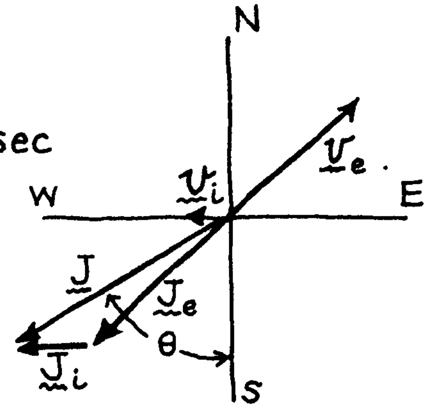
4.1 Electron current

density $= \underset{\sim}{J}_e = \rho_e \underset{\sim}{v}_e = 10^{11}(-e)\underset{\sim}{v}_e$

$J_e = 10^{11} \times 4.8 \times 10^{-10} \times 10^8 = 4.8 \times 10^9$ esu/cm² sec

$J_i = 2 \times 5 \times 10^{10} \times 4.8 \times 10^{-10} \times 10^7 = 4.8 \times 10^8$

$\underset{\sim}{J} = \underset{\sim}{J}_e + \underset{\sim}{J}_i$



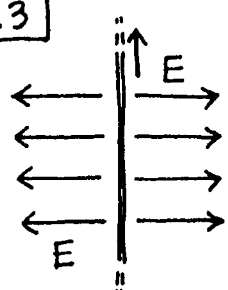
$J_{west} = .707 \times 4.8 \times 10^9 + 4.8 \times 10^8 = 3.87 \times 10^9$

$J_{south} = .707 \times 4.8 \times 10^9 = 3.39 \times 10^9 \qquad \theta = \tan^{-1} \dfrac{3.87}{3.39} = 48.8°$

$J = \sqrt{J_{west}^2 + J_{south}^2} = 5.14 \times 10^9$ esu/cm² sec $= 1.71$ amperes/cm²

4.2 Each electron makes $\dfrac{3 \times 10^8 \text{ meters/sec}}{240 \text{ meters}} = 1.25 \times 10^6$ round trips per second. Charge passing any point, per second, $= 1.25 \times 10^6 \times 10^{11} \times 4.8 \times 10^{-10} = 6 \times 10^7$ esu/sec or .020 amp.

4.3



$E = 40$ statvolts/cm Total σ on belt (both sides) $= 2 \times \dfrac{40}{4\pi} = \dfrac{20}{\pi}$ esu/cm²

Current carried by 30 cm wide belt with speed 2000 cm/sec:

$$I = 30 \times \left(\frac{20}{\pi}\right) \times 2000 = 3.82 \times 10^5 \text{ esu/sec} = 0.127 \text{ milliamp}$$

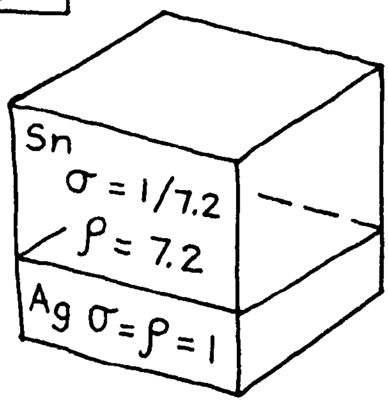
4.4

(a) $R = \dfrac{\text{length}}{\text{area}} \times \text{resistivity} = \dfrac{3 \times 10^8 \text{cm} \times 3 \times 10^{-6} \text{ohm-cm}}{7 \times (\pi/4) \times (.073)^2 \text{ cm}^2}$

$= 30\,000$ ohms

$\boxed{4.6}$ For constant volume $L \propto 1/d^2$ since resistance $R \propto L/d^2$, reducing d by the factor $3/4$, thus increasing L by $16/9$, increases R by $(16/9)^2$ or 3.16. An overall increase in L by the factor 2 would increase R by 4.00.

$\boxed{4.7}$



The ratio of conductivities, $7.2/1$ and the ratio of layer thicknesses, $1/2$, is all that matters. Let's take $\sigma_{Ag} = \rho_{Ag} = 1$ and consider the cube that is $1/3$ silver, $2/3$ tin. For vertical currents the layers are in series and resistances add:

$$\rho_\perp = \frac{1}{3} + \frac{2}{3} \times 7.2 = 5.133 \qquad \sigma_\perp = \frac{1}{\rho_\perp} = 0.1948$$

For horizontal currents the layers are in parallel:

$$\sigma_\parallel = \frac{1}{3} + \frac{2}{3} \times \frac{1}{7.2} = 0.4259 \qquad \sigma_\perp / \sigma_\parallel = 0.4566$$

$\boxed{4.8}$ $R = \dfrac{L\rho}{A} = 10^5\,\text{cm} \times 1.7 \times 10^{-6}\,\text{ohm-cm}/A$

cross-section of wire; not given, but it will cancel out

current density $J = V/AR = V/L\rho = 6/.17\ \text{amp/cm}^2$

$J = ne\bar{v} \quad \bar{v} = J/ne = (6/.17)/(8 \times 10^{22} \times 1.6 \times 10^{-19})$

$= 0.0028\ \text{cm/sec} \quad t = \dfrac{10^5\,\text{cm}}{.0028\ \text{cm/sec}} \approx 3 \times 10^7\ \text{sec}$

≈ 1 year

4.9 $N = 10^6\ cm^{-3}$; $m = 10^{-27}$ gm

τ = mean free time = ℓ/v = 10^{-6} sec

$$\sigma = \frac{e^2 N \tau}{m}$$

$$= \frac{(4.8 \times 10^{-10})^2 \times 10^6 \times 10^{-6}}{10^{-27}} = 2.3 \times 10^8\ sec^{-1}$$

same calculation in SI : $N = 10^{12}\ m^{-3}$; $m = 10^{-30}$ kg ;
$e = 1.6 \times 10^{-19}$ coulomb σ will be in $(\text{ohm}-\text{m})^{-1}$.

$$\sigma = \frac{(1.6 \times 10^{-19})^2 \times 10^{12} \times 10^{-6}}{10^{-30}} = 2.5 \times 10^{-2} (\text{ohm}-\text{m})^{-1}$$

$$= 2.5 \times 10^{-4} (\text{ohm}-\text{cm})^{-1}$$

4.10 Mass of OH^- or OH_3^+ $\approx 3 \times 10^{-23}$ gm

$\sigma = \dfrac{e^2 N \tau}{m}$ gives $\tau = \dfrac{m\sigma}{Ne^2}$ $\sigma = 3.6 \times 10^4\ sec^{-1}$

$$\text{Then } \tau = \frac{3 \times 10^{-23} \times 3.6 \times 10^4}{10^{13} \times 23 \times 10^{-20}} \approx 5 \times 10^{-13}\ \text{sec}$$

Distance travelled at 5×10^4 cm/sec : 2.5×10^{-8} cm

4.11 $R = \dfrac{25 \times 200}{A} = \dfrac{5000}{A}$ ohms

A $\leftarrow$ cm^2

$J = V/RA = 12/5000\ \text{amp}/cm^2$

$= (12/5000)/1.6 \times 10^{-19}$ ions/cm^2/sec $= n\bar{v}$

$n = 2 \times 3 \times 10^{20}\ cm^{-3} = 6 \times 10^{20}\ cm^{-3}$

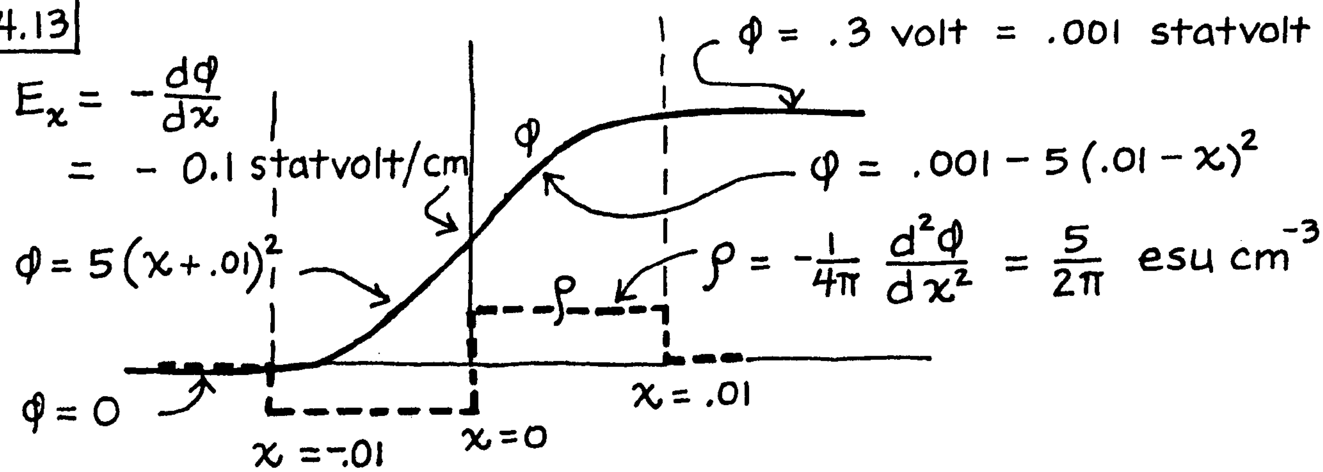
$$\bar{v} = J/ne = \frac{12}{5000 \times 1.6 \times 10^{-19} \times 6 \times 10^{20}}$$

$$= 2.5 \times 10^{-5}\ \text{cm/sec}$$

4.12 From Fig. 4.10 $\sigma = 0.3\,(\text{ohm}-\text{cm})^{-1}$ and $N_+ = N_- = 10^{15}\ \text{cm}^{-3}$. We shall assume that $m_+ = m_- = 10^{-27}$ gm. In CGS units of $\sec^{-1}$, $\sigma = 0.3/1.11 \times 10^{-12}\ \sec^{-1} = 2.7 \times 10^{11}\ \sec^{-1}$. Solving Eq. 20 for τ, we obtain in this case :

$$\tau = \frac{m\sigma}{2e^2N} = \frac{10^{-27} \times 2.7 \times 10^{11}}{2 \times 23 \times 10^{-20} \times 10^{15}} = 6 \times 10^{-13}\ \text{sec}.$$

4.13



4.14

$$\sigma = \frac{2e^2 n\tau}{m} \qquad \tau = \frac{\sigma m}{2e^2 n}$$

in CGS : $0.3\,(\text{ohm}-\text{cm})^{-1} = \dfrac{0.3}{1.1 \times 10^{-12}}\ \sec^{-1}$ $\qquad n = 10^{15}\ \text{cm}^{-3}$

$$\tau = \left(\frac{0.3}{1.1 \times 10^{-12}}\right)\left(\frac{10^{-27}}{2 \times (4.8 \times 10^{-10})^2 \times 10^{15}}\right) = 5.9 \times 10^{-13}\ \text{sec}$$

in SI : $0.3\,(\text{ohm}-\text{cm})^{-1} = 30\,(\text{ohm}-\text{m})^{-1}$ $\qquad n = 10\ \text{m}^{-3}$, $m = 10^{-30}$ kg

$$\tau = \frac{30 \times 10^{-30}}{2 \times (1.6 \times 10^{-19})^2 \times 10^{21}} = 5.9 \times 10^{-13}\ \text{sec}$$

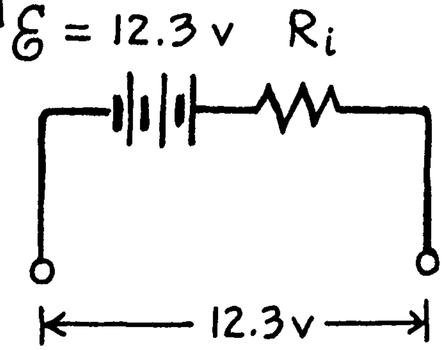
$v\tau = 9 \times 10^{-6}$ cm, more than 300 times the distance between neighboring silicon atoms.

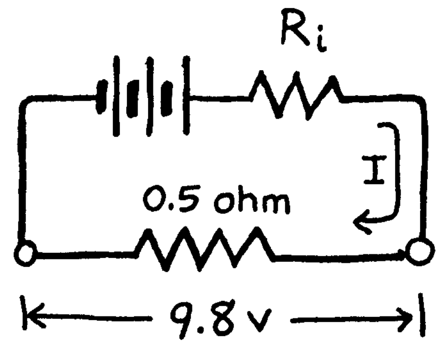
4.16

$$R_0 = \frac{R_1(R_0+R_1)}{R_1 + (R_0+R_1)} + R_1 \qquad \text{Solve for } R_1:$$

$$2\cancel{R_0R_1} + R_0^2 = \cancel{R_0R_1} + R_1^2 + 2R_1^2 + \cancel{R_0R_1} \qquad R_1 = R_0/\sqrt{3}$$

4.17

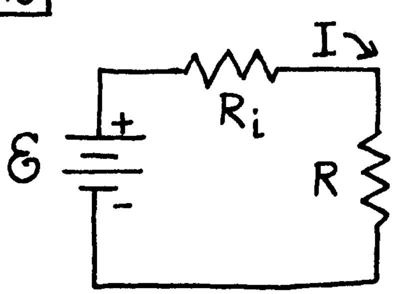




$$9.8\,\text{v} = 0.5 \times I \qquad I = 19.6 \text{ amp}$$

$$(12.3 - 9.8)\,\text{v} = IR_i \qquad R_i = \frac{2.5\,\text{v}}{19.6 \text{ amp}} = 0.128 \text{ ohm}$$

4.18



Let P = power dissipated in resistor R

$$P = I^2R \qquad I = \frac{\mathcal{E}}{R+R_i} \qquad P = \frac{\mathcal{E}^2 R}{(R+R_i)^2}$$

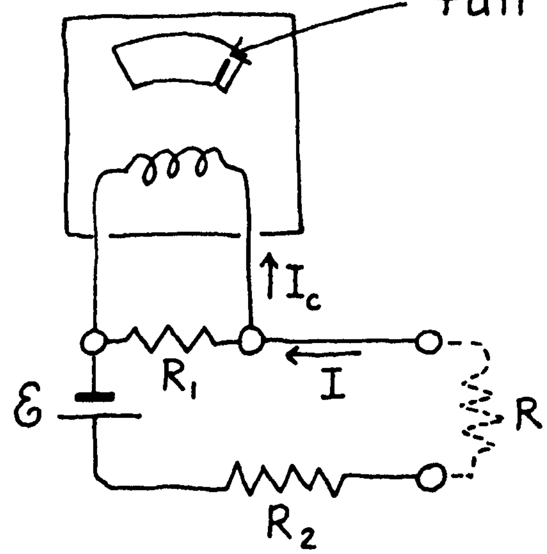
$$\frac{dP}{dR} = \mathcal{E}^2 \frac{(R+R_i)^2 - 2R(R+R_i)}{(R+R_i)^4} = \mathcal{E}^2 \frac{R_i - R}{(R+R_i)^3}$$

For $R = R_i$ $\frac{dP}{dR} = 0$. Also, $\frac{dP}{dR} < 0$ for $R > R_i$, and $\frac{dP}{dR} > 0$ for $R < R_i$. Hence this is condition for maximum P.

4.19

$R_c = 20$ ohms $\quad \mathcal{E} = 1.5$ volt

full scale deflection for $I_c = 50\mu$amp



The current I in the external circuit will be 0.1 amp, in order of magnitude, if adding 15 ohms cuts it in half. This is very much larger than 50μamp. This tells us the resistance R_1 must be $\ll R_c$, so that nearly all of the current I is shunted through R_1.

$I_c/I = R_1/(R_1+R_c) \approx R_1/R_c$ In the external circuit, $R_1 \ll R_2$, so that $I \approx \mathcal{E}/(R_2+R)$. If I is to be half as large for $R = 15$ ohms as for $R = 0$, then R_2 must be 15 ohms and the current is 0.1 amp when $R = 0$. This is to cause $I_c = 50\mu$amp:

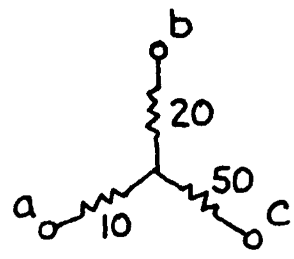
$$R_1/R_c \approx I_c/I = 50 \times 10^{-6}/0.1 = 5 \times 10^{-4}$$

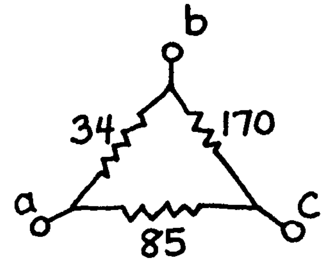
$$R_1 = 20 \times 5 \times 10^{-4} = 0.01 \text{ ohm}.$$

The error introduced by neglecting R_1 in the external circuit could be eliminated by reducing R_2 from 15.00 to 14.99 ohms, while the error in neglecting R_1 compared to R_c would be eliminated by increasing R_1 to .010005. Neither refinement would ordinarily be justified.

$R = 5$ ohms will give $15/(15+5)$, or 3/4 of full scale deflection (to the 37.5μamp mark). $R = 50$ ohms will send $11.5\ \mu$amp through the coil.

4.20

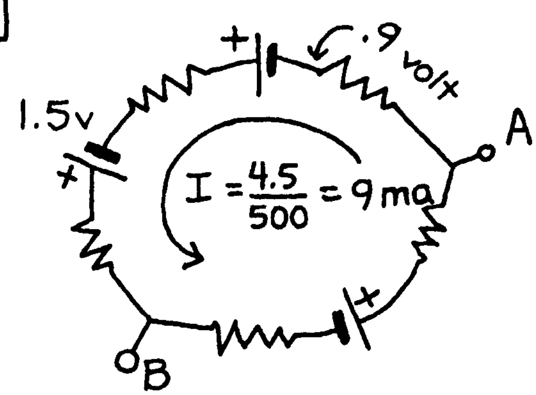


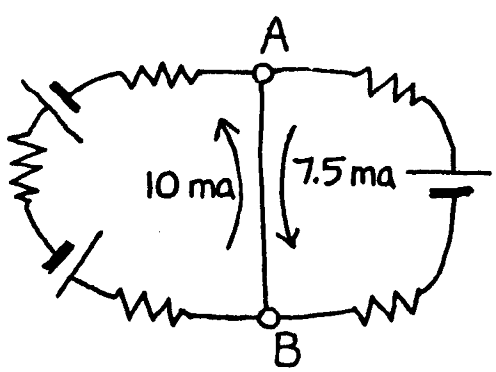


R_{ab}:	$10 + 20 = 30$	$34(170+85)/289 = 30$
R_{bc}:	$20 + 50 = 70$	$170(85+34)/289 = 70$
R_{ca}:	$50 + 10 = 60$	$85(34+170)/289 = 60$

Notice also that the potential assumed by a free terminal when the potentials at the other two terminals are fixed is the same for the two boxes. They are indistinguishable by external electrical measurements.

4.21

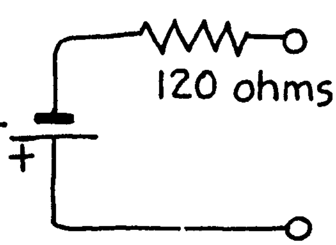




open circuit voltage
$= 1.5 + 1.5 - 3 \times 0.9$
$= 0.3$ volt (B positive)

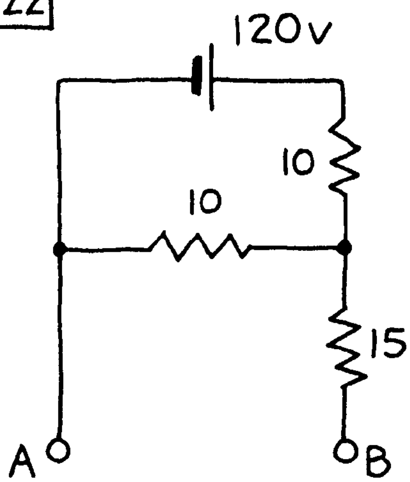
short circuit current
$= 10\ \text{ma} - 7.5\ \text{ma} = 2.5\ \text{ma}$
(from B to A)

$\mathcal{E} = 0.3$ volt

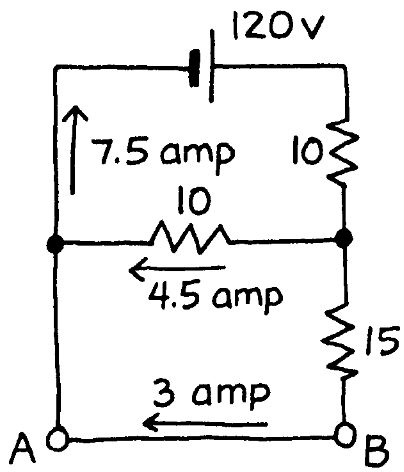


equivalent circuit

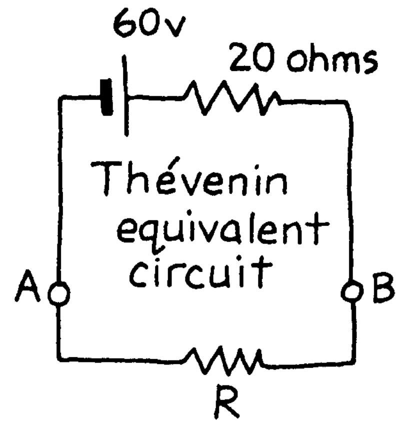
4.22



open circuit
voltage = 60v



short circuit
current = 3 amp

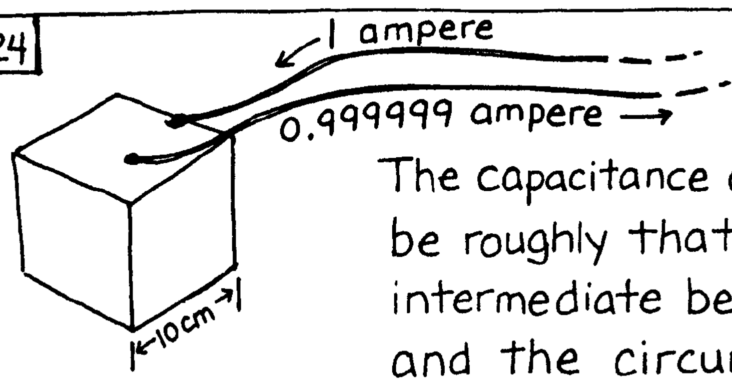


For max. power in R: R = 20 ohms ; P = 45 watts

4.23 If E = 1 statvolt/cm, charge on 1 cm^2 of sheet is $1/4\pi$. It will be neutralized by a displacement d of the electron cloud such that $ned = 1/4\pi$.

$$d = \frac{1}{4\pi n e} = 1.7 \times 10^{-7} \text{ cm}.$$

4.24



The capacitance of the isolated box must be roughly that of a sphere intermediate between the inscribed and the circumscribed sphere — say 6 cm radius. C = 6 cm ≈ 6×10^{-12} farads

This object is acquiring positive charge at the rate of 10^{-6} amperes or 10^{-6} coulomb/sec.

$$\frac{dV}{dt} = \frac{1}{C}\frac{dQ}{dt} = \frac{1}{6\times 10^{-12}} \times 10^{-6} = \frac{1}{6} \times 10^{6} \text{ volts/sec.}$$

Hence the potential will rise by 1000 volts in 6 milliseconds.

4.25

Eq. 34 p. 160 $\quad I = \frac{dQ}{dt} = \frac{V_0}{R} e^{-t/RC}$

At any instant after $t = 0$, the power dissipated in the resistor is $P = I^2 R = \frac{V_0^2}{R} e^{-2t/RC}$

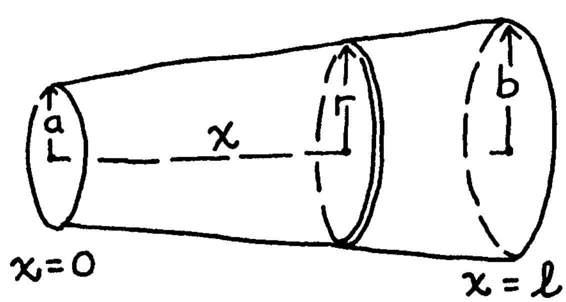
The total energy dissipated is $\int_0^\infty P\,dt = \int_0^\infty \frac{V_0^2}{R} e^{-2t/RC}\,dt$

$$= \frac{V_0^2}{R}\,\frac{RC}{2}\int_0^\infty e^{-x}\,dx = \frac{1}{2} C V_0^2$$

Suppose we have a 1 microfarad capacitor charged to 100 volts. $Q = CV = 10^{-4}$ coulombs. One electron charge is 1.6×10^{-19} coulombs. If $t_0 = RC$ is the time constant, we shall have one electron left when $e^{-t/t_0} = \frac{1.6 \times 10^{-19}}{10^{-4}}$, or $\frac{-t}{t_0} = \ln 1.6 \times 10^{-15}$

This gives $t = t_0 \times 2.3 \log_{10}(6 \times 10^{14}) = 34 t_0$. So if the time constant were ≈ 1 sec, we'd be down to "one electron" in a little over half a minute!

4.26

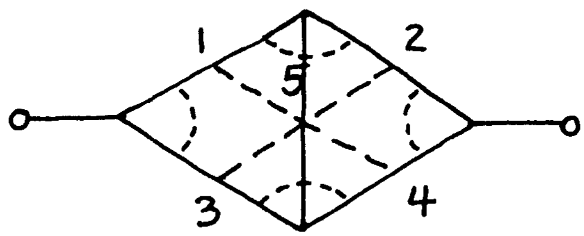


$$R = \int_0^\ell \frac{\rho\,dx}{\pi r^2}$$

$r = a + \left(\frac{b-a}{\ell}\right) x \qquad dx = \frac{\ell}{b-a}\,dr$

$$R = \frac{\rho}{\pi}\,\frac{\ell}{b-a}\int_a^b \frac{dr}{r^2} = \frac{\rho}{\pi}\,\frac{\ell}{b-a}\left(\frac{1}{a} - \frac{1}{b}\right) = \frac{\rho \ell}{\pi a b}$$

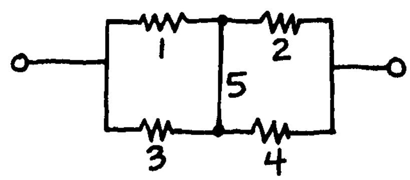
4.27 The first missing term in the numerator must be $R_1R_3R_4$ (the term lacking R_2). The symmetry of the products of pairs can be indicated as follows:



from which we see that the second ? in the numerator is R_1R_4 and the ? in the denominator is R_2R_3. $R_{eq} =$

$$\frac{R_1R_2R_3 + R_1R_2R_4 + R_1R_3R_4 + R_2R_3R_4 + R_5(R_1R_3 + R_2R_3 + R_1R_4 + R_2R_4)}{R_1R_2 + R_1R_4 + R_2R_3 + R_3R_4 + R_5(R_1 + R_2 + R_3 + R_4)}$$

(a) $R_5 = 0$

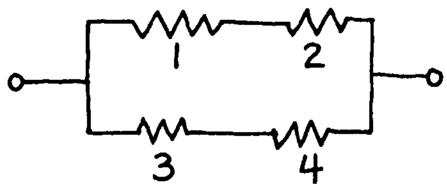


$R = \frac{R_1R_3}{R_1+R_3} + \frac{R_2R_4}{R_2+R_4}$

$$= \frac{R_1R_2R_3 + R_1R_2R_4 + R_1R_3R_4 + R_2R_3R_4}{R_1R_2 + R_1R_4 + R_2R_3 + R_3R_4}\ ,\quad \text{which}$$

agrees with the formula with $R_5 = 0$.

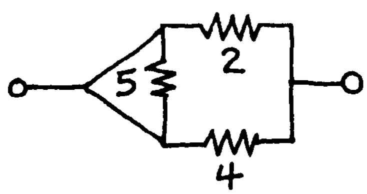
(b) $R_5 = \infty$



$R = \frac{(R_1+R_2)(R_3+R_4)}{R_1 + R_2 + R_3 + R_4}$

which agrees with the formula when $R_5 \to \infty$

(c) $R_1 = R_3 = 0$



$R = \frac{R_2R_4}{R_2+R_4}$ This

agrees with the formula when all terms containing R_1 or R_3 are dropped. The value of R_5 doesn't matter; it is short circuited anyway.