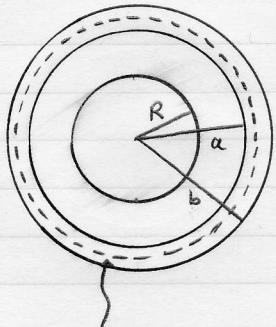


Phys 543

Homework Set 3

Problem 2.35

a)



Gaussian surface with
radius $a < r < b$

The metal sphere radius R is carrying the charge q ; and the electric field inside metals vanish. The charge will be distributed evenly to the surface of the sphere

$$\sigma_R = \frac{q}{4\pi R^2}$$

If we consider the Gaussian surface in the above picture, the flux through it is vanishing since the electric field inside the metals vanish as stated above!

$$\oint \vec{E} \cdot d\vec{s} = 0 \\ = \frac{1}{\epsilon_0} Q_{\text{enclosed}}$$

i.e. the charges on the outer sphere should be distributed such that as long as $a < r < b$, $Q_{\text{enclosed}} = 0$. The total charge distributed on the inner surface is $-q$.

$$\sigma_a = \frac{-q}{4\pi a^2}$$

We are given that the shell does not carry a net charge; i.e. the charge q must be distributed on the outer surface of the shell:

$$\sigma_b = \frac{q}{4\pi b^2}$$

$$b) V(b) - V(a) = - \int_a^b \vec{E} \cdot d\vec{l}$$

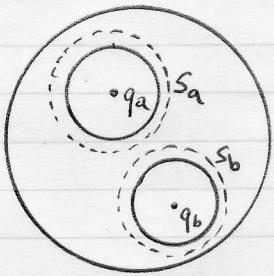
$$\vec{E} = \begin{cases} \frac{q}{4\pi\epsilon_0 r^2} & \text{for } r > b \\ 0 & \text{for } b > r > a \\ \frac{q}{4\pi\epsilon_0 r^2} & \text{for } a > r > R \\ 0 & \text{for } r < a \end{cases} \quad (\text{by Gauss' law})$$

$$\begin{aligned} V(0) - V(\infty) &= - \int_{\infty}^b \frac{q}{4\pi\epsilon_0 r^2} dr - \int_a^R \frac{q}{4\pi\epsilon_0 r^2} dr \\ &= \frac{q}{4\pi\epsilon_0} \left\{ \frac{1}{r} \Big|_{\infty}^b + \frac{1}{r} \Big|_a^R \right\} \\ &= \frac{q}{4\pi\epsilon_0} \left(\frac{1}{b} + \frac{1}{R} - \frac{1}{a} \right) \end{aligned}$$

c) Since the outer surface of the shell is in contact with the ground, $\sigma_b \rightarrow 0$, i.e. $\vec{E} = 0$ for $r > a$, and $V(a) = V(b) = 0$

$$V(0) - V(a) = - \int_a^R \frac{q}{4\pi\epsilon_0 r^2} dr = \frac{q}{4\pi\epsilon_0} \left(\frac{1}{R} - \frac{1}{a} \right)$$

Problem 2.36



a) Since the sphere is conducting, the electric field should vanish inside. Hence,

$$\oint_{S_a} \vec{E} \cdot d\vec{s} = \oint_{S_b} \vec{E} \cdot d\vec{s} = 0$$

the enclosed charge in S_a and S_b is zero. Same arguments as in the previous problem hold:

$$r_a = \frac{-q_a}{4\pi r_a^2} \quad \text{and} \quad r_b = \frac{-q_b}{4\pi r_b^2}$$

If we consider a Gaussian surface including both cavities, we end up with

$$r_R = \frac{-(q_a + q_b)}{4\pi R^2}$$

b) Consider a Gaussian sphere of radius $r > R$

$$\oint \vec{E} \cdot d\vec{s} = \frac{1}{\epsilon_0} Q_{\text{enclosed}}$$

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{q_a + q_b}{r^2} \hat{r}$$

\hat{r} : radial vector from the center of the large sphere

c) If \hat{r}_a and \hat{r}_b are the radial vector from the centers of the cavity a and cavity b, respectively, applying Gauss' theorem tells us

$$\vec{E}_a = \frac{1}{4\pi\epsilon_0} \frac{q_a}{r_a^2} \hat{r}_a \quad \text{and} \quad \vec{E}_b = \frac{1}{4\pi\epsilon_0} \frac{q_b}{r_b^2} \hat{r}_b$$

- d) Since the electric field created, say by q_a , at the point where q_b is located is zero, the force between these two charges is zero.
- e) σ_R changes (but not σ_a or σ_b), \vec{E} outside changes (but not \vec{E}_a and \vec{E}_b); force between the charges is still zero.

Problem 3.2

A stable equilibrium is a point of local minimum in the potential energy. Here the potential energy is qV . But we know that Laplace's equation allows no local minima for V . What looks like a minimum, in the figure, must in fact be a saddle point, and the box "leaks" through the center of each face.

Problem 3.3

Laplace's equation in spherical coordinates, for V dependent only on r , reads:

$$\nabla^2 V = \frac{1}{r^2} \frac{d}{dr} \left(r^2 \frac{dV}{dr} \right) = 0 \Rightarrow r^2 \frac{dV}{dr} = c$$

$$V = -\frac{c}{r} + k$$

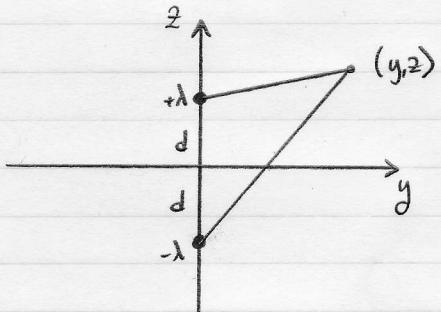
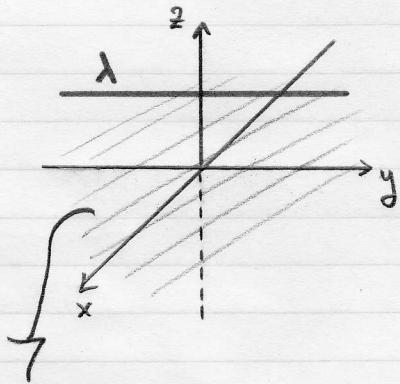
In cylindrical coordinates:

$$\nabla^2 V = \frac{1}{s} \frac{d}{ds} \left(s \frac{dV}{ds} \right) = 0 \Rightarrow s \frac{dV}{ds} = c$$

$$V = c \ln s + k$$

Problem 3.9

a) Apply the method of images:



$$V(y, z) = \frac{2\lambda}{4\pi\epsilon_0} \ln(s_-/s_+) = \frac{\lambda}{4\pi\epsilon_0} \ln(s_-^2/s_+^2)$$

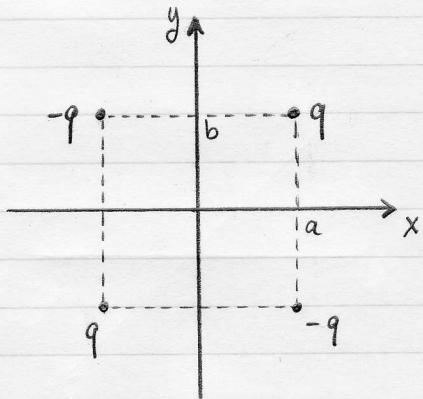
$$= \frac{\lambda}{4\pi\epsilon_0} \ln \left\{ \frac{y^2 + (z+d)^2}{y^2 + (z-d)^2} \right\}$$

$$b) \sigma = -\epsilon_0 \frac{\partial V}{\partial n} \Rightarrow \frac{\partial V}{\partial n} = \frac{\partial V}{\partial z} \Big|_{z=0}$$

$$\sigma(y) = -\epsilon_0 \frac{\lambda}{4\pi\epsilon_0} \left\{ \frac{1}{y^2 + (z+d)^2} 2(z+d) - \frac{1}{y^2 + (z-d)^2} 2(z-d) \right\} \Big|_{z=0}$$

$$= -\frac{\lambda d}{\pi(y^2 + d^2)}$$

Problem 3.10



$$V(x,y) = \frac{q}{4\pi\epsilon_0} \left\{ \frac{1}{\sqrt{(x-a)^2 + (y-b)^2 + z^2}} + \frac{1}{\sqrt{(x+a)^2 + (y+b)^2 + z^2}} - \frac{1}{\sqrt{(x+a)^2 + (y-b)^2 + z^2}} \right. \\ \left. - \frac{1}{\sqrt{(x-a)^2 + (y+b)^2 + z^2}} \right\}$$

For this to work, θ must be an integer divisor of 180° . Thus $180^\circ, 90^\circ, 60^\circ, 45^\circ$, etc. are OK, but no others. The reason this doesn't work for arbitrary angles is that you are eventually forced to place an image charge within the original region of interest, that's not allowed - all images must go outside the region, or you're no longer dealing with the same problem.