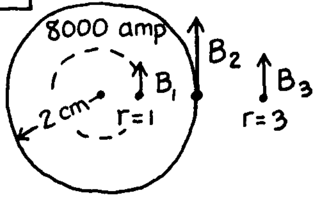
6.1 $B = \dfrac{2I}{cr} = \dfrac{2 \times 6 \times 10^{10}}{3 \times 10^{10} \times 5} = 0.8$ gauss

Force per cm $= IB/c = 6 \times 10^{10} \times 0.8/3 \times 10^{10} = 1.6$ dyne/cm

6.2



8000 amp $= 24 \times 10^{12}$ esu/sec

current inside $r=1$ is 6×10^{12} esu/sec

$$B_1 = \frac{2I}{rc} = \frac{2 \times 6 \times 10^{12}}{3 \times 10^{10} \times 1} = 400 \text{ gauss}$$

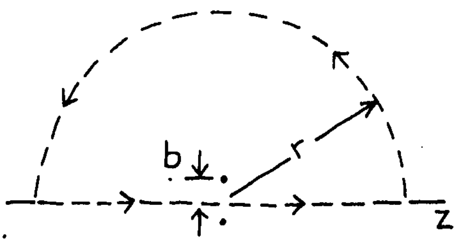
$$B_2 = \frac{2 \times 24 \times 10^{12}}{3 \times 10^{10} \times 2} = 800 \text{ gauss}$$

$$B_3 = \frac{2 \times 24 \times 10^{12}}{3 \times 10^{10} \times 3} = 533 \text{ gauss}$$

6.3 $B_z = \dfrac{2\pi b^2 I}{c(b^2+z^2)^{3/2}}$ $\qquad \displaystyle\int_{-\infty}^{\infty} B_z\,dz = \frac{2\pi b^2 I}{c}\int_{-\infty}^{\infty}\frac{dz}{(b^2+z^2)^{3/2}}$

$$= \frac{2\pi b^2 I}{c}\left[\frac{z}{b^2(b^2+z^2)^{1/2}}\right]_{-\infty}^{\infty} = \frac{2\pi b^2 I}{c}\cdot\frac{2}{b^2} = \frac{4\pi I}{c}$$

To see why the return path can be ignored in the limit $z \to \infty$, consider the finite path out to $z = r$, returning by way of the large semicircle.



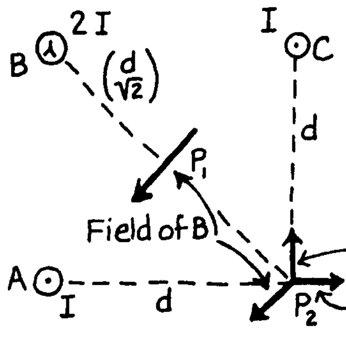
On the axis $B_z \sim \frac{1}{z^3}$, for $z \gg b$ and we may infer that, going out in any direction from the ring, $|B| \sim \frac{1}{r^3}$ as $r \to \infty$. Since the length of the semicircle is proportional to r, the integral $\int \mathbf{B}\cdot d\mathbf{s}$ over the semicircle must vanish at least as fast as $1/r^2$ as $r \to \infty$. [In fact, it vanishes just that fast.]

6.4

$$B = \underbrace{\frac{1}{2}\left(\frac{2I}{rc}\right) + \frac{1}{2}\left(\frac{2I}{rc}\right)}_{\text{Each straight section contributes half the field of an infinite wire}} + \underbrace{\frac{1}{2}\left(\frac{2\pi I}{rc}\right)}_{\text{Half the field of a complete ring}}$$

$$B = (2+\pi)\frac{I}{rc} = 5.1416\,\frac{I}{rc}$$

6.5



Field of A = $\frac{2I}{cd}$

Field of C

At P_1 the field of wires A and C cancel. Field of wire B at P_1 is $\frac{2\times 2I}{c\left(\frac{d}{\sqrt{2}}\right)} = \frac{4\sqrt{2}\,I}{cd}$

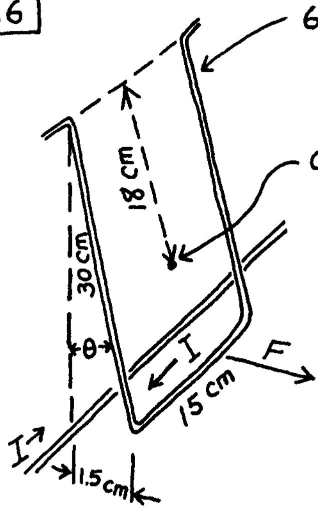
Field of B at P_2 = $\frac{2\sqrt{2}\,I}{cd}$

The vector sum of the 3 fields at P_2 is zero.

6.6

600 dynes weight (75 cm of wire at 8 dynes/cm)*

center of gravity



$$F \times 30\text{ cm} = 600 \times 18 \sin\theta$$
$$= 600 \times 18 \times \frac{1.5}{30}$$

$$F = 18\text{ dynes}$$

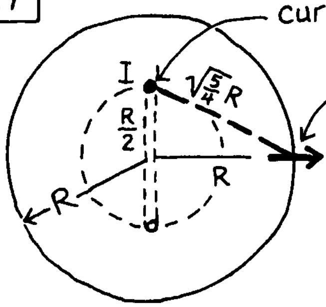
$$F = \frac{2I^2}{c^2 \times 1.5} \times 15 \qquad I = c\sqrt{\frac{18}{20}}$$

$$I = 2.85 \times 10^{10}\text{ esu/sec} = 9.5\text{ amperes}$$

The equilibrium is stable.

* For 1 mm diameter copper wire this is not realistic. It would weigh nearly 80 dynes per cm.

6.7



$R = 6 \times 10^8$ cm

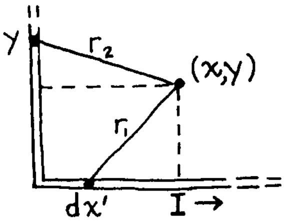
$$B = \frac{I \times 2\pi (R/2)}{c\left(\sqrt{\frac{5}{4}}R\right)^2} \cdot \frac{(R/2)}{\left(\sqrt{\frac{5}{4}}R\right)}$$

$$= \frac{1.1\, I}{c R} = 0.5 \text{ gauss}$$

$$I = \frac{0.5}{1.1} cR = \frac{0.5}{1.1}\, 3 \times 10^{10} \times 6 \times 10^{8} \text{ esu/sec}$$

$$= 9 \times 10^{18} \text{ esu/sec} = 3 \times 10^{9} \text{ amp}$$

6.8



$$B = \frac{I}{c}\int_0^\infty \frac{dx'}{r_1^2} \cdot \frac{y}{r_1} + \frac{I}{c}\int_0^\infty \frac{dy'}{r_2^2}\frac{x}{r_2}$$

$$= \frac{I}{c}\int_0^\infty \frac{y\, dx'}{[y^2 + (x - x')^2]^{3/2}} + \boxed{\text{same with } x \rightleftarrows y}$$

6.9 Evidently the magnetic field of the current in the wire was .2 gauss at a distance of roughly 2 cm. The current must have been about 2 amperes.

6.10 For 10^7 watts at 5×10^4 volts, $I = 200$ amp.

Field in teslas at 1 meter $= \dfrac{\mu_0 I}{2\pi r}$ ← amp, ← meters

$$= \frac{4\pi \times 10^{-7}}{2\pi} \times \frac{200}{1} = 4 \times 10^{-5}\,\text{T} = .4 \text{ gauss}$$

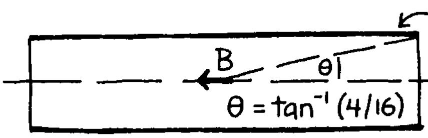
Other wire causes equal field : $B = 8 \times 10^{-5}$ T $= 0.8$ gauss

6.11 Average diameter of turn $= 8 + 2 \times 0.163 = 8.3$ cm

Total length of wire $= \pi \times 8.33 \times 8 \times 32 = 67$ meters

Resistance $= 0.67$ ohm $\quad I = 50/.67 = 75$ amps

Power $= 50 \times 75 = 3750$ watts

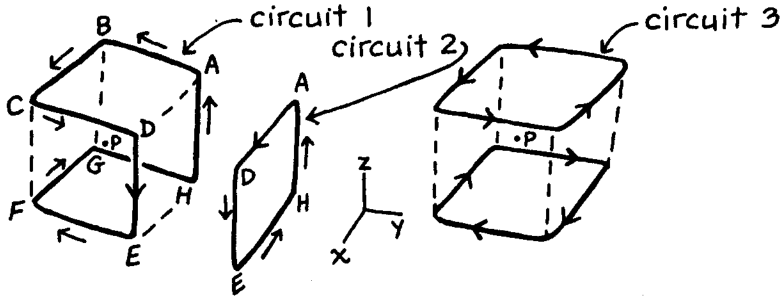


8×75 amp turns/cm

Field in infinitely long coil would be $.4\pi \times 600$ or 754 gauss.

$B = 754 \times \cos(\tan^{-1}\frac{1}{4}) = 731$ gauss

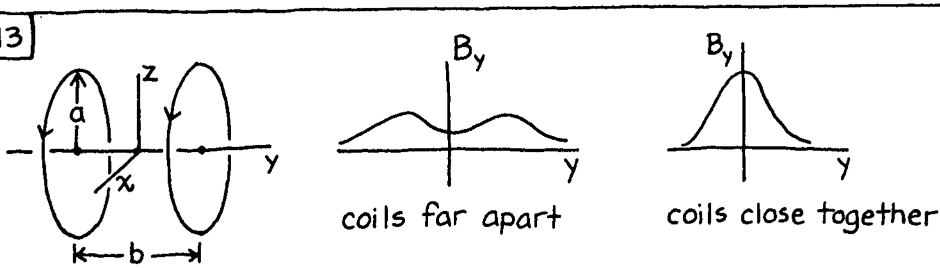
6.12



(a) In circuit 1 the contributions of AB and EF to the field at P cancel, as do those of CD and GH. The pair BC and FG make a field at P in the y direction, and so does the pair HA and ED.

(b) <u>Reverse</u> the current in circuit 2 and add it to circuit 1, thereby creating circuit 3 in which the field at P must vanish by symmetry. It follows that circuit 2 <u>not</u> reversed must produce the same field at P as circuit 1.

6.13

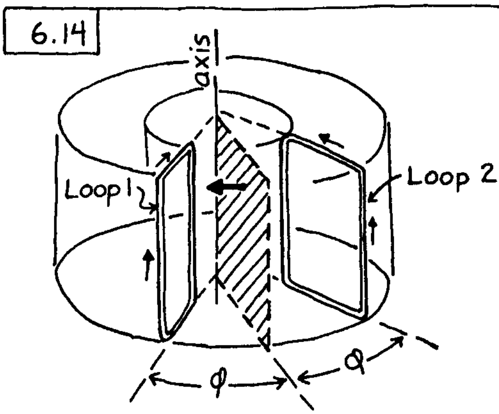


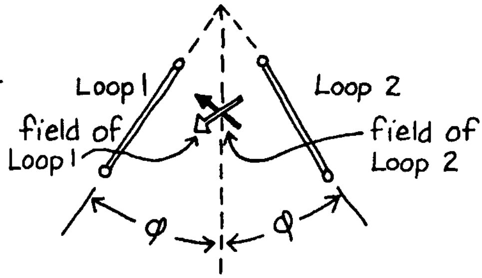
$$B_y \propto \frac{b/2-y}{[a^2+(b/2-y)^2]^{3/2}} + \frac{b/2+y}{[a^2+(b/2+y)^2]^{3/2}}$$ Differentiate twice and set $\frac{d^2 B_y}{dy^2} = 0$ at $y=0$. This gives: $b=a$

Note that $\frac{d^3 B_y}{dy^3} = 0$ at $y=0$ just from symmetry.

With $b=a$ we have $B_y = B_y(0) + \text{constant} \times y^4 + \cdots$
Two coils thus arranged are called Helmholtz coils.

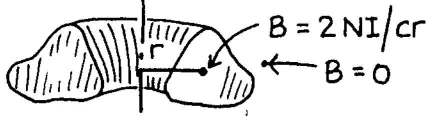
6.14



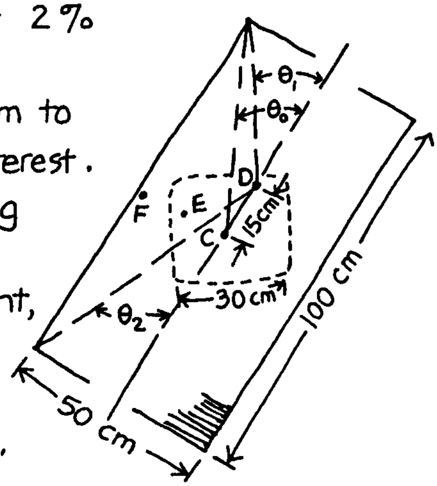


Consider two current loops symmetrically located with respect to a plane through the axis of the toroid. At any point on this plane the vector sum of the field of Loop 1 and the field of Loop 2 is a vector normal to the plane. (This is true for two similar loops of any shape carrying equal currents in the same direction.) Now the entire coil can be decomposed into pairs of loops symmetrical about a chosen plane. Hence the total magnetic field at <u>any</u> point must be perpendicular to the plane through that point and the axis.

If the magnitude of the field is B at any point P a distance r from the axis, it must have the same value everywhere on the circle of radius r in the plane through P perpendicular to the axis. The line integral of B around this path $= 4\pi/c$ times the current enclosed. If P is outside the toroid the current enclosed is zero. Therefore $\mathbf{B} = 0$ everywhere outside the toroid. If P is inside the toroid the current enclosed is NI. Therefore at any point inside the toroid $B = 2NI/cr$. This holds for a toroid of any cross-section:



$\boxed{6.15}$ Since 10 milligauss is about 2% of the earth's field, we need a compensating field that is uniform to about 2% over the region of interest. Let's try a solenoid 1 meter long and 50 cm in diameter. To see whether it meets the requirement, compare the field at the center C with the field on the axis 15 cm from the center, at D. From Eq. 44:



$$\frac{\text{field at C}}{\text{field at D}} = \frac{2\cos\theta_0}{\cos\theta_1 + \cos\theta_2}$$

$\theta_0 = \tan^{-1}\frac{25}{50}$ $\qquad \cos\theta_0 = 0.8944$

$\theta_1 = \tan^{-1}\frac{25}{35}$ $\qquad \cos\theta_1 = 0.8138$

$\theta_2 = \tan^{-1}\frac{25}{65}$ $\qquad \cos\theta_2 = 0.9334$

$$\frac{2\cos\theta_0}{\cos\theta_1 + \cos\theta_2} = \frac{1.7888}{1.7472} = 1.024 .$$

This is a little too large for comfort, especially as we have no easy way to estimate the deviation at off-axis points, such as E. Let's lengthen the solenoid to 120 cm. Then $\theta_0 = \tan^{-1}\frac{25}{60}$, $\theta_1 = \tan^{-1}\frac{25}{45}$ and $\theta_2 = \tan^{-1}\frac{25}{75}$.

This gives: $\dfrac{2\cos\theta_0}{\cos\theta_1 + \cos\theta_2} = \dfrac{1.8462}{1.8229} = 1.012$

We may expect the departure from uniformity in the radial direction to be of the same magnitude, roughly, as the variation in the axial direction — although it has the opposite sign, of course, the field strength at E, being <u>greater</u> than that at C. [Note: an exact calculation of the field strength at F, which involves an elliptic integral, shows that it is 1.4% greater than the field strength at C, in the 120 cm long solenoid.]

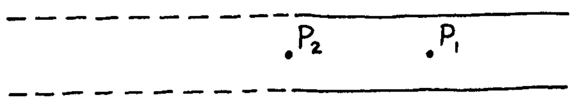
The ampere-turns, NI, required to make the field of the solenoid at C equal to the earth's field, 0.55 gauss, are computed as follows : $\left(\frac{4\pi}{10}\right)\frac{NI}{120}\cos\theta_0 = 0.55$

$$NI = \frac{0.55 \times 120 \times 10}{4\pi \times 0.923} = 57.0 \text{ ampere turns}$$

6.16 If the hole were filled with a copper rod carrying a current of 300 amperes, complete symmetry would be restored and the field at P would surely be zero. So the actual field at P must be the negative of the field of the rod just described. Its magnitude in gauss is $(2/10)I/r$ with $I = 300$ amperes and $r = 2$ cm, or 30 gauss, and it points to the left. A more remarkable fact, not too hard to prove : The field is 30 gauss pointing to the left not only at P but <u>everywhere within the cylindrical hole</u> !

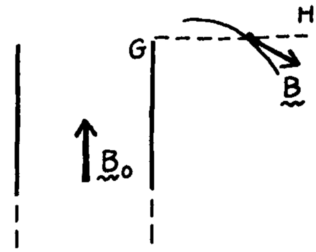
6.17

(a)

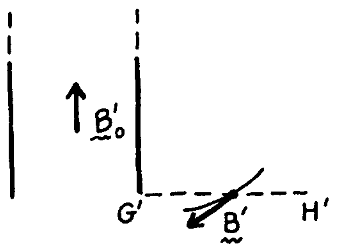


Imagine adding a similar solenoid on the left, as shown. This exactly doubles the field strength B at P_2. But now the field strengths at P_2 and P_1 are approximately equal, for both points lie well inside a fairly long solenoid, the field at P_2 being slightly stronger. Therefore the original field at P_2 must have been slightly more than half the field at P_1.

(b) Suppose the field does have a vertical component somewhere along the line GH, that is, in the plane of the end of the coil, as shown at the right.



Now imagine a similar solenoid, extending upward, as shown at the left.

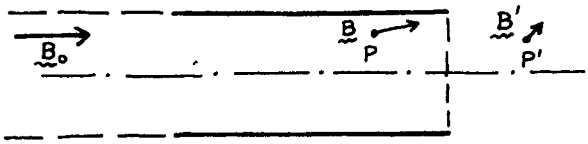


The field $\mathbf{B}'$ at the corresponding point must have a downward component, if $\mathbf{B}$ does and if $\mathbf{B}'_0$ and $\mathbf{B}_0$ are in the same direction. Now join the two semi-infinite solenoids end to end, and superpose their fields. The resulting infinitely long solenoid must have zero external field. But the addition of $\mathbf{B}$ and $\mathbf{B}'$ cannot give zero. Hence $\mathbf{B}$ cannot have a vertical component. Therefore $\mathbf{B}$ is horizontal at all points on the line GH, and GH is a field line.

(c) The argument used in (a), applied to the semi-infinite solenoid, shows that the axial component of the field, at any point on the end face is $\frac{1}{2}B_0$. For adding another semi-infinite solenoid just doubles the axial field component and produces the purely axial uniform field B_0. In calculating the flux through the end face, only the axial field component is involved. Therefore the flux must be just half the interior flux.

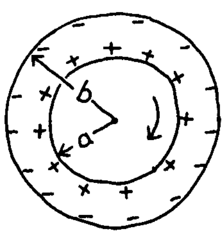
(d) For the same reason, the flux tube on the surface of which the field line CDE lies must flare out as it approaches the end face so that its cross-section there becomes a circle of twice the area, containing the same amount of flux. $\pi r^2 = 2\pi r_0^2$, or $r = \sqrt{2}\, r_0$.

The arguments used in (b) and (c) lead to a more general statement about the field of the semi-infinite solenoid :



At two corresponding points P and P', symmetrically located with respect to the end plane and equidistant from the axis, the fields $\underset{\sim}{B}$ and $\underset{\sim}{B}'$ are related as follows : The radial components of $\underset{\sim}{B}$ and $\underset{\sim}{B}'$ are equal. The sum of the axial components of $\underset{\sim}{B}$ and $\underset{\sim}{B}'$ is equal to B_0, if P lies inside the coil, or to zero, if P lies outside the coil. The conclusions of (b) and (c) follow in the special case in which P and P' coincide.

6.18



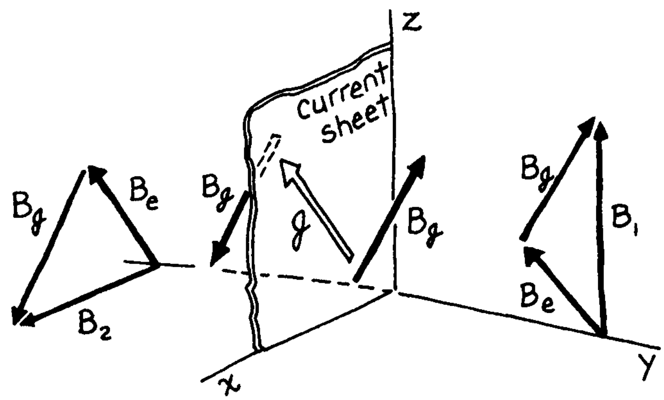
capacitance of coaxial cylinders, length L: $C = \dfrac{L}{2\ln\frac{b}{a}}$. Charge per unit length on inner cylinder (assumed positive) $= \phi C/L$ $= \phi/2\ln\frac{b}{a} = 50/2\ln(4/3) = 87$ esu/cm

If inner cylinder rotates at 30 rev/sec it is a solenoidal surface current of density $\mathcal{J} = 30 \times 87$ $= 2610$ esu cm^{-1} sec^{-1}. Inside that cylinder $B = 4\pi\mathcal{J}/c$ $= 1.09 \times 10^{-6}$ gauss (into paper), $r < a$; $B = 0$, $r > a$.

If both cylinders rotate at 30 rev/sec, $B = 1.09 \times 10^{-6}$ gauss (out of paper) $a < r < b$ and $B = 0$, $r < a$, $r > b$.

$\boxed{6.19}$ The analysis in Sec. 5.9 showed that a test charge moving parallel to a wire carrying current experiences a force which, as observed in the rest frame of the test charge, is due to an electric field. To understand why the introduction of a conductor, such as a metal plate, between the wire and the test charge has no effect, let us view the situation from the rest frame of the test charge. In that frame, the conducting plate, which is stationary in the lab frame, is moving. It is moving through a magnetic field and an electric field and these are related precisely so as to make the total force on any charge in the plate zero. Hence there is no redistribution whatever of the electrons in the plate. On the other hand, if we caused the plate to move along with the same velocity as the test charge, it would make a difference. An observer in the test-charge frame would say that we have introduced a stationary plate into an electrostatic field, with a consequent redistribution of charge on the plate and a resulting alteration of the total electric field. An observer in the lab frame, where there was no electric field before, would say that the electrons in the plate have redistributed themselves under the influence of the $q\frac{\mathbf{v}}{c}\times\mathbf{B}$ force, and the new charge distribution itself produces an electric field.

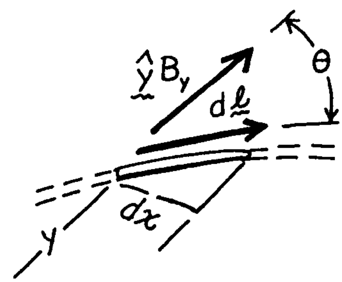
6.20



The statement on p. 234 that $(B_1^2 - B_2^2)/8\pi$ is the force per cm² would imply that the force is zero in this case where $B_1^2 = B_2^2$ although $\mathbf{B}_1 \neq \mathbf{B}_2$. Indeed that is true, for what we have here is a superposition of the field of some external source $\mathbf{B}_e$, the same on both sides of the sheet, and $\mathbf{B}_{\mathcal{J}}$, the field of a current $\boldsymbol{\mathcal{J}}$ flowing in the sheet. If $\boldsymbol{\mathcal{J}}$ is parallel to $\mathbf{B}_e$ there can be no force on the sheet.

6.21 If the conduction electrons are forced closer to the axis there will be uncompensated negative charge near the axis. This will cause a radial electric field E_r pushing outward on the electrons, preventing further constriction when $E_r = (v/c)B$. The field B within the conducting rod is $2\pi r J/c$, where the conduction current density J is nev, n being the number density and v the mean drift velocity of the conduction electrons. Suppose the electron cloud at radius r is squeezed inward by a small distance Δr. The cylinder of radius r will now contain, per unit length, an excess of negative charge in amount $(ne)(2\pi r \Delta r)$ causing an electric field $E_r = 4\pi ne\Delta r$. In equilibrium, then, $4\pi ne\Delta r = (v/c)B = 2\pi rne\,(v/c)^2$, or $\Delta r/r = \frac{1}{2}(v/c)^2$. In solid conductors we always find $v/c \ll 1$. In metallic conduction v/c is seldom much greater than 10^{-10}, and $\Delta r/r \approx 10^{-20}$ is too small to detect. In highly ionized gases however, the "Pinch effect", as it is called, can be not only detectable but important.

6.22 $d\mathbf{f} = \dfrac{I\, d\boldsymbol{\ell} \times \mathbf{B}}{c}$



The z-component of $\mathbf{B}$ produces a force in the plane of the coil, which contributes nothing to the torque and can therefore be ignored in this calculation.

$d\boldsymbol{\ell} \times \hat{y} B_y = \hat{z}\, d\ell\, B_y \sin\theta$ But $d\ell \sin\theta = dx$

Hence $dF_z = \frac{I}{c} B_y\, dx$ The torque about the x-axis is $y \cdot dF_z = \frac{I B_y}{c}\, y\, dx = dN$. We have to integrate this around the loop to find the total torque.

$\displaystyle\int_{\text{loop}} y\, dx$ = area of loop $\equiv a$ (See fig. 11.4, p. 406)

Hence $N = \dfrac{B_y I a}{c}$. The torque vector $\mathbf{N}$ is in the $\hat{x}$ direction, and if we define $\mathbf{m} \equiv \frac{I a}{c}$ as in the Figure, our result can be written in the more general form: $\mathbf{N} = \mathbf{m} \times \mathbf{B}$.

If $\mathbf{B}$ is the same at all points on the loop, the net force on the loop is zero, for

$\int d\mathbf{F} = \int \frac{I}{c}\, d\boldsymbol{\ell} \times \mathbf{B} = -\frac{I}{c}\mathbf{B} \times \int d\boldsymbol{\ell}$ and $\int d\boldsymbol{\ell}$ over the whole loop is zero.

6.23 $\gamma = 2 \quad \beta = 0.866$. Travelling with the ion we see an electric field $E' = \gamma\beta \times B$. If this is to be $\leqslant 1.5 \times 10^4$ statvolts/cm we must have

$$B \leqslant \frac{1.5 \times 10^4}{\beta\gamma} = 8660 \text{ gauss}$$

6.24 $\text{current} = e \cdot \dfrac{v}{2\pi r} \qquad B = \dfrac{2\pi I}{cr} = \dfrac{\beta e}{r^2}$

$$= \frac{.01 \times 4.8 \times 10^{-10}}{10^{-16}} = 4.8 \times 10^4 \text{ gauss}$$

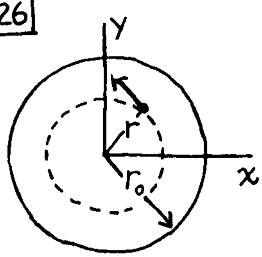
6.25 $B_x = 0 \quad B_y = 0 \quad B_z = B_0$ with $\underset{\sim}{B} = \nabla \times \underset{\sim}{A}$

$$\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z} = 0 \qquad \frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x} = 0 \qquad \frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} = B_0$$

One obvious choice is: $A_x = \dfrac{-B_0 y}{2}$, $A_y = \dfrac{B_0 x}{2}$, $A_z = 0$.

Equally obvious is: $A_y = B_0 x \quad A_x = A_z = 0$. To make others, add any vector function with zero curl.

6.26



Current inside $r = Ir^2/r_0^2$

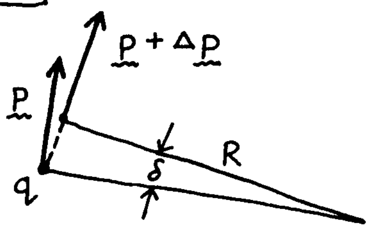
$$B = \frac{2Ir^2}{cr_0^2 r} = \frac{2Ir}{cr_0^2}$$

$$B_x = \frac{-y}{r} B = \frac{-2yI}{cr_0^2} \qquad B_y = \frac{2xI}{cr_0^2}$$

If $\underset{\sim}{A} = A_0 \hat{\underset{\sim}{z}} (x^2+y^2)$, $\nabla \times \underset{\sim}{A} = \hat{\underset{\sim}{x}} \dfrac{\partial A_z}{\partial y} - \hat{\underset{\sim}{y}} \dfrac{\partial A_z}{\partial x}$

$$= 2y A_0 \hat{\underset{\sim}{x}} - 2x A_0 \hat{\underset{\sim}{y}} \qquad A_0 = -\frac{I}{cr_0^2}$$

$\boxed{6.27}$



$$p = \beta\gamma mc \qquad \Delta p = q\beta B\Delta t$$

$$\delta = \frac{\beta c\Delta t}{R} = \frac{\Delta p}{p} = \frac{qB\Delta t}{\gamma mc}$$

$$\boxed{R = \frac{\beta\gamma mc^2}{qB} = \frac{pc}{qB}}$$

for one revolution: $T = \dfrac{2\pi R}{\beta c} = \dfrac{2\pi\gamma mc^2}{qB}$

$\boxed{6.28}$ $\gamma = 10^7 \qquad \beta = 1 \qquad m = 1.6 \times 10^{-24}$

$q = e \qquad B = 3\times 10^{-6}$ gauss $\quad mc^2 = 1.44 \times 10^{-3}$ erg

$$R = \frac{10^7 \times 1.44 \times 10^{-3}}{4.8\times 10^{-10} \times 3\times 10^{-6}} = 1.0 \times 10^{19}\ \text{cm}$$

$$T = \frac{2\pi \times 10^{-19}}{3\times 10^{10}} = 2.1 \times 10^9\ \text{sec} \approx 70\ \text{years}$$

$\boxed{6.29}$ $\gamma = 3.00 \quad \beta = \sqrt{8/9} = 0.943$

10^{-3} amp $= 3\times 10^6$ esu/sec $= I \qquad \lambda = I/v$

$$\lambda = \frac{3\times 10^6}{3\times 10^{10}\times\beta} = 1.06 \times 10^{-4}\ \text{esu/cm}$$

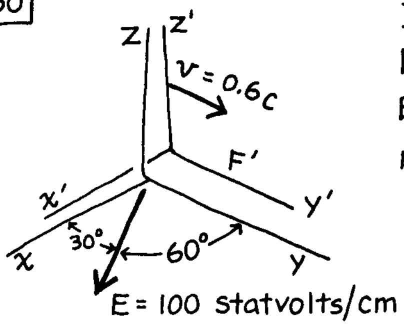
(a) $E = \dfrac{2\lambda}{r} = \dfrac{2.12\times 10^{-4}}{1} = 2.12 \times 10^{-4}$ statvolt/cm

(b) $B = \dfrac{2\lambda\beta}{r} = 2.00 \times 10^{-4}$ gauss

In F' there is no current ; $B = 0$

$\lambda' = \lambda/\gamma$, so $E' = \dfrac{E}{\gamma} = 0.71 \times 10^{-4}$ statvolts/cm

6.30



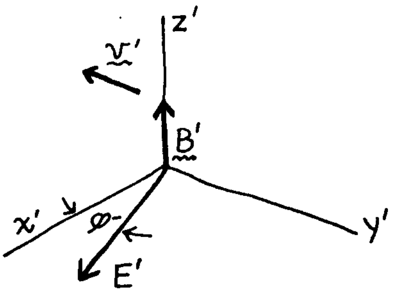
In frame F, $E_x = 100 \cos 30^\circ = 86.6$; $E_y = 100 \sin 30^\circ = 50$; $E_z = 0$; $\underset{\sim}{B} = 0$

Eq. 58 is the transformation for motion of F' in the x-direction.

For F' moving in the y-direction, with $\underset{\sim}{B} = 0$ in F, we have:

$E'_y = E_y \quad E'_x = \gamma E_x \quad E'_z = \gamma E_z = 0$

Here $\gamma = \dfrac{1}{\sqrt{1 - 0.6^2}} = 1.25 \qquad E'_y = 50 \qquad E'_x = 86.6 \times 1.25 = 108.3$



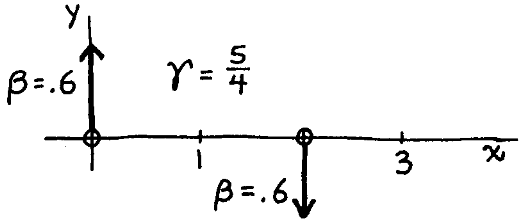
$\varphi' = \tan^{-1} \dfrac{50}{108.3} = 24.8^\circ$

$E' = \sqrt{108.3^2 + 50^2} = 119.3 \ \dfrac{\text{statvolts}}{\text{cm}}$

To find $\underset{\sim}{B}'$ we can use Eq. 62 with $\underset{\sim}{v}' = -.6c\,\hat{\underset{\sim}{y}}'$

$\underset{\sim}{B}' = \dfrac{\underset{\sim}{v}'}{c} \times \underset{\sim}{E}'$ $\quad \underset{\sim}{B}'$ is in the direction of $\hat{\underset{\sim}{z}}'$, with magnitude $0.6\, E'_x = 0.6 \times 108.3 = 65$ gauss

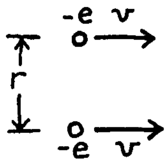
6.31



(a) at (3,0,0) $E = \gamma \dfrac{e}{1^2} + \gamma \dfrac{e}{9^2} = \dfrac{25e}{18}$; $\underset{\sim}{E} = \dfrac{25}{18} e\, \hat{\underset{\sim}{x}}$

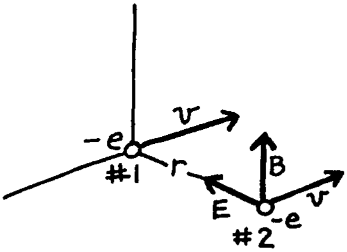
(b) at (3,0,0) $B = \beta\gamma \dfrac{e}{1^2} - \dfrac{\beta\gamma e}{3^2} = \dfrac{2}{3} e$; $\underset{\sim}{B} = \dfrac{2}{3} e\, \hat{\underset{\sim}{z}}$

6.32



(a) In a frame of reference moving with the electrons we have two charges at rest, r cm apart. The force is repulsive and its magnitude is simply $\frac{e^2}{r^2}$. Transforming the force to the lab frame, using $\frac{dp_\perp}{dt} = \frac{1}{\gamma}\frac{dp'_\perp}{dt'}$, as given on p. 191 we have

$$F = \frac{1}{\gamma}\frac{e^2}{r^2}.$$



(b) In the lab frame, consider the fields at the location of electron #2 arising from electron #1

$E = \frac{\gamma e}{r^2}$ (from Eq. 5.12, p. 184, with $\theta' = \frac{\pi}{2}$)

$B = \beta E = \beta\gamma\frac{e}{r^2}$ (applying Eq. 61, p. 240)

The resulting force on electron #2 is made up of:

1) an outward force $eE = \gamma\frac{e^2}{r^2}$

2) an inward force $\frac{ev}{c}B = \frac{e^2\beta^2\gamma}{r^2}$

The net force is $F = \gamma\frac{e^2}{r^2} - \gamma\beta^2\frac{e^2}{r^2} = \frac{1}{\gamma}\frac{e^2}{r^2}$

which agrees with what we found by method (a).

(c) As $v \to c$, $\gamma \to \infty$ and the force, as observed in the lab frame, approaches zero.

6.33 If we move in the $\hat{y}$ direction at the constant speed of 10 cm/microsecond, we see the ion simply revolving on a circular path that remains stationary. In this frame of reference there is no electric field. If $\mathbf{E}$ was the field in the original frame:

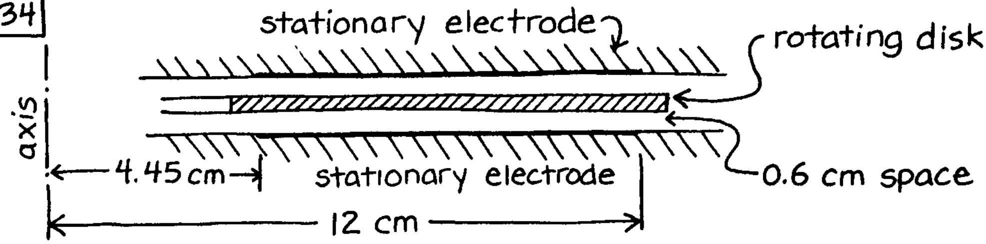
$\mathbf{E}' = 0 = \mathbf{E} + \frac{\mathbf{v}}{c}\times\mathbf{B}$ (We can neglect β^2

in the transformation, since v/c is only 1/3000.)

$\underset{\sim}{v} = 10^7 \hat{\underset{\sim}{y}}$; $\underset{\sim}{B} = 6000\, \hat{\underset{\sim}{z}}$ Hence $\underset{\sim}{E}$ must have

been $\quad -\dfrac{6\times 10^{10}}{3\times 10^{10}}\, \hat{\underset{\sim}{y}} \times \hat{\underset{\sim}{z}} = -2\hat{\underset{\sim}{x}}$ statvolt/cm

6.34

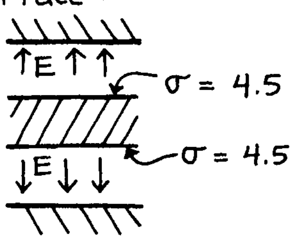


With the disk at 10 kv, the electric field strength in the space above and in the space below the disk is:

$E = \dfrac{10^4}{300 \times 0.6} = 55$ statvolts/cm.

Density of surface charge on each surface:

$\sigma = \dfrac{E}{4\pi} = 4.5$ esu/cm^2

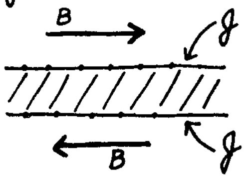


At the mean radius $\bar{r} = \dfrac{12 + 4.4}{2}$ $= 8.2$ cm, the velocity is

$2\pi \times 8.2 \times 61 = 3150$ cm/sec.

The equivalent surface current density is

$\mathcal{J} = \sigma v = 4.5 \times 3150 = 1.4 \times 10^4$ esu/sec/cm.



The two current sheets each of surface density $\mathcal{J}$, produce a field $B = \frac{4\pi}{c}\mathcal{J}$ both above and below.

$B = \dfrac{4\pi}{c}\mathcal{J} = \dfrac{4\pi \times 1.4 \times 10^4}{3 \times 10^{10}} = 5.9 \times 10^{-6}$ gauss

We have used the velocity at the mean radius to estimate the field strength B immediately above the disk in that region. For a more accurate calculation of the field strength to be expected at the location of the magnetometer needle, one could divide the disk into equivalent circular current rings and integrate over the whole distribution. That is what Rowland did.

6.35

$$\text{Resistance of ribbon} = \frac{1.6 \text{ ohm-cm} \times 0.5 \text{ cm}}{.001 \text{ cm}^2} = 800 \text{ ohms}$$

$V = 1$ volt $\quad I = 1.25$ milliamp.

$J = 1.25$ amp/cm^2 $= 1.25 \times 10$ amp/m^2

Let's use S.I.: $\quad v = J/ne$; $\quad n = 2 \times 10^{21}$ per m^3

$e = 1.6 \times 10^{-19}$ coulomb $\qquad v = \dfrac{1.25 \times 10^4}{2 \times 10^{21} \times 1.6 \times 10^{-19}} = 39$ m/sec

$B = 0.1$ tesla $\qquad E_t = vB = 3.9$ volt/m

Across the 0.2 cm width of the ribbon the Hall voltage $= 7.8$ millivolts.

6.36 The relation between current density J and charge carrier velocity v is $J = nqv$, with v in m/sec, J in amp/m^2, n in m^{-3} and q in coulomb. Force on charge carrier is $q(\mathbf{E}_t + \mathbf{v} \times \mathbf{B})$ which is zero if

$$\mathbf{E}_t = \frac{-\mathbf{J} \times \mathbf{B}}{nq}$$

6.37 The resistance of the winding of the small solenoid will be 10 times that of the large coil. (The wire is 1/10 as long, with 1/100 the cross-sectional area.) If we apply the same voltage, 120 volts, we'll get 1/10 the current. That is just what will be needed to produce a magnetic field equal to that in the large coil, because the small coil has 10 times as many turns per unit length. The power is down by the factor 10, but the small coil has only 1/100 the surface area. It will be much harder to keep it cool.

6.38 The grain in Problem 2.22 had a radius of 3×10^{-7} m and was charged to a potential V of 0.15 volt. Its charge $q = 4\pi\epsilon_0 r V = 0.5 \times 10^{-17}$ coulomb. Moving through a magnetic field B the grain experiences a transverse force qvB. If its path is a circle of radius R, around which it moves with angular speed $\omega = v/R$, setting $mR\omega^2 = BqR\omega$ gives us the usual "cyclotron" relation:

$$\omega = Bq/m$$

Given $B = 3 \times 10^{-10}$ tesla, $m = 10^{-16}$ kg, $q = 0.5 \times 10^{-17}$ coulomb, we find $\omega = 1.5 \times 10^{-11}$ sec. The period of one revolution is $2\pi/\omega$ or 4×10^{11} sec, about 1300 years.