - M\frac{dI_1}{dt}$$

Had we assigned the opposite positive directions for I_2 and $\mathcal{E}_2$, the signs before M in both equations above would be +.

For the circuit in (b):

$$\mathcal{E} = \mathcal{E}_1 + \mathcal{E}_2 \qquad I = I_1 = I_2$$

$$\mathcal{E} = -L_1\frac{dI}{dt} - M\frac{dI}{dt} - L_2\frac{dI}{dt} - M\frac{dI}{dt} = (L_1 + L_2 + 2M)\frac{dI}{dt}$$

This is equivalent to a single coil with $L' = L_1 + L_2 + 2M$

For the circuit in (c):

$$\mathcal{E} = \mathcal{E}_1 - \mathcal{E}_2 \qquad I = I_1 = -I_2$$

$$\mathcal{E} = -L_1\frac{dI_1}{dt} - M\frac{dI_2}{dt} + L_2\frac{dI_2}{dt} + M\frac{dI_1}{dt}$$

$$= -L_1\frac{dI}{dt} + M\frac{dI}{dt} - L_2\frac{dI}{dt} + M\frac{dI}{dt} = -(L_1 + L_2 - 2M)\frac{dI}{dt}$$

This is equivalent to a single coil with $L'' = L_1 + L_2 - 2M$. Evidently, $L' > L''$. A circuit with $L < 0$ would violate Lenz's Law – it would be unstable. This means that L'' must be ≥ 0, which implies $M \leq \frac{L_1 + L_2}{2}$ for any pair of circuits. [An even stronger inequality, $M^2 \leq L_1 L_2$, can be derived by considering the coils connected in parallel.]

7.12 $\frac{v}{c}B = \frac{100}{3\times 10^{10}} \times 0.35 = 1.17 \times 10^{-9}$ statvolts/cm

or 3.5×10^{-7} volts/cm.

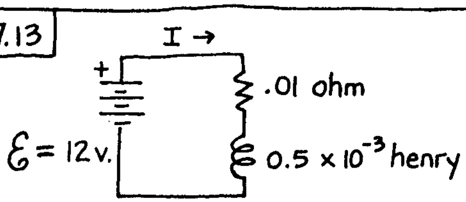
$J = E\sigma = 3.5 \times 10^{-7} \times 0.04 = 1.4 \times 10^{-8}$ amp/cm²

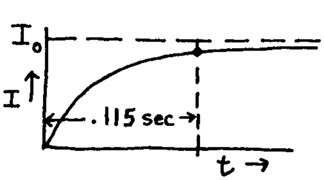
$= 1.4 \times 10^{-4}$ amp/m²

If a bottle of sea water were carried at this speed, a current would flow only long enough to separate enough charge to establish an electric field equal and opposite to $\frac{v}{c} \times \mathbf{B}$. To find how long this takes consider the charge that would pile up on one cm² of surface in τ seconds, with $J = 1.4 \times 10^{-8}$ amp/cm² $= 1.4 \times 10^{-8} \times 3 \times 10^{9}$ esu/sec $= 42$ esu/sec/cm². The resulting field will be $E = 4\pi \times 42\tau$, which will equal 10^{-9} statvolts/cm when

$\tau = \frac{10^{-9}}{4\pi \times 42} = 2 \times 10^{-12}$ sec.

7.13





$I_0 = \frac{\mathcal{E}}{R}$

$= 1200$ amp.

$\frac{R}{L} = 20$ sec^{-1}.

$I = I_0\left(1 - e^{-\frac{R}{L}t}\right) = 1200\left(1 - e^{-20t}\right)$

$I = 0.9 I_0$ for $e^{-20t} = 0.1$

$20t = \ln_e 10 = 2.30 \qquad t = 0.115$ sec

$I = 1080$ amp at $t = 0.115$

Magnetic field energy $= \frac{1}{2}LI^2 = \frac{1}{2}(0.5 \times 10^{-3})(1080)^2 = 292$ joules

Energy supplied by battery between $t = 0$ and $t = .115$

$$= \int_0^{.115} \mathcal{E} I\,dt = \mathcal{E} I_0 \int_0^{.115} \left(1 - e^{-\frac{R}{L}t}\right) dt = \frac{\mathcal{E} I_0 L}{R} \int_0^{2.3} \left(1 - e^{-x}\right) dx$$

$$= \frac{\mathcal{E} I_0 L}{R}\left[x + e^{-x}\right]_0^{2.3} = \frac{\mathcal{E} I_0 L}{R}\left[2.3 + 0.1 - 1.0\right] = 1.4\,\frac{\mathcal{E} I_0 L}{R}$$

$$= \frac{1.4 \times 12 \times 1200}{20} = 1008 \text{ joules}$$

7.14 Let v be the instantaneous velocity of the bar.

$$|\mathcal{E}| = \left|\frac{1}{c}\frac{d\Phi}{dt}\right| = \frac{1}{c} Bbv \qquad I = \frac{|\mathcal{E}|}{R} = \frac{Bbv}{Rc}$$

The force on the bar : $F = \frac{IbB}{c} = \frac{b^2 B^2 v}{Rc^2}$

$F = -m\frac{dv}{dt}$ (minus because the force opposes the motion)

$\frac{dv}{v} = -\frac{B^2 b^2}{Rmc^2} dt$ Integrating both sides :

$\ln v = -\frac{B^2 b^2}{Rmc^2} t + \text{constant}.$ If $v = v_0$ at $t = 0$,

we have : $v = v_0 e^{-t/T}$, where $T = \frac{Rmc^2}{B^2 b^2}$

The velocity decreases exponentially — in that sense, the rod never stops moving. But the distance it travels is finite : $x = \int_0^\infty v\,dt = \int_0^\infty v_0 e^{-t/T} dt = T v_0$

The initial kinetic energy of the rod, $\frac{1}{2} m v_0^2$, is transferred to the resistor as heat :

$$\frac{1}{2} m v_0^2 = \int_0^\infty R I^2 dt$$, as can be easily verified :

$$I = I_0 e^{-t/T} = \frac{Bbv_0}{Rc} e^{-t/T}$$

$$\int_0^\infty R I^2 dt = \frac{B^2 b^2 v_0^2}{Rc^2} \int_0^\infty e^{-2t/T} dt = \frac{B^2 b^2 v_0^2}{Rc^2} \cdot \left(\frac{T}{2}\right) = \frac{m v_0^2}{2}$$

7.15 Max. velocity of wire is $\omega \times .03$ cm

$= 2\pi \times 2000 \times .03 = 380$ cm/sec

$$\left(\frac{d\Phi}{dt}\right)_{max} = 380 \frac{\text{cm}}{\text{sec}} \times 1.8\text{cm} \times 5000 \text{gauss} = 3.4 \times 10^6 \frac{\text{gauss cm}^2}{\text{sec}}$$

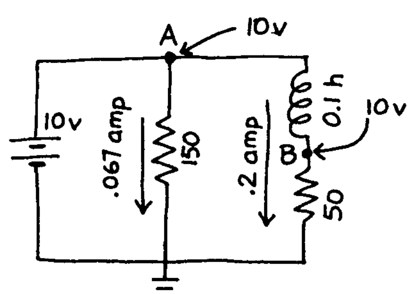
$$\mathcal{E}_{(\text{volts})} = 10^{-8} \left(\frac{d\Phi}{dt}\right)_{\frac{\text{gauss cm}^2}{\text{sec}}} = 0.034 \text{ volts}.$$

7.16 The current I in the moving frame is proportional to $\mathcal{E}/R$. The resistance R is proportional to resistivity/(rod diameter)^2. For given B, the electromotive force $\mathcal{E}$ is proportional to the frame's velocity v, and the force F which must be applied to maintain that velocity is proportional to I. Thus, other things being constant,

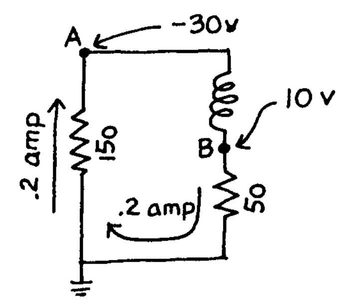
$$F \propto v/R \propto v \times \frac{(\text{rod diameter})^2}{\text{resistivity}}$$

A force of 2 N will pull the frame out of the field in 0.5 sec. A brass frame of the same dimensions would be pulled out in 1 sec by 0.5 N. The aluminum frame of 1 cm diameter rod would be pulled out in 1 sec by a force of 4 N.

7.17

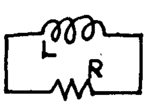


before switch opened

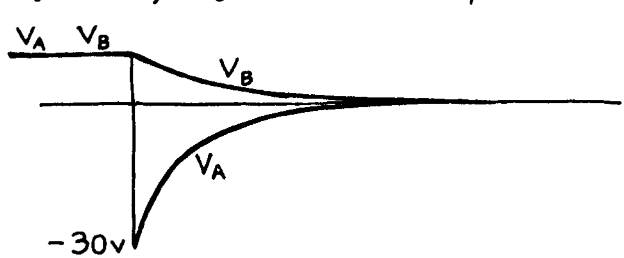


just after switch opened

Current through inductance cannot change abruptly!

Circuit on right is  with $L = 0.1$ h and $R = 200$ ohms

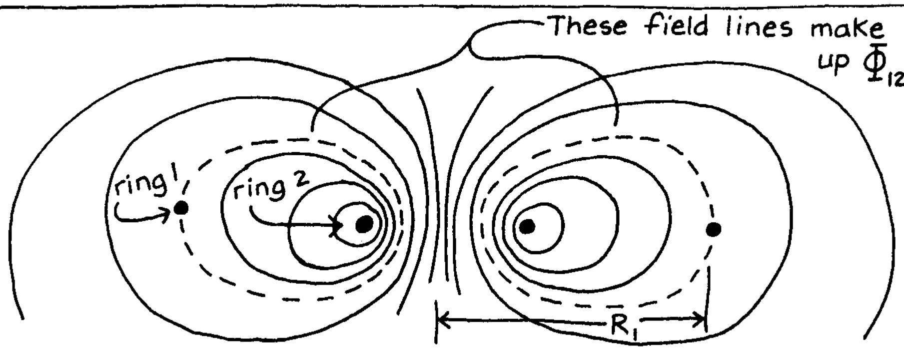
$I = I_0 e^{-\frac{R}{L}t}$; $I_0 = .2$ amp. $L/R = 0.5$ millisec.



7.18 $\Phi = N\pi a^2 B \quad \mathcal{E} = -\frac{1}{c}\frac{d\Phi}{dt} = -\frac{N\pi a^2}{c}\frac{dB}{dt}$

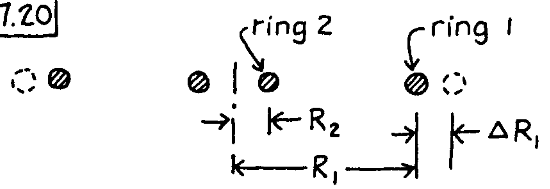
$$I = \mathcal{E}/R \qquad Q = \int I\,dt = -\frac{N\pi a^2}{cR}\int_{B_0}^{0} dB = \frac{N\pi a^2 B_0}{cR}$$

7.19



The point is that the flux which links ring 2 is determined by the lines of force, in the field of ring 1, which return outside ring 2. That is why Φ_{12} decreases as R_1 increases, holding R_2 and I_2 constant.

7.20



With current I in the outer ring the flux through the inner ring, for $R_1 \gg R_2$, is:

$$\Phi_{21} = \pi R_2^2 \cdot \frac{2\pi I}{cR_1} \quad \text{or} \quad \Phi_{21} = \frac{2\pi^2 I}{c}\,\frac{R_2^2}{R_1}.$$ Suppose we change R_1 to $R_1 + \Delta R_1$, by expanding the outer ring while holding I constant. The resulting change in Φ_{21} is:

$$\Delta\Phi_{21} = \frac{\partial \Phi_{21}}{\partial R_1}\Delta R_1 = -\frac{2\pi^2 I}{c}\,\frac{R_2^2}{R_1^2}\,\Delta R_1$$

Now consider a current I in the inner ring, ring 2. Let B be the field strength at the radius of the outer ring, R_1. If we now expand the outer ring by ΔR_1 the flux Φ_{12} decreases by just the amount of flux between the circle of radius R_1 and the circle of radius $R_1 + \Delta R_1$. (Problem 7.19 explained why it is a <u>dec</u>rease.) The change in flux is $\Delta \Phi_{21} = -B \cdot 2\pi R_1 \Delta R_1$ since $2\pi R_1 \Delta R_1$ is the area between the circles.

Our theorem $\Phi_{12} = \Phi_{21}$ guarantees that $\Delta\Phi_{12} = \Delta\Phi_{21}$

Hence: $$-\frac{2\pi^2 I}{c}\frac{R_2^2}{R_1^2}\Delta R_1 = -B \cdot 2\pi R_1 \Delta R_1$$

Solving for B: $B = \dfrac{\pi R_2^2 I}{c R_1^3}$ or more generally, $B = \dfrac{\pi R_2^2 I}{c r^3}$ at any point in the plane of the ring where $r \gg R_2$.

7.21 Assume current I flows in the outer solenoid. The field inside, approximately uniform in the region occupied by the inner solenoid, is $B = \frac{4\pi I}{c}\frac{N_2}{b_2}$. We have assumed here that $\frac{b_2}{a_2}$ is so large that we can use the formula for an infinite solenoid. We can refine this by using Eq. 6.44, page 228, to calculate the field at the center of a finite solenoid of length b_2, radius a_2. The correction factor is simply $\cos\left(\tan^{-1}\frac{2a_2}{b^2}\right)$ or $b_2/\sqrt{b_2^2 + 4a_2^2}$. This will still not
lead us to an exact result, for the inner coil includes a finite volume in which the field strength varies somewhat. But for the proportions shown in the Figure, the approximation will be pretty good. The flux linking the inner coil is

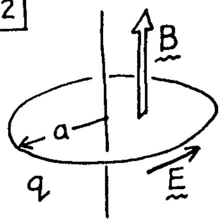
$$\Phi_{12} = \pi a_1^2 B N_1 \quad \text{or} \quad \frac{4\pi^2 I}{c}\frac{N_1 N_2 a_1^2}{b_2}\left(\frac{b_2}{\sqrt{b_2^2 + 4a_2^2}}\right)$$

Since $\mathcal{E}_{12} = -\frac{1}{c}\frac{d\Phi_{12}}{dt} = -M\frac{dI}{dt}$, we have, in CGS units, $$M = \frac{4\pi^2 N_1 N_2 a_1^2}{c^2\, b_2}\left(\frac{b_2}{\sqrt{b_2^2 + 4a_2^2}}\right)$$

To express M in henrys, replace the constant $\frac{1}{c^2}$ by 10^{-9}:

$$M = \frac{4\pi^2 \times 10^{-9} N_1 N_2 a_1^2}{\sqrt{b_2^2 + 4a_2^2}} \quad \text{henrys}$$

7.22



$\Phi = \pi a^2 B \quad \mathcal{E} = \frac{1}{c}\frac{d\Phi}{dt} = \frac{\pi a^2}{c}\frac{dB}{dt}$

(worry about signs later)

$\mathcal{E} = \oint \mathbf{E}\cdot d\mathbf{s} = 2\pi a E \qquad E = \frac{a}{2c}\frac{dB}{dt}$

Torque on ring $= qEa$

angular momentum acquired by ring $= \int qEa\,dt$

$= \frac{qa^2}{2c}\int_{B_0}^{0}\frac{dB}{dt}dt = \frac{qa^2B_0}{2c}$ If q is positive

angular momentum is in direction of $\mathbf{B}_0$.

$\omega =$ angular momentum $/ ma^2 = \frac{qB_0}{2mc}$. Note that $\mathbf{E}$ in diagram has correct sign for $\mathbf{B}$ decreasing. Check with Lenz's law.

7.23 Energy density $= \frac{B^2}{8\pi} = 3.6\times 10^{-13}$ erg cm^{-3}

volume $= 8\times 10^{66}$ cm^3 ; Total field energy $\approx 3\times 10^{54}$ erg

If total radiation of starlight in Galaxy is 10^{44} erg/sec, this equals 1000 years of starlight.

7.24 Energy density in J m^{-3} $= \frac{B^2}{2\mu_0}$ (B ← tesla, μ_0 ← $4\pi\times 10^{-7}$)

For $B = 0.4$ tesla, $U = \frac{(0.4)^2}{2\times 4\pi\times 10^{-7}} = 6.4\times 10^4$ J m^{-3}

To estimate $\int U\,dv$ we'll assume B is uniform through the interior of the solenoid and zero outside. The volume is $(\pi/4)\times .81$ m^2 $\times 2.2$ m or 1.40 m^3, giving 9×10^4 J for the total stored energy. [A more accurate estimate could be made, if it were needed, by consulting a table of the inductance of finite solenoids, calculating the current required to produce the given central field, and then computing $LI^2/2$. The result, when that is carried out in this case, is 8.8×10^4 J, fortuitously close to the approximate estimate.]

7.25

$$U = \frac{B^2}{8\pi} = \frac{10^{24}}{8\pi} = 4 \times 10^{22}\ \text{erg cm}^{-3} = 45\ \text{gram cm}^{-3}$$

7.26 Assume the speed of the tidal current is 1 meter/sec (about 2 knots).

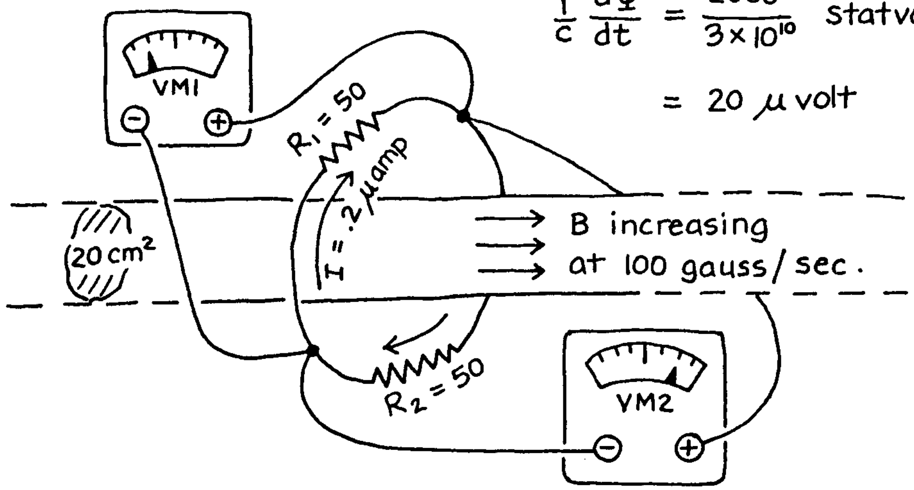
$vB = 1\ \text{m s}^{-1} \times 5 \times 10^{-5}\ \text{tesla} = 5 \times 10^{-5}\ \text{volt m}^{-1}$

$960\ \text{ft} = 300\ \text{m} \quad 300 \times 5 \times 10^{-5} = 15\ \text{millivolts}$

7.27

$$\frac{1}{c}\frac{d\Phi}{dt} = \frac{2000}{3\times 10^{10}}\ \text{statvolts}$$

$$= 20\ \mu\text{volt}$$



The loop that includes R_1 and VM1 encloses no changing flux. The potential drop across R_1 is $10\mu v$, with the more positive end connected to ⊖ terminal on VM1. Hence VM1 will read $-10\ \mu$volt. The loop that involves R_1 and VM2 encloses changing flux. The line integral of $\mathbf{E}$ around that loop includes $+20\mu v$ in addition to the -10 across R_1. VM2 will read $+10\ \mu$volt.

7.28 $R = \dfrac{\pi}{a\sigma} \qquad B = \dfrac{2\pi I}{c\,(a/2)} = \dfrac{4\pi I}{ca}$

$\dfrac{B^2}{8\pi} = \dfrac{2\pi I^2}{c^2 a^2} \qquad \text{volume} = 2\pi \times \dfrac{a}{2} \times a^2 = \pi a^3$

$$\left.\begin{aligned} &\text{stored energy} = \frac{2\pi^2 I^2 a}{c^2} \\ &\text{ohmic dissipation} = I^2 R = \frac{\pi I^2}{a\sigma} \end{aligned}\right\}\ \text{decay time} = \frac{2\pi a^2 \sigma}{c^2}$$

$\dfrac{a}{c} = \dfrac{3 \times 10^8\ \text{cm}}{3 \times 10^{10}\ \text{cm sec}^{-1}} = 10^{-2}\ \text{sec} \qquad \sigma = 10^{16}\ \text{sec}^{-1}$

time $= 2\pi \times 10^{12}$ sec $=$ 2000 centuries

7.29 Neglecting the inductance and resistance of the two rings and the leads, the charge in the capacitor C_2 at any time t is $Q = \mathcal{E}_0 (\cos 2\pi f t)\, C_2$. Since $I = \dfrac{dQ}{dt}$, $I = -2\pi f \mathcal{E}_0 C_2 \sin 2\pi f t$. The two rings are in series and this current flows in each. Assume $h \ll b$ so that we are justified in computing the force between the rings as if they were parallel straight wires, with force per unit length $= \dfrac{2I^2}{c^2 h}$. The length is $2\pi b$, so the force pulling the upper ring down (note currents are in same direction) is $F_m = \dfrac{4\pi b}{c^2 h} I^2$ or

$$F_m = \frac{4\pi b}{c^2 h} \left(2\pi f \mathcal{E}_0 C_2\right)^2 \sin^2 2\pi f t$$

The time-average of $\sin^2 2\pi f t$ is simply $\frac{1}{2}$
Hence the average force is

$$\bar{F}_m = \frac{8\pi^3 b f^2 \mathcal{E}_0^2 C_2^2}{h c^2}$$

In the capacitor at the left the electric field strength is: $E = \dfrac{\mathcal{E}_0 \cos 2\pi f t}{s}$. The downward force on the upper plate is $\dfrac{E^2}{8\pi} \times$ area,

$$\text{or} \quad F_e = \frac{E^2}{8\pi} \cdot \pi a^2 = \frac{\mathcal{E}_0^2 a^2}{8 s^2} \cos^2 2\pi f t$$

The time average of $\cos^2 2\pi ft$ is $\frac{1}{2}$, so

$$\overline{F}_e = \frac{a^2\,\mathcal{E}_0^2}{16\,s^2}$$

This can be expressed in terms of the capacitance C_1, which is $\frac{\pi a^2}{4\pi s}$ or $\frac{a^2}{4s}$. Substituting for s:

$$\overline{F}_e = \frac{\mathcal{E}_0^2\,C_1^2}{a^2}$$

When the forces are balanced (which might be brought about by varying C_2), we have $\overline{F}_c = \overline{F}_m$:

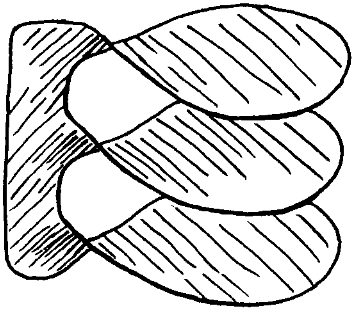
$$\frac{\cancel{\mathcal{E}_0^2}\,C_1^2}{a^2} = \frac{8\pi^3 b f^2 \cancel{\mathcal{E}_0^2}\,C_2^2}{h\,c^2}$$

Solving for c:

$$c = (2\pi)^{\frac{3}{2}}\; a\left(\frac{b}{h}\right)^{\frac{1}{2}} f\left(\frac{C_2}{C_1}\right)$$

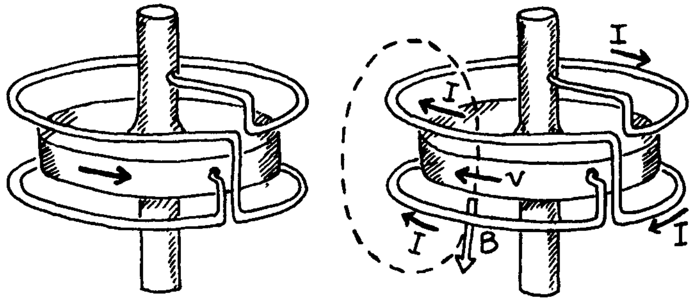
$\boxed{7.30}$ Surface (a) has two sides. Surface (b) has only one side; it is a "Möbius strip"

The two-sided surface is the one you must use to calculate the flux through the loop. The two-sided surface for a three-turn coil looks like this →



(or could be deformed to look like this). The extension to N turns is obvious.

7.31



The device on the right is the dynamo. Suppose a current I is flowing in the coil, in the direction indicated. Such a current would produce a magnetic field $\mathbf{B}$ going downward through the disk. With $\mathbf{v}$ the velocity of any part of the rotating disk, $\mathbf{v} \times \mathbf{B}$ is a vector pointing radially outward. Positive charges in the disk would be pushed outward, negative charges pushed inward. Either effect would cause current to flow in the direction postulated. Had we assumed the opposite direction for the coil current I, both $\mathbf{B}$ and $\mathbf{v} \times \mathbf{B}$ would have been reversed and the force would again be in the direction to sustain or increase the current. The conclusion is independent of the sign of the mobile charges. See if you can formulate an unambiguous rule to distinguish the potential dynamo on the right from the non-dynamo on the left, a rule which refers only to the relation of disk rotation to coil configuration. Would a mirror image of the figure on the left represent a dynamo?

A dynamo of this kind runs equally well with current in either direction. The current can also be zero. However, in any circuit not at absolute zero there are slight random motions of charge, or randomly fluctuating currents. Some fluctuation, tremendously amplified by the "positive feedback" of the dynamo action, becomes the steady dynamo current. It retains the direction of its initial excitation. (In a conventional d-c generator there is some residual magnetic field in the iron poles, even at zero current, which suffices to determine the eventual polarity.)

The magnitude of the current in this purely ohmic dynamo would be determined by the input mechanical power. It would be such that ohmic loss in coil and disk would precisely equal applied torque × shaft speed.

7.32 The resistance of the current path in the dynamo will scale as $(\sigma d)^{-1}$. Ignoring dimensionless factors, let us set $R = 1/\sigma d$. To maintain current I we need an electromotive force $\mathcal{E} = IR$. We can set $\mathcal{E} = Ed$, where $E = \frac{v}{c}B$. The field B produced by current I is of magnitude $B = I/cd$, and $v = \omega d$, if ω is the angular velocity of the rotor. Collecting these relations:

$$R = \frac{1}{\sigma d} \quad ; \quad \mathcal{E} = IR = \left(\frac{\omega d}{c}\right) B d \quad ; \quad R = \frac{I}{cd}$$

Upon eliminating I and B we are left with

$$\omega = \frac{c^2}{\sigma d^2} \quad , \text{ or } \quad \omega = \frac{Kc^2}{\sigma d^2}$$

where the dimensionless factor K makes up for all the dimensionless factors we had ignored. For copper at room temperature $\sigma \approx 4 \times 10^{17}\ \text{sec}^{-1}$ and if $d \approx 100$ cm, say, this doesn't look so bad. $c^2/\sigma d^2 \approx .2\ \text{sec}^{-1}$. But actually the factor K is generally much larger than 1. In the dynamo we met in Problem 7.31 the resistance R is larger than $1/d\sigma$ by something like d^2/A where A is the cross-sectional area of the wire of the "coil". And it will not be easy to make sliding contacts with resistance not much larger than $(\sigma d)^{-1}$.