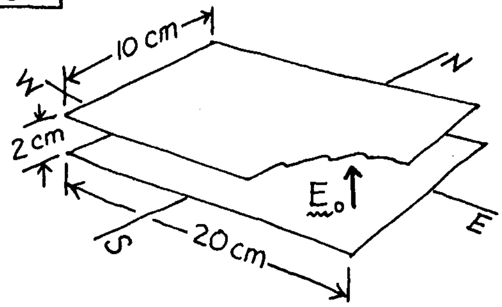
5.1



In rest frame of plates:

$$E_0 = \frac{1 \text{ statvolt}}{2 \text{ cm}}$$

$$= 0.5 \text{ statvolts/cm}$$

$$\sigma_0 = \frac{E_0}{4\pi}$$

$$Q_0 = \sigma_0 A_0$$

Number of excess electrons on negative plate $= \frac{Q_0}{e}$

$$= \frac{E_0 A_0}{4\pi e} = \frac{0.5 \times 10 \times 20}{4\pi \times 4.8 \times 10^{-10}} = 1.66 \times 10^{10} \text{ electrons}$$

Frame F_1 is moving east at 0.6 C. In F_1, the plates are moving west at 0.6 C. Their east-west dimension is $20\sqrt{1-.6^2} = 16$ cm. NS dimension is unchanged, as is the vertical separation. The number of electrons on the negative plate is the same, 1.66×10^{10}. But σ_1 and E_1 are greater, by the factor $\gamma = \frac{1}{\sqrt{1-.6^2}} = 1.25$

$$E_1 = \gamma E_0 = 0.625 \frac{\text{statvolts}}{\text{cm}}$$

Frame F_2 is moving upward at 0.6 C. In F_2, the plate dimensions are 20×10 cm, the vertical separation is $2\sqrt{1-.6^2} = 1.6$ cm. The number of electrons is unchanged. $\sigma_2 = \sigma_0$ and $E_2 = E_0 = 0.5$ statvolts/cm.

5.2

$$Q = 5 \times 10^8 \times 4.8 \times 10^{-10} = 0.24 \text{ esu}$$

$$\lambda = Q/\ell = 0.06 \text{ esu/cm}$$

(a) $E = \frac{2\lambda}{r} = \frac{.12}{.005} = 24$ statvolts/cm

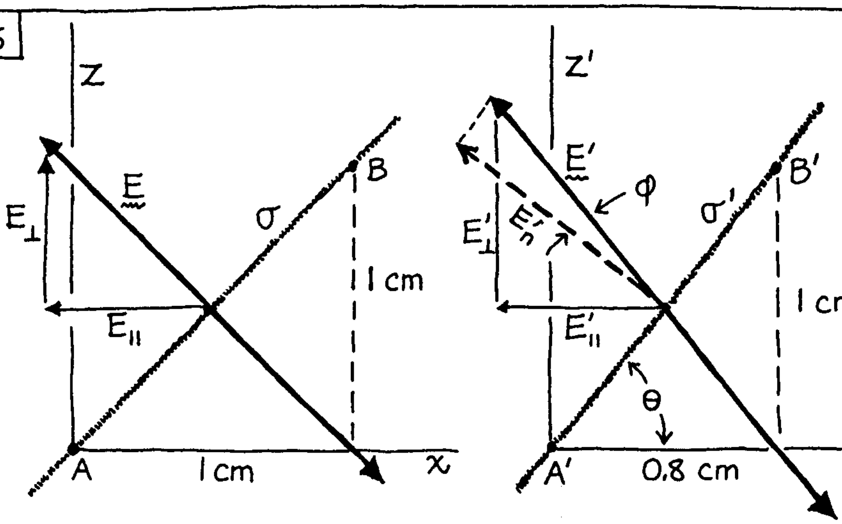
(b) $\gamma = \frac{1}{\sqrt{1-.9^2}} = 2.29$

$$E' = \gamma E = 55 \text{ statvolts/cm}$$

5.3 (a) Number of electrons passing per second $= .05\text{ amp}/1.6 \times 10^{-19}\text{ coulomb} = 3.1 \times 10^{11}\text{ sec}^{-1}$. velocity $= 3 \times 10^{10}\text{ cm sec}^{-1}$. Mean distance between electrons $= 3 \times 10^{10}\text{ cm sec}^{-1}/3 \times 10^{11}\text{ sec}^{-1} = 0.1\text{ cm}$. $\lambda = 4.8 \times 10^{-10}/0.1 = 4.8 \times 10^{-9}\text{ esu cm}^{-1}$. Electric field 1 cm from beam $= 2\lambda/r = 9.6 \times 10^{-9}$ statvolt/cm.

(b) In electron rest frame distance between successive electrons is $\gamma \times 0.1$ cm, or 2 cm. $\lambda' = \lambda/\gamma$ and the electric field 1 cm from the beam is $9.6 \times 10^{-9}/\gamma$, or 4.8×10^{-10} statvolt/cm. This is the <u>average</u> of the radial field component, along a line parallel to the beam. Because the electrons are so far apart there is a large variation in field with position along this line.

5.5



$\beta = .6 \quad \gamma = 1.25 \quad 1/\gamma = .8 \quad \theta = \tan^{-1}\gamma = 51.34°$

$E = 2\pi\sigma \quad E_{\parallel} = E_{\perp} = \sqrt{2}\,\pi\sigma$

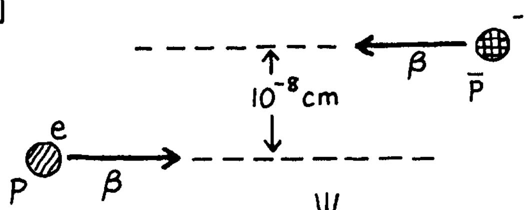
$E'_{\parallel} = E_{\parallel} \quad E'_{\perp} = \gamma E_{\perp} \qquad E' = \sqrt{\frac{1+\gamma^2}{2}}\,E = 1.132\,E$

Charge between A and B is same as charge between A' and B'. $\sigma'\sqrt{1+.8^2} = \sigma\sqrt{2} \qquad \sigma' = 1.1043\,\sigma$

$\phi = 2(\theta - 45°) = 12.68° \quad E'_n = E'\cos\phi = 1.1043\,E$

Gauss's law still holds.

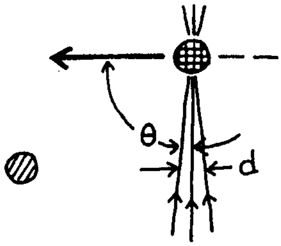
5.6



$\gamma = 100$

$\gamma = 100$

In proton rest frame



$$\beta' = \frac{2\beta}{1+\beta^2} \qquad \gamma' = \frac{1}{(1-\beta'^2)^{1/2}}$$

$$\gamma' = 2\gamma^2 - 1 = 2 \times 10^4 - 1$$

$$\gamma' \approx 2\gamma^2 \quad \text{if } \gamma \gg 1$$

In Eq. 12 replace Q by $-e$ and β by β'. Let $\phi = \frac{\pi}{2} - \theta$

$$E_r = \frac{-e}{r^2} \frac{1-\beta'^2}{(1-\beta'^2\cos^2\phi)^{3/2}}$$ which is maximum for

$\phi = 0$, with value $\gamma' \frac{e}{r^2}$ $\quad E_{max} = 9.6 \times 10^{10}$ statvolt/cm.

For $\phi \ll 1$ $\quad (1-\beta'^2\cos^2\phi)^{3/2} = \left(1-\beta'^2\left(1-\frac{\phi^2}{2}\right)\right)^{3/2} = \left(\frac{1}{\gamma'^2} + \frac{\phi^2}{2}\right)^{3/2}$

As ϕ increases from zero this doubles its value when $\phi = 1.08/\gamma$. At $r = 10^{-8}$ cm the angle $\pm\phi$ includes a distance $d \approx 2r\phi = 10^{-12}$ cm, which is traversed in 3×10^{-23} sec.

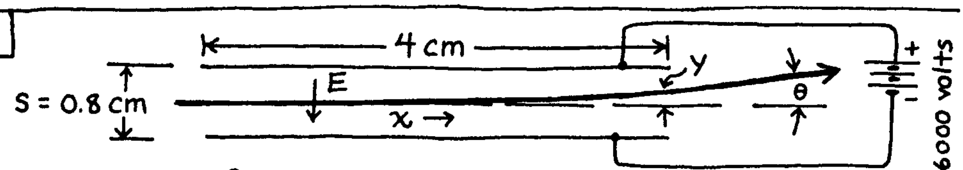
5.7

$$r^2 = \gamma \frac{\overset{\text{coulomb}}{e}}{4\pi\epsilon_0 \underset{\text{volt/m}}{E}} = 10^{10} \frac{1.6 \times 10^{-19}}{1.11 \times 10^{-10} \times 1} = 14 \text{ m}^2$$

$r = 3.8$ m $\qquad 2r/\gamma = 4 \times 10^{-10}$ m $\approx$ thickness at that distance.

5.8 The field of the proton at the position of the pion is e/r^2 ; the force on the pion is Ee.
$F = e^2/r^2 = 2.3 \times 10^{-15}$ dyne. The field of the pion at the position of the proton is $(1-\beta^2)e/r^2$ (Eq. 12 with $\theta = 0$). The force on the proton is $0.64 \times 2.3 \times 10^{-15}$ dyne, not equal to the force on the pion. There is momentum in the field and the field is changing. Only the total momentum, proton momentum + pion momentum + field momentum, is conserved. This is not a two-body system!

5.9



Kinetic energy of electron is 250 kev.
Rest energy + kinetic energy = (500 + 250) kev = γ (500 kev)

$\gamma = 750/500 = 1.5 \qquad \beta^2 = 1 - \frac{1}{\gamma^2} = 0.555$

$\beta = 0.745 \qquad P_x = \gamma\beta mc = 1.118\ mc$

$$t = \frac{4\text{ cm}}{\beta c} = \frac{4}{0.745 \times 3 \times 10^{10}} = 1.79 \times 10^{-10}\text{ sec}$$

$$P_y = Eet \qquad \frac{P_y}{mc} = \frac{Eet}{mc} = \frac{Ve}{s}\,\frac{tc}{mc^2}$$

Ve has dimensions of energy, as does mc^2, so both can be in kev : $Ve/mc^2 = 6/500$

$$\frac{P_y}{mc} = \frac{6 \times 1.79 \times 10^{-10} \times 3 \times 10^{10}}{500 \times 0.8} = 0.0805 = \gamma \frac{v_y}{c}$$

$$v_y = \frac{0.0805}{\gamma} c = 1.61 \times 10^9\text{ cm/sec} \quad \text{(at exit)}$$

average transverse velocity $\overline{v_y} = 0.805 \times 10^9$ cm/sec

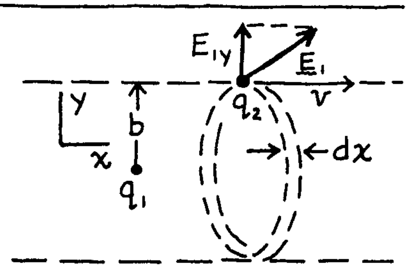
$y = \overline{v_y}\, t = 0.805 \times 10^9 \times 1.79 \times 10^{-10} = 0.144$ cm at exit.

$$\theta = P_y/P_x = \frac{0.0805}{1.118} = 0.072\text{ radian} = 4.13^\circ$$

In a frame in which the electron is initially at rest, the plates are moving to the left with speed $\beta c = 0.745c$. The length of the plates is $4/\gamma$ cm $=$ 2.67 cm. The plates are above and below the electron for a time $t' = \dfrac{2.67 \text{ cm}}{0.745\, c}$, during which the electron is accelerated upward in the field $E' = \gamma E = \gamma$ V/s. The upward momentum acquired is the same as in the other frame: $E'et' = Eet = 0.0805$ mc. In this frame the electron is non-relativistic, to a good approximation. $v_{y'} = .0805\,c$, $\bar{v}_{y'} = .0402\,c$ and $y' = (.0402\,c)t' = (.0402\,c) \times 2.67/(0.745\,c)$

$= 0.144$ cm

5.10 Let E_{1y} be the y-component of the electric field of q_1 at the location of q_2. Let P_{2y} be the y-component of momentum of the particle q_2.



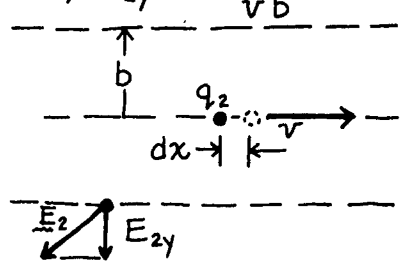
$$\frac{dP_{2y}}{dt} = E_{1y} q_2 \qquad dt = \frac{dx}{v} \qquad P_{2y} = 0 \text{ at } x = -\infty$$

for $x = +\infty$, $\displaystyle P_{2y} = \int_{-\infty}^{\infty} E_{1y} q_2\, dt = \frac{q_2}{v}\int_{-\infty}^{\infty} E_{1y}\, dx$

The surface integral of $\mathbf{E}_1$ over the infinitely long cylinder of radius b is $4\pi q_1$ by Gauss's law, and this integral is just $\displaystyle\int_{-\infty}^{\infty} E_{1y} \cdot 2\pi b\, dx$. Hence

$$\int_{-\infty}^{\infty} E_{1y}\, dx = \frac{4\pi q_1}{2\pi b} = \frac{2q_1}{b}, \text{ from which, } P_{2y} = \frac{2q_1 q_2}{vb}$$

To calculate in the same way the momentum acquired by q_1, consider a cylinder drawn with the path of q_2 as its axis. At any instant, the flux through the cylinder being $4\pi q_2$, the integral $\displaystyle\int_{-\infty}^{\infty} E_{2y}\, dx = \frac{2q_2}{b}$. Now q_1 is stationary;



it remains at a fixed x. But clearly $\int_{-\infty}^{\infty} E_{2y}\,dt$,

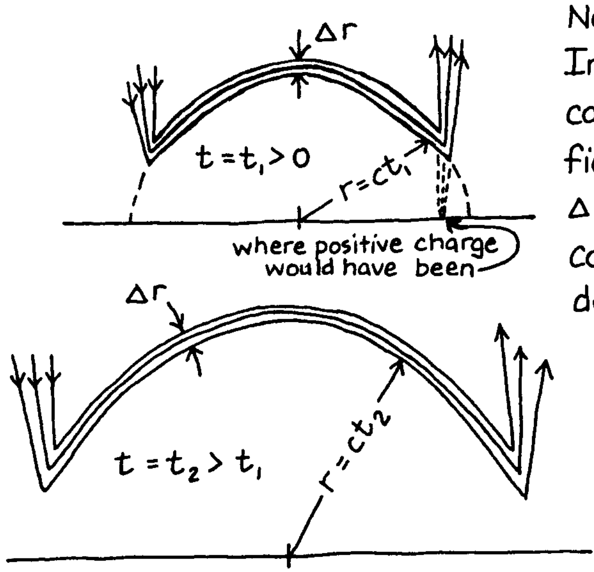
computed at the fixed point x, is equal to $\frac{1}{v}\int_{-\infty}^{\infty} E_{2y}\,dx$ if q_2 slides along at the constant speed v. Hence $P_{1y} = \dfrac{-2q_1q_2}{vb}$.

$\boxed{5.13}$ (a) The electron had been traveling along the negative x-axis toward the origin, where it rather suddenly stopped at $t = -5 \times 10^{-10}$ sec. (By the time $t = 0$ light has travelled 15 cm.) If the electron hadn't stopped it would be at $x = 12$ cm, so its speed was 0.8c.

(b) At $t = -7.5 \times 10^{-10}$, which was 2.5×10^{-10} sec before the electron stopped, it must have been at $x = -2.5 \times 10^{-10} \times 0.8 \times 3 \times 10^{10} = -6.0$ cm.

(c) At that moment the field strength at the origin caused by this electron was $(1-\beta^2)e/r^2$ or $0.36 \times 4.8 \times 10^{-10}/36 = 4.8 \times 10^{-12}$ statvolt/cm.

$\boxed{5.14}$



No charge remains. Incoming field lines connect to outgoing field lines. Thickness Δr of shell containing connecting field is determined by duration of deceleration period. Δr remains constant. As sphere expands, E within this shell $\sim \frac{1}{r}$. This radiation is called Bremsstrahlung.

$\boxed{5.15}$ Given $\beta_0 = 0.8 \quad \gamma = 1.2$

$$\beta = \sqrt{1 - 1/\gamma^2} = .553 \qquad \beta_0' = \frac{\beta_0 - \beta}{1 - \beta\beta_0} = 0.445$$

$$\lambda' = \gamma\lambda_0 - \gamma(1 - \beta\beta_0)\lambda_0 = \gamma\beta\beta_0\lambda_0 = 0.531\,\lambda_0$$

$\boxed{5.16}$ If $\beta = \beta_0$ then $\gamma = \gamma_0$ and

positive charge density $\gamma\lambda_0$ becomes $\gamma_0\lambda_0$;

negative charge density $-\gamma(1 - \beta\beta_0)$ becomes λ_0/γ_0 .

$\boxed{5.17}$ In the rest frame of the two protons the force of repulsion is simply e^2/r^2. The force in the lab frame must be $(1/\gamma)(e^2/r^2)$. (Remember, the force is always largest in the rest frame of the particle on which it acts.) This <u>is</u> the correct total force in the lab frame. But the electrical force $e\mathbf{E}$ in the lab frame is $\gamma e^2/r^2$. However, in the lab frame the electric field of each proton, $E = \gamma e/r^2$ at the location of the other proton, is accompanied by a magnetic field of strength $\beta\gamma e/r^2$ at the same location. And that other proton <u>is moving with speed βc through that field B</u>. So we must add to the electrical repulsive force $\gamma e^2/r^2$ the magnetic force $\beta e \cdot B$ or $\gamma\beta^2 e^2/r^2$. This force is attractive in sign. The total force is now $\gamma e^2/r^2 - \gamma\beta^2 e^2/r^2$ which reduces simply to $(1/\gamma)e^2/r^2$, the answer we obtained more directly by transforming from the rest frame of the protons.

One might have been tempted to argue that the proton is not "moving through" the B field of the other proton because that field is "moving right along with it". That would be wrong. In the force law which is the fundamental definition of $\underset{\sim}{B}$: $\underset{\sim}{F} = q\underset{\sim}{E} + q(\underset{\sim}{v}/c) \times \underset{\sim}{B}$, $\underset{\sim}{B}$ is the field at the position of the charge q at an instant of time, both position and time measured in the frame in which we are measuring the force on q. What the "source" of B may be doing at that instant is irrelevant.

$\boxed{5.18}$

$$I = c\sum_k n_k q_k \beta_k \qquad \lambda = \sum_k n_k q_k$$

In the new frame, $\beta_k' = \dfrac{\beta_k + \beta}{1 + \beta_k \beta}$ $\quad n_k' = n_k \dfrac{\gamma_k'}{\gamma_k}$

$$\gamma_k' = \frac{1}{\sqrt{1 - \left(\frac{\beta_k + \beta}{1+\beta_k\beta}\right)^2}} = \frac{1 + \beta_k\beta}{\sqrt{1 + 2\beta_k\beta + \beta_k^2\beta^2 - \beta_k^2 - 2\beta_k\beta + \beta^2}}$$

$$= \frac{1+\beta_k\beta}{\sqrt{(1-\beta^2)(1-\beta_k^2)}} = \gamma\gamma_k(1 + \beta_k\beta)$$

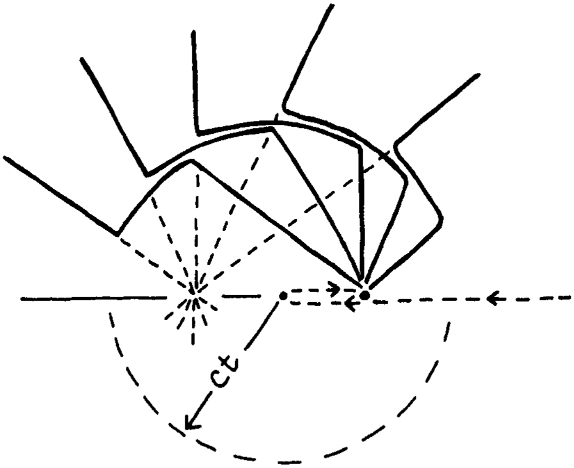
$$I' = c\sum_k n_k' q_k \beta_k' = c\sum_k q_k \frac{n_k}{\cancel{\gamma_k}}\gamma\cancel{\gamma_k}\frac{\cancel{(1+\beta_k\beta)}(\beta_k\beta)}{\cancel{(1+\beta_k\beta)}}$$

$$= c\sum_k \gamma q_k n_k \beta_k + c\beta\sum_k \gamma q_k n_k = \gamma[I + c\beta\lambda]$$

$$\lambda' = \sum_k n_k' q_k = \sum_k q_k n_k \frac{1}{\cancel{\gamma_k}}\gamma\cancel{\gamma_k}(1 + \beta_k\beta)$$

$$= \sum_k \gamma q_k n_k + \beta\sum_k \gamma n_k q_k \beta_k = \gamma\left[\lambda + \frac{\beta I}{c}\right]$$

5.19



at time t

Outside a sphere of radius ct, centered on the origin, the field at time t is that of a uniformly moving charge which would have been at this time, at $x = -vt$.