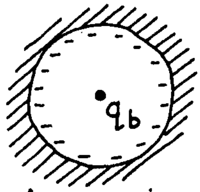
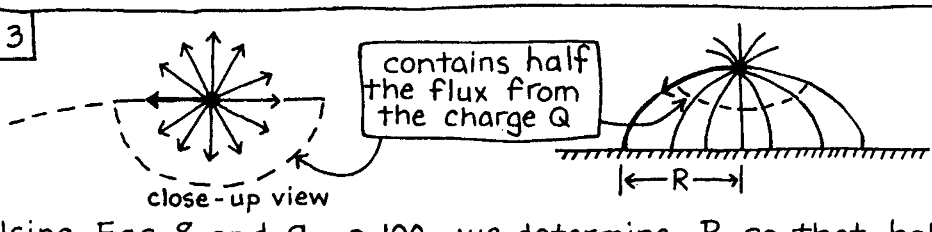
3.1 The force on q_b is zero. The field inside the spherical cavity is quite independent of anything outside. A charge $-q_b$ is uniformly distributed over the conducting surface. The same goes for q_c.



Since the total charge on conductor A is zero, a charge of amount $q_b + q_c$ must be distributed over its external surface. If q_d were absent, the field outside A would be the symmetrical, radial field, $E = \frac{q_b + q_c}{r^2}$, the same as the field of a point charge located where the center of the sphere is located. The influence of q_d will slightly alter the distribution of charge on A, but without affecting the total amount. Hence for large r, the force on q_d will be approximately $\frac{q_d(q_b + q_c)}{r^2}$. The force on A must be precisely equal and opposite to the force on q_d. [The <u>exact</u> value of the force on q_d is the sum of the force just given, $q_d(q_b + q_c)/r^2$, and the force that would act on q_d if the total charge <u>on</u> <u>and</u> <u>within</u> A were zero.]

3.3

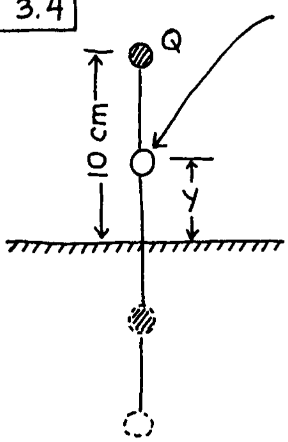


Using Eqs. 8 and 9, p. 100, we determine R so that half of $-Q$, the induced charge on the plane, is contained within the circle of radius R:

$$-\frac{Q}{2} = \int_0^R \sigma \cdot 2\pi r\,dr, \qquad \text{or } \frac{1}{2} = \int_0^R \frac{hr\,dr}{(h^2+R^2)^{3/2}} = \left[\frac{-h}{\sqrt{h^2+R^2}}\right]_0^R = 1 - \frac{h}{\sqrt{h^2+R^2}}$$

Then $\frac{h}{\sqrt{h^2+R^2}} = \frac{1}{2}$, or $h^2 + R^2 = 4h^2$, or $R = \sqrt{3}\,h$

3.4



Calling upward force positive, the force on this charge due to the upper charge Q and the two image charges below the plane is

$$Q^2\left[\frac{1}{(10-y)^2} - \frac{1}{(2y)^2} + \frac{1}{(10+y)^2}\right]$$

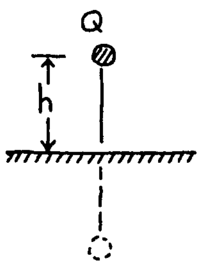
setting this equal to zero we get

$$\frac{1}{4y^2} = \frac{200+2y^2}{(100-y^2)^2},$$ which is

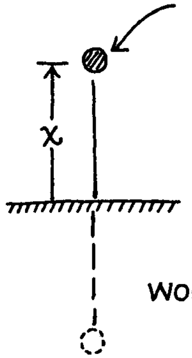
a quadratic equation in y^2: $7y^4 + 1000y^2 - 10000 = 0$

with a positive root $y^2 = 9.38$, giving $y = 3.06$

3.5



initial state



Force required to move this charge upward $= Q^2/(2x)^2$

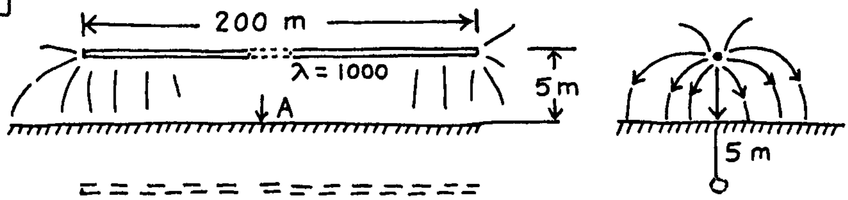
The second student calculates as follows:

$$\text{work} = \int F\,dx = \int_h^\infty \frac{Q^2}{(2x)^2}\,dx = \frac{Q^2}{4h}$$

This is the correct answer.

Note that if two real charges Q and $-Q$ were being pulled apart symmetrically, the <u>total</u> work done would be $Q^2/2h$, but the agency moving Q would supply only half of it.

3.6



Because the wire is very long compared with its height, the field at A will be practically that of a wire of infinite length, $2\lambda/r$ or in this case

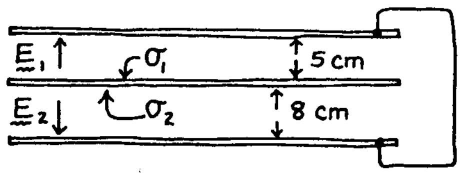
$\dfrac{2 \times 1000 \text{ esu/cm}}{500 \text{ cm}}$, or 4 statvolts/cm. To that must be added the field that would be caused by the image, making the total field at A 8 statvolt/cm.

The electrical force on the wire is that which would be exerted by the field of the image acting on the charge on the wire. The total charge on the wire is 20000 cm × 1000 esu/cm, or 2×10^7 esu. Over nearly the whole length of the wire the field of the image is $2\lambda/1000$ cm or 2 dyne/esu. Neglecting the decrease in field very near the ends of the wire, the force is 2×10^7 esu × 2 dyne/esu, or 4×10^7 dynes. It is downward, of course. [It is not hard to show that the fractional error we make in neglecting the end effects is in first order just h/L, where h is the height of the wire and L its length. Here $h/L = 1/40$. So we have over-estimated the force by 2.5 percent.]

3.7 Consider a path that runs from conductor B across to conductor D, then through the interior of the wire that connects D to C, then across the gap to A, thence via the other wire, down to B. The line integral of $\mathbf{E}$ around any closed path must be zero, if $\mathbf{E}$ is a static electric field. But if the fields are as shown in (c) the line integral over the closed path just described is not zero. Each gap makes a positive contribution, while in the conductors, including the connecting wires, $\mathbf{E}$ is zero. So (c) cannot represent a static field or charge distribution. If it did, by the way, we could contrive to violate the Uniqueness Theorem too!

3.8 Given: $\sigma_1 + \sigma_2 = 10$ esu/cm^2

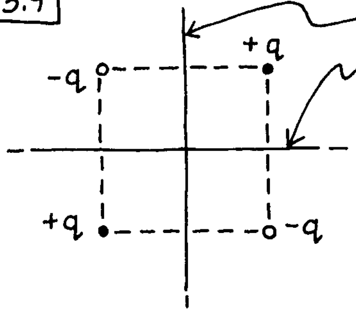
$E_1 = 4\pi\sigma_1 \quad E_2 = 4\pi\sigma_2$



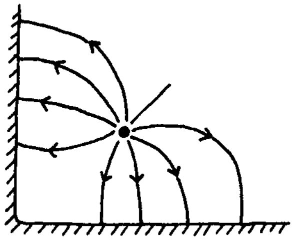
Since top and bottom plates are at the same potential,
$E_1 \cdot 5\text{cm} = E_2 \cdot 8\text{ cm}$, or $5\sigma_1 = 8\sigma_2$

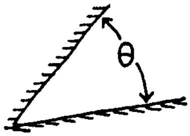
$$\left.\begin{aligned} \sigma_1 + \sigma_2 &= 10 \\ 5\sigma_1 - 8\sigma_2 &= 0 \end{aligned}\right\} \quad \text{Solving for } \sigma_1 \text{ and } \sigma_2: \quad \sigma_1 = 6.15, \quad \sigma_2 = 3.85 \ \frac{\text{esu}}{\text{cm}^2}$$

3.9



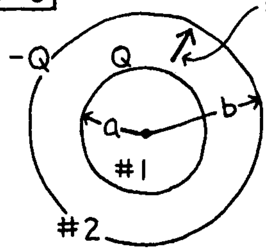
Planes perpendicular to paper. $\varphi = 0$ at every point on these two planes, because for any such point, $\sum \frac{q}{r} = 0$, by symmetry.





The method won't work unless $\theta = \pi/N$, N an integer. We have to divide space into an even number of similar wedges, in order to have an alternating ring of images. It won't work, for example, for $\theta = 120°$

3.10



$E = \frac{Q}{r^2}$

$$\varphi_{12} = \int_a^b \frac{Q}{r^2}\,dr = Q\left(\frac{1}{a} - \frac{1}{b}\right)$$

$$C = \frac{Q}{\varphi_{12}} = \frac{1}{\frac{1}{a} - \frac{1}{b}} = \frac{ab}{b-a}$$

If $b - a \ll a$ the capacitor is approximately equivalent to a flat-plate capacitor of area $4\pi r^2$, where $r \approx a \approx b$, with plate separation $b - a$. Indeed, if we use for r the <u>geometric</u> mean of a and b, the equivalence is exact :

$$\frac{1}{4\pi} \times \frac{4\pi(\sqrt{ab})^2}{b-a} = \frac{ab}{b-a} = C \text{ as calculated above.}$$

$\boxed{3.11}$ $Q = C_1 V_1 = 10^{-10}$ farad $\times$ 100 volt $= 10^{-8}$ coulomb.
When the same charge Q is shared between C_1 and C_2 connected in parallel, $Q = (C_1 + C_2) V_2$

$(C_1 + C_2)/C_1 = V_1/V_2 = 100/30 = 3.33$

$C_1 + C_2 = 333\,pF \qquad C_2 = 233\,pF$

Energy stored was $\frac{1}{2} Q V_1 = 0.5 \times 10^{-6}$ Joules
For same charge at 30 volts, energy is 0.15×10^{-6} J.
0.35×10^{-6} J of energy has been lost. That much energy has to go somewhere before the system can settle down to static equilibrium. If it is not stored anywhere else (for instance, in a weight lifted by a motor driven by the current from C_1 to C_2) it will eventually be dissipated in circuit resistance, no matter how small that resistance may be.

$\boxed{3.12}$ $C = \epsilon_0 A/s \qquad \epsilon_0 = 8.854 \times 10^{-12}$

$A = (\pi/4) d^2 = (\pi/4)(0.15)^2 = 1.767 \times 10^{-2}\ m^2$

$s = 0.04$ mm $= 4 \times 10^{-5}$ m

$C = (8.854 \times 10^{-12}) \times (1.767 \times 10^{-2})/4 \times 10^{-5} = 3910\ pF$

$\boxed{3.13}$ Assume the capacitance is that of a conducting sphere 1 meter in diameter.
Then $C \approx 50$ cm. 2 kilovolts $\approx$ 7 statvolts

$U = \frac{1}{2} CV^2 = \frac{50}{2} \times 7^2 = 1200$ erg. So 10^3 ergs would be a reasonable estimate.

3.14 Energy stored $= \frac{Q^2}{2C}$. If capacitance of conducting disk is $2a/\pi$, $U = \frac{\pi}{4}\frac{Q^2}{a}$

For uniformly charged non-conducting disk we found, in Problem 2.27, $U = \frac{8}{3\pi}\frac{Q^2}{a}$

$(8/3\pi)/(\frac{\pi}{4}) = 32/3\pi^2 = 1.081$. The field of the uniform charge distribution has 8 percent more energy. On the conductor the charge has distributed itself so as to minimize the energy.

3.15 If λ is the charge per unit length on the inner cylinder the field between the cylinders (except close to the ends, a correction we shall ignore) is $2\lambda/r$. The potential difference between the cylinders

is $\varphi_2 - \varphi_1 = \int_{r_1}^{r_2} \frac{2\lambda}{r}\,dr = 2\lambda \ln(r_2/r_1)$

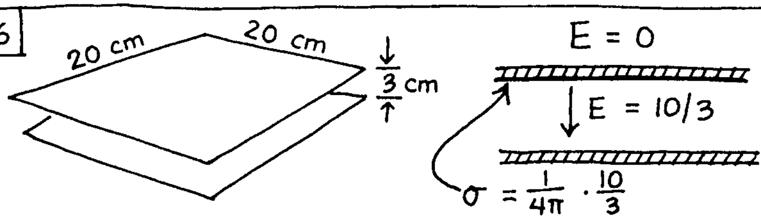
If L is the length of the cylinders the total charge Q is $L\lambda$. The capacitance C is

$\frac{Q}{\varphi_2-\varphi_1}$, or $\frac{L}{2\ln(r_2/r_1)} = \frac{30}{2\ln(4/3)} = 52.1$ cm

45 volts $= .15$ statvolts

Energy stored $= \frac{1}{2}CV^2 = \frac{52.1}{2} \times (0.15)^2 = 0.59$ erg.

3.16



Force on unit area $= \sigma \cdot \frac{E}{2} = \frac{1}{8\pi}E^2 = 0.442$ dyne cm^{-2}

Force on entire plate $= 400 \times 0.442 = 177$ dynes

$F \times 3$ cm $= 530$ erg $=$ energy stored in field.

3.17 Let a be the radius of the outer sphere, ka that of the inner sphere. Then the capacitance C is (see Problem 3.10)
$$C = \frac{ka^2}{a - ka} = \frac{k}{1-k}\,a$$

If Q is the charge on the inner sphere the field E_0 at its surface is $Q/(ka)^2$. The energy stored in the capacitor is

$$U = \frac{1}{2}\frac{Q^2}{C} = \frac{k^4 a^4 E_0^2}{2\left(\frac{ka}{1-k}\right)} = \frac{k^3(1-k)}{2} E_0^2 a^3$$

For fixed a and E_0 this is a maximum if

$$\frac{d}{dk}(k^3 - k^4) = 3k^2 - 4k^3 = 0 \quad : \quad k = 3/4$$

For $k = 3/4$ $\quad U_{max} = \frac{27}{512} E_0^2 a^3$

3.18 We shall neglect edge fields on the assumption that the gap width s is much smaller than y and b. The charge Q on sheet A will be distributed over the area yb on both sides of the sheet. The field E on each side of the sheet A will be

$$E = 4\pi\underbrace{\left(\frac{Q}{2yb}\right)}_{\boxed{\sigma \text{ on each side of sheet}}} = 2\pi Q/yb$$

$V = \phi_A - \phi_B = Es = \dfrac{2\pi Qs}{yb}$. The stored energy

$U = \frac{1}{2} QV = \dfrac{\pi Q^2 s}{yb}$. If A is allowed to move

downward, increasing y by Δy while Q remains constant, the stored energy decreases:

$$-\nabla U = \frac{\pi Q^2 s}{b}\,\frac{\Delta y}{y^2}$$ and the difference equals the

work $F\Delta y$ done by the force F pulling down on something else. From which we find:

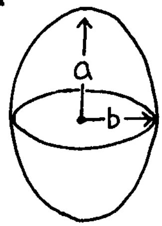
$$F = \frac{\pi Q^2 s}{b y^2} = \frac{V^2 b}{4\pi s}$$

[Note that Q remains constant here as y increases, whereas V does not. If sheet A were connected to a constant voltage source, Q would increase with increasing y and U also would increase. But in that case the voltage source would supply energy for both the increase in U and the external work. The force F would be exactly the same. See the solution of Problem 3.23 for a more complete discussion of this point.]

3.19 The kinetic energy of the ion, as it moves along the circular path is $\frac{1}{2}mv^2 = eV_0$. To keep it on the circular path of radius r_0 requires a force mv^2/r_0, to be provided by a radial electric field $E_r = 2V_0/r_0$. We know that the field between concentric cylinders is proportional to $1/r$ so it must be $E_r = 2V_0/r$, which has the required value for $r = r_0$. We want to set $V = 0$ at $r = r_0$, so the ion will experience no further acceleration. Then the potentials of the two electrodes must be:

$$V_A = \int_{r_0}^{a} \frac{2V_0}{r}\,dr = 2V_0 \ln\frac{a}{r_0} \quad \text{and} \quad V_B = \int_{r_0}^{b} \frac{2V_0}{r}\,dr = 2V_0 \ln\frac{b}{r_0}.$$

3.20



$$C = \frac{2a\epsilon}{\ln\left(\frac{1+\epsilon}{1-\epsilon}\right)} \qquad \epsilon = \sqrt{1 - b^2/a^2}$$

called the "eccentricity"

If $b \approx a$, $\epsilon \ll 1$ and $\ln(1+\epsilon) \approx \epsilon$

Then $C \approx 2a\epsilon/2\epsilon = a$, the value of C for the sphere of radius a.

Let C_0 be the capacitance of a sphere of unit radius, $a = b = 1$, and C the capacitance of a prolate spheroid of equal volume, that is, with $b = 1/\sqrt{a}$.

Then $$\frac{C}{C_0} = \frac{2a\epsilon}{\ln\left(\frac{1+\epsilon}{1-\epsilon}\right)} \quad \text{with} \quad \epsilon = \sqrt{1 - \frac{1}{a^3}}$$

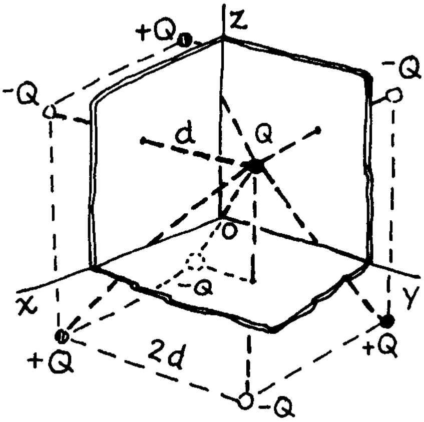
To see whether C is greater or less than C_0 it would suffice to calculate one case with $a > 1$, but it is interesting to consider both large and small eccentricity:

a:	1.01	1.1	2	20
C/C_0:	1.0000198	1.00182	1.1005	3.85

For large a, $C/C_0 \approx 2a/\ln(4a^3)$

A long wire can have a capacitance very much greater than that of a sphere of equal volume. Consider a spheroid with $a = 1$ km and $b = 1$ mm. Its volume is that of a sphere of radius 10 cm but its capacitance is that of a sphere of radius 69 meters.

3.21 "Image" charges are at the other seven corners of the cube of side $2d$. From the symmetry it is obvious that the direction of the resultant force on Q is parallel to the "body diagonal" OQ. Hence we need compute only force components in that direction:



(a) 3 negative charges at distance $2d$; direction makes angle $\cos^{-1}\frac{1}{\sqrt{3}}$ with OQ:

$$F = 3\cdot\frac{Q^2}{(2d)^2}\cdot\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{4}\frac{Q^2}{d^2} \quad \text{(toward origin)}$$

(b) 3 positive charges at distance $2\sqrt{2}d$; direction makes angle $\cos^{-1}\sqrt{\frac{2}{3}}$ with OQ:

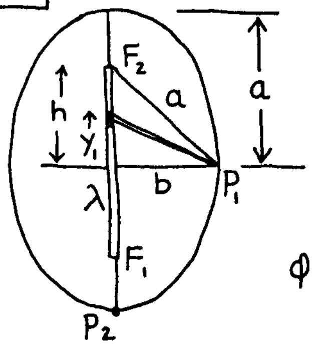
$$F = 3\,\frac{Q^2}{(2\sqrt{2}d)^2}\sqrt{\frac{2}{3}} = \frac{\sqrt{3}}{4\sqrt{2}}\,\frac{Q^2}{d^2} \quad \text{(away from origin)}$$

(c) 1 negative charge at opposite corner; distance $\sqrt{3}\cdot 2d$:

$$F = \frac{Q^2}{(2\sqrt{3}d)^2} = \frac{Q^2}{12\,d^2} \quad \text{(toward origin)}$$

Total force on $Q = \frac{Q^2}{d^2}\left(\frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4\sqrt{2}} + \frac{1}{12}\right) = \frac{0.210\,Q^2}{d^2}$ toward origin.

3.22



From the geometry of the ellipse: $h^2 + b^2 = a^2$. The rod extends from one focus to the other and has uniform linear charge density λ. The potential at P_1 is

$$\phi = \int_{-h}^{h} \frac{\lambda\, dy}{\sqrt{y^2 + b^2}} = \lambda \ln \frac{\sqrt{h^2+b^2} + h}{\sqrt{h^2+b^2} - h}$$

$$= \lambda \ln \frac{a + \sqrt{a^2-b^2}}{a - \sqrt{a^2-b^2}} = \lambda \ln \frac{1+\epsilon}{1-\epsilon}, \text{ where } \epsilon = \sqrt{1 - \frac{b^2}{a^2}}$$

The total charge inside the surface is

$$Q = 2\lambda h = 2\lambda\sqrt{a^2-b^2} = 2\lambda a\epsilon$$

The capacitance $C = \frac{Q}{\phi} = \frac{2\cancel{\lambda} a\epsilon}{\cancel{\lambda} \ln \frac{1+\epsilon}{1-\epsilon}}$ Q.E.D.

We could have used any other point, such as P_2, with the same result.

3.23 (a) Neglecting end effects, we assume charge Q is uniformly distributed along cylinder. Then field E is that of an axial line charge of density $\lambda = Q/L$. That is, $E = \frac{2\lambda}{r} = \frac{2Q}{rL}$. The potential difference ϕ_{ab} is:

$$\phi_{ab} = \int_a^b \frac{2Q}{L}\frac{dr}{r} = \frac{2Q}{L}\ln\frac{b}{a}.$$

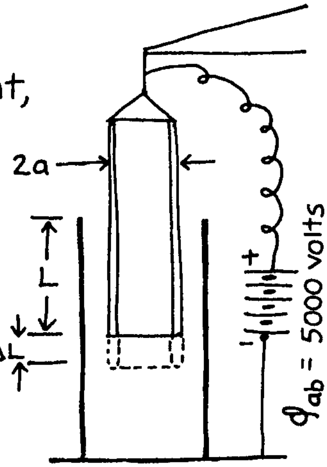
Since $Q = C\phi_{ab}$, the capacitance C is given by $C = \frac{L}{2\ln(\frac{b}{a})}$. If $b-a \ll a$,

$\ln(\frac{b}{a}) = \ln(1 + \frac{b-a}{a}) \approx \frac{b-a}{a}$ Then $C \approx aL/2(b-a)$

But $2\pi aL$ is the area of the inner cylinder and $b-a$ is the plate separation, so this is just what we would get using the formula for the parallel plate capacitor: $C = \frac{1}{4\pi}\frac{\text{area}}{\text{separation}}$.

(b) Consider the energy changes involved in a downward displacement, by ΔL, of the inner cylinder. The capacitance increases from

$$C = \frac{L}{2 \ln \frac{b}{a}} \quad \text{to} \quad C + \Delta C = \frac{L + \Delta L}{2 \ln \frac{b}{a}} .$$



With constant potential difference φ_{ab}, the stored electrical energy $\frac{1}{2} C \varphi_{ab}^2$ increases by $\frac{\Delta C}{2} \varphi_{ab}^2$.

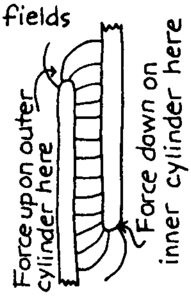
At the same time, an amount of charge $\Delta Q = \varphi_{ab} \Delta C$ flows into the capacitor. The battery thereby does work, in amount $\varphi_{ab} \Delta Q = \varphi_{ab}^2 \Delta C$. This is twice the increase in stored energy in the field. The difference is the work done against the external force F which balances the electrical attraction of the cylinders. That is:

$$\underset{\boxed{\text{work done by battery}}}{\varphi_{ab}^2 \Delta C} \;\begin{cases} \nearrow \; \frac{1}{2} \varphi_{ab}^2 \Delta C \;\; \boxed{\text{increase in field energy}} \\ \searrow \; F \Delta L \;\; \boxed{\text{work done against external force}} \end{cases}$$

Hence $F \Delta L = \frac{1}{2} \varphi_{ab}^2 \Delta C$ from which we get $F = \frac{1}{2} \varphi_{ab}^2 \frac{\Delta C}{\Delta L}$. This is a quite general formula. In the case at hand $\frac{\Delta C}{\Delta L} = \frac{1}{2 \ln \frac{b}{a}}$. With $b/a = 3/2$ and $\varphi_{ab} = 5000$ volts $= 16.7$ statvolts

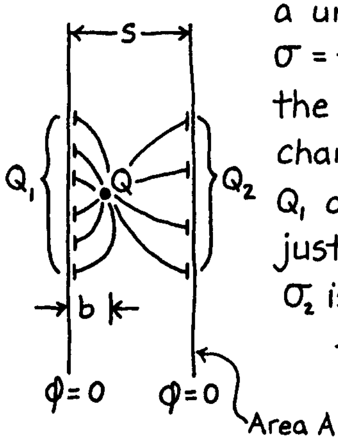
we get $F = \frac{1}{2} (16.7)^2 \frac{1}{.812} = 171$ dynes

Note: End effects do not spoil the accuracy of this simple result, for the downward displacement of the inner cylinder leaves the end fields themselves unaltered; it simply lengthens the region where the field is nicely cylindrical. That is, we can safely ignore the end fields in calculating dC/dL, even when they would seriously affect C itself. Nevertheless, the origin of the force just calculated lies in the very end fields that our method permits us to ignore, for it is only at the ends that we find vertical components of the electric field.

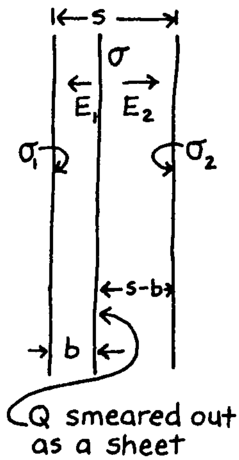


$\boxed{3.24}$

Imagine charge Q smeared out in a uniform sheet of density $\sigma = \frac{Q}{A}$. This cannot affect the total amount of induced charge on each plate, Q_1 and Q_2. In fact σ_1 is just Q_1 smeared out and σ_2 is Q_2 smeared out.



Thus : $\dfrac{Q_1}{Q_2} = \dfrac{\sigma_1}{\sigma_2}$



But from Gauss's law, $\sigma_1 = E_1/4\pi$ and $\sigma_2 = E_2/4\pi$. Also, because the two plates are at the same potential, $E_1 b = E_2(s-b)$. Hence :

$$\frac{Q_1}{Q_2} = \frac{\sigma_1}{\sigma_2} = \frac{E_1}{E_2} = \frac{s-b}{b}.$$

Therefore, with due regard to sign, we have : $Q_1 = -Q\left(\frac{s-b}{s}\right) \quad Q_2 = -Q\frac{b}{s}$.

$\boxed{3.25}$

$$(\text{Potential})^2 \sim \left(\frac{\text{charge}}{\text{distance}}\right)^2 \sim \frac{\text{charge}^2}{\text{distance}^2} \sim \text{force}$$

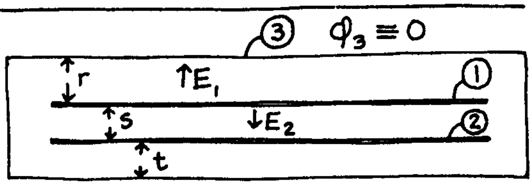
$$(1 \text{ statvolt})^2 = \frac{(1 \text{ esu})^2}{(1 \text{ cm})^2} = 1 \text{ dyne}$$

$$(1 \text{ megavolt})^2 = \left(\frac{10^6}{300} \text{ statvolt}\right)^2 = \frac{10^8}{9} \text{ dynes}$$

$$= \frac{10^8}{9 \times 453 \times 980} \text{ pounds} = 25 \text{ pounds}$$

3.26

$$Q_1 = C_{11}\phi_1 + C_{12}\phi_2$$
$$Q_2 = C_{21}\phi_1 + C_{22}\phi_2$$



Make $\phi_2 = 0$ by connecting ② to ③ Then $E_1 = \phi_1/r$ and $E_2 = \phi_1/s$ $\quad 4\pi Q_1 = AE_1 + AE_2$ $= A\phi_1\left(\frac{1}{r} + \frac{1}{s}\right)$ Hence $C_{11} = \frac{A}{4\pi}\left(\frac{1}{r} + \frac{1}{s}\right)$

By similar argument, $C_{22} = \frac{A}{4\pi}\left(\frac{1}{t} + \frac{1}{s}\right)$

When $\phi_2 = 0$, $Q_2 = C_{21}\phi_1$, and here $Q_2 = -\frac{E_2}{4\pi}A = -\frac{A\phi_1}{4\pi s}$.

Thus $C_{21} = -\frac{A}{4\pi s}$. Clearly a similar argument with 1's and 2's interchanged will give $C_{12} = -\frac{A}{4\pi s}$.

3.27 $Q_1 = C_{11}\phi_1 + C_{12}\phi_2 \qquad Q_2 = C_{21}\phi_1 + C_{22}\phi_2$

Process (a): Step 1: $\phi_2 = 0$: ϕ_1 goes from 0 to ϕ_{1f}.

$dQ_1 = C_{11}d\phi_1$ Work $= \int_0^{\phi_{1f}} \phi_1 C_{11}\, d\phi_1 = \frac{1}{2}C_{11}\phi_{1f}^2$

Step 2: $\phi_1 = \phi_{1f}$; ϕ_2 goes from 0 to ϕ_{2f}. $dQ_1 = C_{12}d\phi_2$

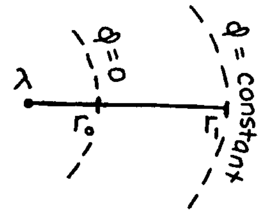
$dQ_2 = C_{22}d\phi_2$ Work $= \int_0^{\phi_{2f}} (\phi_{1f}C_{12} + \phi_2 C_{22})\,d\phi_2 = C_{12}\phi_{1f}\phi_{2f} + \frac{1}{2}C_{22}\phi_{2f}^2$

Total work done $= \frac{1}{2}C_{11}\phi_{1f}^2 + \frac{1}{2}C_{22}\phi_{2f}^2 + C_{12}\phi_{1f}\phi_{2f}$

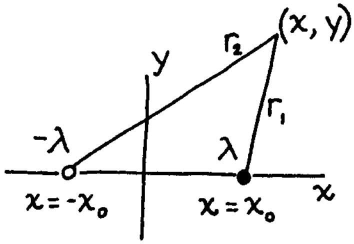
Process (b): In process (b) 1's and 2's are interchanged throughout. Hence the expression for total work done must be $\frac{1}{2}C_{11}\phi_{1f}^2 + \frac{1}{2}C_{22}\phi_{2f}^2 + C_{21}\phi_{1f}\phi_{2f}$. Since the final state is the same and there is no dissipation of energy in the charging process, the work done must be the same, if energy is to be conserved.

It follows that $C_{12} = C_{21}$.

3.28 Consider the potential ϕ for a single line charge of density λ, in esu/cm. Let $\phi = 0$ at $r = r_0$. Then since $\underset{\sim}{E} = \frac{2\lambda}{r}\hat{\underset{\sim}{r}}$, the potential at a distance r_1 from the line is:



$$\phi = -\int_{r_0}^{r_1} \frac{2\lambda}{r}\,dr = 2\lambda \ln \frac{r_0}{r_1}$$



With two line charges located as shown,

$$\phi = 2\lambda \ln \frac{x_0}{r_1} + 2(-\lambda) \ln \frac{x_0}{r_2}$$

$$\phi = 2\lambda \ln \frac{r_2}{r_1}$$

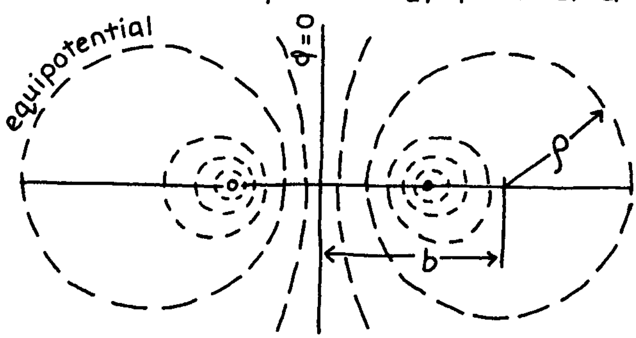
Now to show that $\frac{r_2}{r_1}$ = constant defines a circle:

Let $\frac{r_2}{r_1} = k > 1$ $\quad r_2^2 = (x + x_0)^2 + y^2 \quad r_1^2 = (x - x_0)^2 + y^2$

$(x + x_0)^2 + y^2 = k^2[(x - x_0)^2 + y^2]$ which reduces to

$$x^2 + y^2 - 2\frac{k^2+1}{k^2-1} x_0 x + x_0^2 = 0$$

A circle of radius ρ, centered at $x = b$, $y = 0$ has the equation $x^2 + y^2 - 2bx + b^2 - \rho^2 = 0$ which is the same as the preceding equation if we take $b = \frac{k^2+1}{k^2-1} x_0$ and $\rho^2 = b^2 - x_0^2$. This all applies in the right half-plane, where $r_2/r_1 > 1$. In the left half-plane $r_2/r_1 < 1$ and $\phi < 0$.



$\boxed{3.29}$ $$\phi(x_0+\delta, y_0, z_0) = \phi(x_0,y_0,z_0) + \delta\frac{\partial\phi}{\partial x} + \frac{\delta^2}{2}\frac{\partial^2\phi}{\partial x^2} + \frac{\delta^3}{6}\frac{\partial^3\phi}{\partial x^3} + \dots$$

$$\phi(x_0-\delta, y_0, z_0) = \phi(x_0,y_0,z_0) - \delta\frac{\partial\phi}{\partial x} + \frac{\delta^2}{2}\frac{\partial^2\phi}{\partial x^2} - \frac{\delta^3}{6}\frac{\partial^3\phi}{\partial x^3} + \dots$$

$$\phi(x_0, y_0+\delta, z_0) = \phi(x_0,y_0,z_0) + \delta\frac{\partial\phi}{\partial y} + \frac{\delta^2}{2}\frac{\partial^2\phi}{\partial y^2} + \frac{\delta^3}{6}\frac{\partial^3\phi}{\partial y^3} + \dots$$

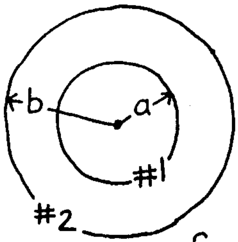
etc.

The average of the six values of ϕ is

$$\bar{\phi} = \frac{1}{6}\left[6\phi_0(x_0,y_0,z_0) + \delta^2\left(\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} + \frac{\partial^2\phi}{\partial z^2}\right) + \delta^4(\cdots) + \cdots\right]$$

If $\nabla^2\phi = 0$, we are left with $\bar{\phi} = \phi(x_0,y_0,z_0) + \delta^4(\cdots) + \cdots$

$\boxed{3.32}$



$$Q_1 = C_{11}\phi_1 + C_{12}\phi_2 \qquad (1)$$

$$Q_2 = C_{21}\phi_1 + C_{22}\phi_2 \qquad (2)$$

For the spherical capacitor we found in Problem 3.10 $C = ab/(b-a)$.

The charge Q on the inner sphere is :

$$Q_1 = \frac{ab}{b-a}(\phi_1 - \phi_2) = \frac{ab}{b-a}\phi_1 - \frac{ab}{b-a}\phi_2$$

Comparing with (1) we see that $C_{11} = \frac{ab}{b-a}$; $C_{12} = \frac{-ab}{b-a}$

The total charge on #2 is the charge $-Q$, <u>inside</u>, plus $b\phi_2$ on the <u>outside</u>. That is,

$$Q_2 = -\frac{ab}{b-a}(\phi_1 - \phi_2) + b_2\phi_2 = -\frac{ab}{b-a}\phi_1 + \left(\frac{ab}{b-a} + b\right)\phi_2$$

Comparing with (2) we see that $C_{21} = C_{12}$ as expected, and that $C_{22} = \frac{ab}{b-a} + b = \frac{b^2}{b-a}$