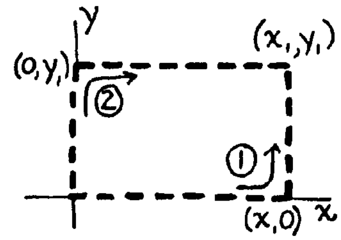
2.1

Path ①: $$\int_{(0,0)}^{(x_1,y_1)} \underset{\sim}{E}\cdot d\underset{\sim}{s} = \int_0^{x_1} E_x(x,0)\,dx + \int_0^{y_1} E_y(x_1,y)\,dy$$



$$= 0 + \int_0^{y_1} (3x_1^2 - 3y^2)\,dy = 3x_1^2 y_1 - y_1^3$$

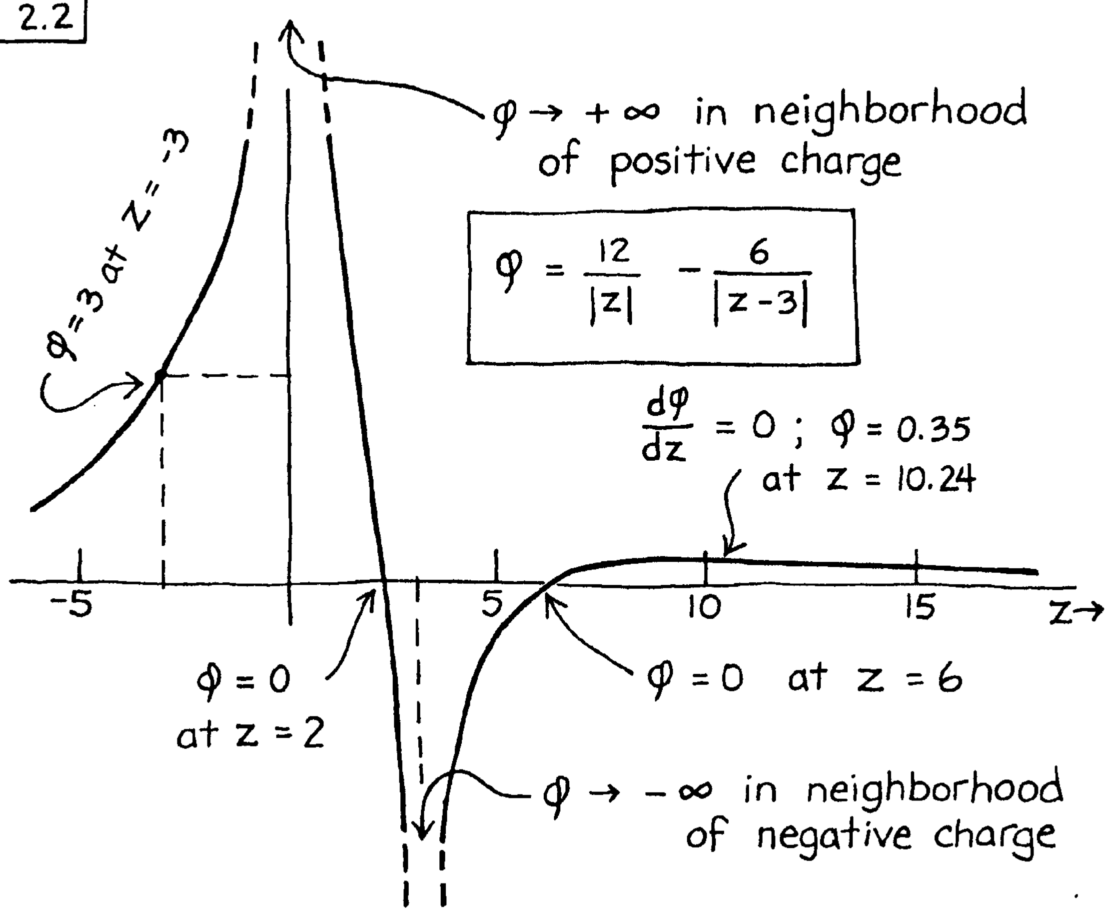
Path ②: $$\int_{0,0}^{x,y_1} \underset{\sim}{E}\cdot d\underset{\sim}{s} = \int_0^{y_1} E_y(0,y)\,dy + \int_0^{x_1} E_x(x,y_1)\,dx$$

$$= \int_0^{y_1} -3y^2dy + \int_0^{x_1} 6xy_1\,dx = -y_1^3 + 3x_1^2 y_1$$

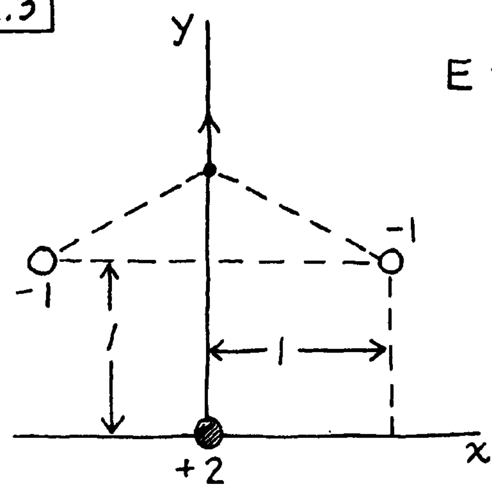
same result

The electric potential φ, if taken as zero at $(0,0)$, is just the negative of this, since we define φ by $-\int \underset{\sim}{E}\cdot d\underset{\sim}{s}$, or $\underset{\sim}{E} = -\nabla\varphi$. That is, $\varphi = y^3 - 3x^2y$

2.2



2.3



$$E = \frac{2}{y^2} - \frac{2}{(y-1)^2+1} \times \frac{y-1}{\sqrt{(y-1)^2+1}}$$

$$= 0 \quad \text{if} \quad \frac{1}{y^4} = \frac{(y-1)^2}{(y^2-2y+2)^3}$$

$$y^4 = \frac{(y^2-2y+2)^3}{(y-1)^2}$$

Solve by iteration: We know y must be >1, so let's start with $y=2$ on the right. After 6 iterations we converge to $y = 1.6207\ldots$ with no further change in the 5th figure.

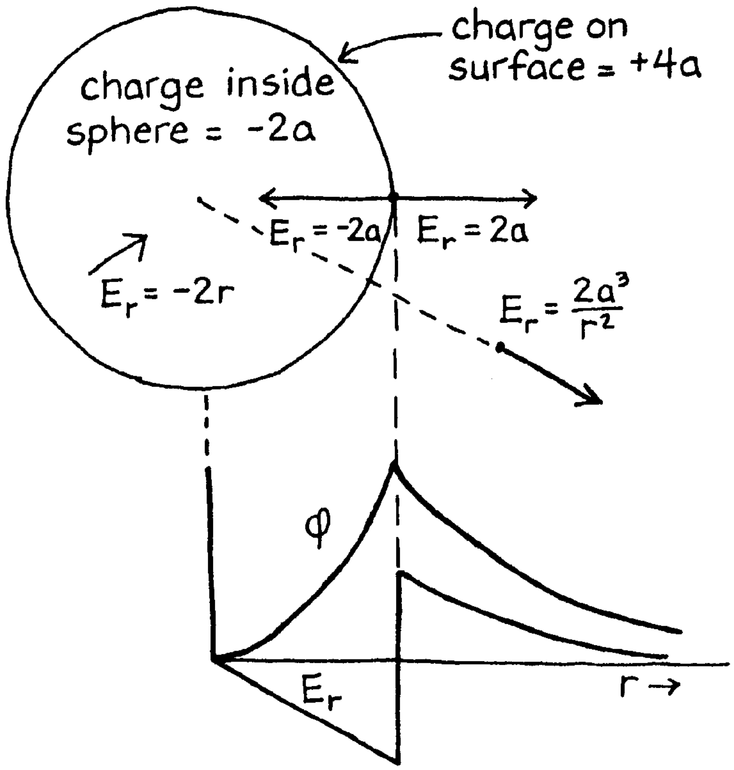
2.4 This is the potential of a spherical charge distribution, more briefly described by

$$\phi = r^2 \text{ for } r < a; \quad \phi = -a^2 + 2a^3/r \text{ for } r > a.$$

For $r = (x^2+y^2+z^2)^{1/2} < a$, $\nabla^2\phi = \frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} + \frac{\partial^2\phi}{\partial z^2}$

$= 2+2+2 = 6$. Using $\nabla^2\phi = -4\pi\rho$, we find $\rho = -3/2\pi$ for $r < a$. [In spherical polar coordinates

$\nabla^2\phi = \frac{1}{r}\frac{d}{dr}r\frac{d\phi}{dr}$ when ϕ is a function of r only. This gives the same answer.] Outside the sphere of uniform charge density ρ is zero, as we find by computing $\nabla^2(1/r)$, or just by recognizing $2a^3/r$ as the potential of point charge $2a^3$ at the origin. The electric field E_r at radius $r < a$ is that of a charge $(4\pi/3)r^3\rho$ divided by r^2: $E_r = (4\pi/3)\rho r = -2r$, $r < a$. This tells us there is a surface charge σ on the sphere: $4\pi\sigma = 2a - (-2a)$. $\sigma = a/\pi$. The total surface charge is $4\pi a^2\sigma$, or $4a^3$. This is positive and twice as large as the negative charge $-2a^3$ distributed through the interior of the sphere. Thus the external field of the sphere as a whole is that of a positive charge $2a^3$.



$\boxed{2.5}$ Assume the diameter is about 1 foot, 30 cm. 1000 volts is 3.3 statvolts. This is Q/r, where $r = 15$ cm. The charge Q is therefore 15×3.3 or -50 esu. The number of extra electrons per cm² is

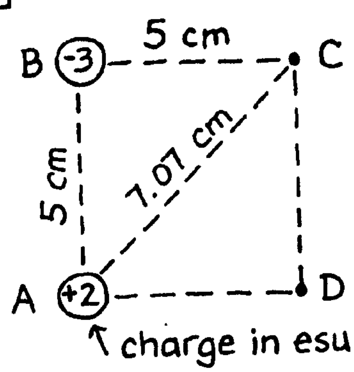
$$\frac{50}{4\pi \times 15^2 \times 4.8 \times 10^{-10}} = 3.7 \times 10^7$$

$\boxed{2.6}$ Earth radius $= 6 \times 10^6$ m ; $4\pi\epsilon_0 = 1.11 \times 10^{-10}$

$$E = \frac{Q^{\leftarrow \text{coulomb}}}{4\pi\epsilon_0 r^2} \qquad \phi = \frac{Q}{4\pi\epsilon_0 r} \qquad Q = 1 \text{ coulomb}$$

$$E = 2.5 \times 10^{-4} \text{ volt/m} \qquad \phi = 1500 \text{ volts}$$

2.7



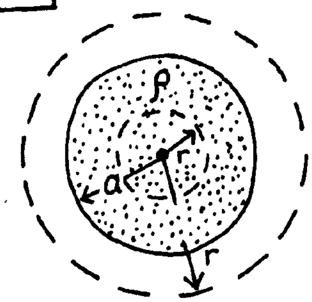
$$\Phi_C = -\frac{3}{5} + \frac{2}{7.07} = -0.317$$

$$\Phi_D = \frac{2}{5} - \frac{3}{7.07} = -0.024$$

$$\int_C^D \underset{\sim}{E} \cdot d\underset{\sim}{s} = \Phi_C - \Phi_D = -0.293$$

statvolts

2.8



(a) Consider a cylinder of unit length, of radius $r < a$. Charge contained is $\pi r^2 \rho$. Area of surface is $2\pi r$; flux through surface is $2\pi r E$. Gauss's law says:

$$\underbrace{2\pi r E}_{\text{flux}} = 4\pi \underbrace{(\pi r^2 \rho)}_{\text{charge}}, \text{ from which } E = 2\pi\rho r.$$

Considering a cylinder of radius $r > a$, which contains an amount of charge $\pi a^2 \rho$, we find

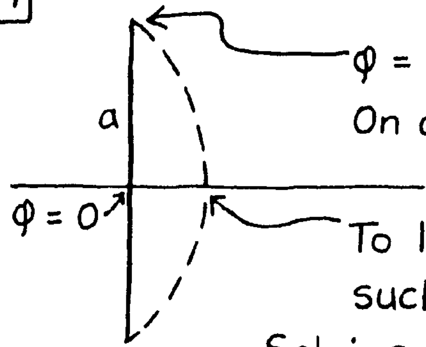
$$2\pi r E = 4\pi(\pi a^2 \rho), \text{ or } E = \frac{2\pi\rho a^2}{r}$$

(b) Take $\phi = 0$ at $r = 0$:

for $r < a$, $\phi = \int_0^r -2\pi\rho r' dr' = -\pi\rho r^2$

for $r > a$, $\phi = -\pi\rho a^2 - \int_a^r \frac{2\pi\rho a^2 dr'}{r'} = -\pi\rho a^2 - 2\pi\rho a^2 \ln\frac{r}{a}$

2.9



$\varphi = 4\sigma a$ (see Eq. 24, p. 53)

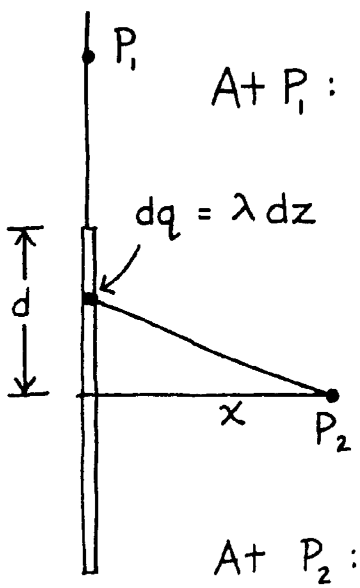
On axis, $\varphi = 2\pi\sigma\left[\sqrt{y^2+a^2} - y\right]$ (Eq. 20)

To locate this point, we must find y such that $4\sigma a = 2\pi\sigma\left[\sqrt{y^2+a^2} - y\right]$

Solving for y :

$$\left(\frac{4a}{2\pi} + y\right)^2 = y^2 + a^2 \quad \text{gives} \quad y = a\,\frac{1-\left(\frac{4}{2\pi}\right)^2}{\left(\frac{8}{2\pi}\right)} = 0.466\,a$$

2.10



$$\text{At } P_1:\ \varphi = \int_{-d}^{d} \frac{\lambda\,dz}{2d-z} = -\lambda \ln(2d-z)\Big]_{-d}^{d}$$

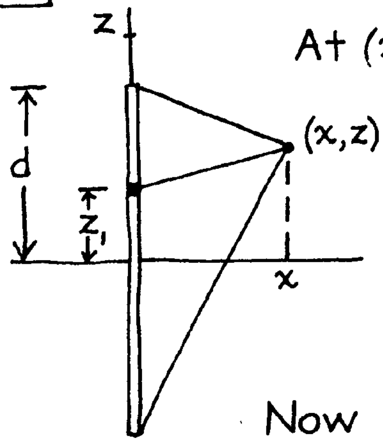
$$= -\lambda \ln\frac{d}{3d} = \lambda \ln 3$$

$$\text{At } P_2:\ \varphi = \int_{-d}^{d} \frac{\lambda\,dz}{\sqrt{x^2+z^2}} = \lambda \ln \frac{\sqrt{x^2+d^2}+d}{\sqrt{x^2+d^2}-d}$$

$$\text{Setting} \quad \frac{\sqrt{x^2+d^2}+d}{\sqrt{x^2+d^2}-d} = 3$$

gives $x = \sqrt{3}\,d$.

2.11



At (x,z): $\quad \phi = \int_{-d}^{d} \frac{\lambda\, dz_1}{\sqrt{x^2+(z-z_1)^2}}$

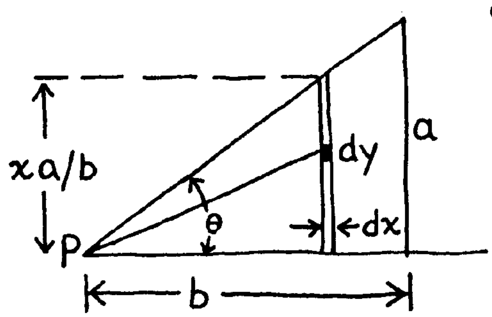
$$= \lambda \ln \frac{\sqrt{(-z+d)^2+x^2}\; -z+d}{\sqrt{(-z-d)^2+x^2}\; -z-d}$$

Now let $\frac{x^2}{a^2-d^2} + \frac{z^2}{a^2} = 1$ Then

$x^2 = \frac{(a^2-z^2)(a^2-d^2)}{a^2}$ and $x^2+(d-z)^2 = \frac{1}{a^2}(a^2-zd)^2$

$\phi = \lambda \ln \frac{a^2-zd+a(d-z)}{a^2+zd-a(d+z)} = \lambda \ln \frac{a+d}{a-d}$. That is, the value of ϕ is the same at every point x,z on the ellipse defined by $x^2/(a^2-d^2)+z^2/a^2=1$.

2.12



$$\phi_P = \sigma \int_0^b dx \underbrace{\int_0^{\frac{xa}{b}} \frac{dy}{\sqrt{x^2+y^2}}}$$

$$\ln\left(y+\sqrt{x^2+y^2}\right)\Big]_0^{\frac{xa}{b}}$$

$$= \ln\left(\frac{a}{b}+\sqrt{1+\frac{a^2}{b^2}}\right)$$

$$\phi_P = \sigma \ln\left(\frac{a}{b}+\sqrt{1+\frac{a^2}{b^2}}\right)\int_0^b dx = \sigma b \ln\left(\frac{a}{b}+\sqrt{1+\frac{a^2}{b^2}}\right)$$

$\frac{a}{b} + \frac{\sqrt{a^2+b^2}}{b} = \frac{\sin\theta+1}{\cos\theta} \qquad \phi_P = \sigma b \ln\left(\frac{1+\sin\theta}{\cos\theta}\right)$

2.13 $E_x = 6xy \quad E_y = 3x^2 - 3y^2 \quad E_z = 0$

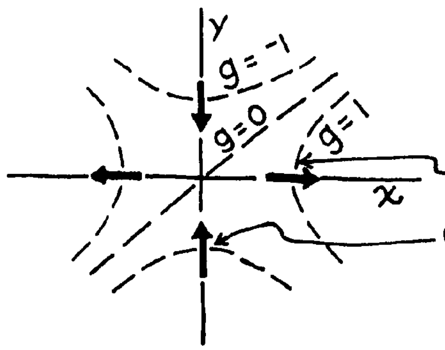
$$(\text{curl } \mathbf{E})_x = \frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} = 0 \qquad (\text{curl } \mathbf{E})_y = \frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} = 0$$

$$(\text{curl } \mathbf{E})_z = \frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = 6x - 6x = 0$$

$$\text{div } \mathbf{E} = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} = 6y - 6y = 0$$

2.14 $f(x,y) = x^2 + y^2 \quad \nabla^2 f = 2 + 2 \neq 0$

$g(x,y) = x^2 - y^2 \quad \nabla^2 g = 2 - 2 = 0$



$\nabla g = 2x\hat{\mathbf{x}} - 2y\hat{\mathbf{y}}$

at $(1,0)$ $\nabla g = +2\hat{\mathbf{x}}$

at $(0,-1)$ $\nabla g = +2\hat{\mathbf{y}}$

2.15 (a) $\mathbf{F} = \hat{\mathbf{x}}(x+y) + \hat{\mathbf{y}}(-x+y) + \hat{\mathbf{z}}(-2z)$

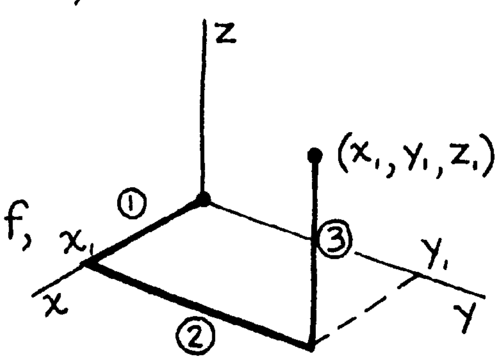
$$\nabla \times \mathbf{F} = \hat{\mathbf{x}}(0+0) + \hat{\mathbf{y}}(0+0) + \hat{\mathbf{z}}(-1-1) = -2\hat{\mathbf{z}}$$

$$\nabla \cdot \mathbf{F} = 1 + 1 - 2 = 0$$

(b) $\mathbf{G} = \hat{\mathbf{x}}(2y) + \hat{\mathbf{y}}(2x+3z) + \hat{\mathbf{z}}(3y)$

$$\nabla \times \mathbf{G} = \hat{\mathbf{x}}(3-3) + \hat{\mathbf{y}}(0+0) + \hat{\mathbf{z}}(2-2) = 0$$

$$\nabla \cdot \mathbf{G} = 0 + 0 + 0 = 0$$



Since $\nabla \times \mathbf{G} = 0$ there exists an f such that $\mathbf{G} = \nabla f$. To determine f, compute the line integral of $\mathbf{G}$ from a fixed point, say $(0,0,0)$, to a general point x_1, y_1, z_1, over any path. Using the path composed of lines ①, ② and ③ :

$$G\int_{(000)}^{(x_1 y_1 z_1)} \underset{\sim}{G}\cdot d\underset{\sim}{s} = \int_0^{x_1} G_x(x,0,0)dx + \int_0^{y_1} G_y(x_1,y_1,0)dy + \int_0^{z_1} G_z(x_1,y_1,z)dz$$

$$= \int_0^{x_1} 0\,dx + \int_0^{y_1} 2x_1 dy + \int_0^{z_1} 3y_1 dz = 2x_1y_1 + 3y_1z_1 = f$$

x_1, y_1, z_1 was a general point, so we can drop the subscripts and write : $f = 2xy + 3yz$.

To check that $\nabla f = \underset{\sim}{G}$:

$$\nabla f = 2y\hat{\underset{\sim}{x}} + (2x+3z)\hat{\underset{\sim}{y}} + 3y\hat{\underset{\sim}{z}} = \underset{\sim}{G}$$

(c) $\underset{\sim}{H} = \hat{\underset{\sim}{x}}(x^2-z^2) + \hat{\underset{\sim}{y}}(2) + \hat{\underset{\sim}{z}}(2xz)$

$$\nabla\times\underset{\sim}{H} = x(0+0) + \hat{\underset{\sim}{y}}(-2z-2z) + \hat{\underset{\sim}{z}}(0+0) = -4z\hat{\underset{\sim}{y}}$$

$$\nabla\cdot\underset{\sim}{H} = 2x + 0 + 2x = 4x$$

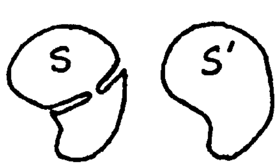
$\boxed{2.16}$ (a) $\nabla\cdot(\nabla\times\underset{\sim}{A}) = \frac{\partial}{\partial x}(\nabla\times\underset{\sim}{A})_x + \frac{\partial}{\partial y}(\nabla\times\underset{\sim}{A})_y + \frac{\partial}{\partial z}(\nabla\times\underset{\sim}{A})_z$

$$= \frac{\partial}{\partial x}\left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right) + \frac{\partial}{\partial y}\left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right) + \frac{\partial}{\partial z}\left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)$$

$$= \frac{\partial^2 A_z}{\partial x\partial y} - \frac{\partial^2 A_y}{\partial x\partial z} + \frac{\partial^2 A_x}{\partial y\partial z} - \frac{\partial^2 A_z}{\partial y\partial x} + \frac{\partial^2 A_y}{\partial z\partial x} - \frac{\partial^2 A_x}{\partial z\partial y} = 0,$$

because $\frac{\partial^2 A_z}{\partial x\partial y} = \frac{\partial^2 A_z}{\partial y\partial x}$ for any function A_z with continuous derivatives ; likewise for A_x and A_y .

(b) If $\underset{\sim}{A}$ is any vector field, $\int_C \underset{\sim}{A}\cdot d\underset{\sim}{s} \to 0$ over a curve such as C which consists of <u>adjacent</u> paths running in opposite directions. This path bounds the shaded surface S. Hence, by Stokes' theorem, $\int_S(\nabla\times\underset{\sim}{A})\cdot d\underset{\sim}{a} = 0$



But the slit is a negligible part of the whole <u>closed</u> surface S', so the same conclusion must apply to S' : $\int_{S'} (\nabla \times \underset{\sim}{A}) \cdot d\underset{\sim}{a} = 0$

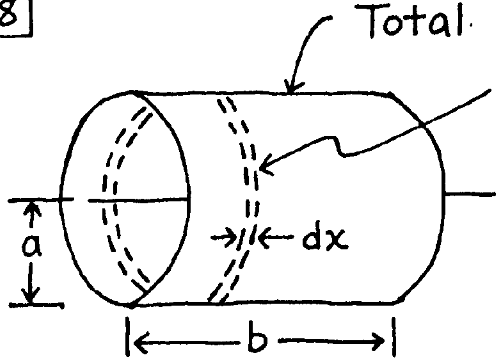
S' could have been any surface, so we have concluded that the vector function $(\nabla \times \underset{\sim}{A})$ has zero surface integral over any arbitrary closed surface. It follows from Gauss's theorem that $\operatorname{div}(\nabla \times \underset{\sim}{A}) = 0$ everywhere, for if the divergence were different from zero in any small region, a surface surrounding just that region would necessarily have a non-zero surface integral. (Of course, we can apply such an argument only to "well-behaved" functions.)

$\boxed{2.17}$ To show that $U = \frac{1}{8\pi}\int E^2 dv$ and $U = \frac{1}{2}\int \rho\varphi\, dv$ are equivalent, using the identity $\nabla(\varphi\nabla\varphi) = (\nabla\varphi)^2 + \varphi\nabla^2\varphi$. $E^2 = (\nabla\varphi)^2$ and $-4\pi\rho = \nabla^2\varphi$ (Poisson's Eq.) $(\nabla\varphi)^2 = -\varphi\nabla^2\varphi + \nabla(\varphi\nabla\varphi) = 4\pi\rho\varphi + \nabla(\varphi\nabla\varphi)$

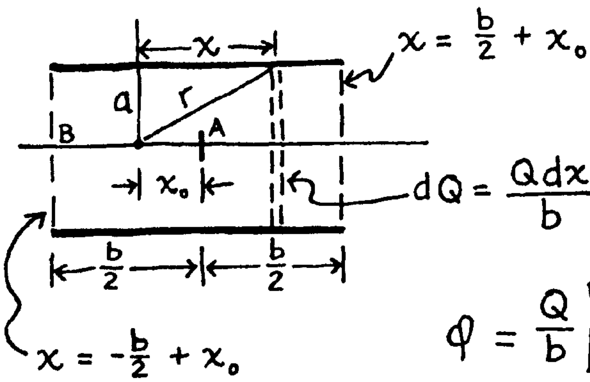
$$\frac{1}{8\pi}\int E^2 dv = \frac{1}{2}\int \rho\varphi\, dv + \frac{1}{8\pi}\int \nabla(\varphi\nabla\varphi)\, dv$$

The last integral is the volume integral of the divergence of the vector $\varphi\nabla\varphi$, or $-\varphi\underset{\sim}{E}$. We can replace it with a surface integral over an enclosing surface S. Suppose all sources lie within a finite region, say a sphere of radius r. Then let S be a gigantic sphere of radius $R \gg r$. As $R \to \infty$ E will vanish at least as fast as $1/R^2$ and φ will vanish at least as fast as $1/R$. So the surface integral over S will vanish as $R \to \infty$ and the equivalence is proved. (But if the sources were not confined to a finite region we could not be sure that any of these integrals would converge when extended over all space.)

2.18



Total charge Q uniformly distributed. Charge in ring of width dx $= dQ = Q\frac{dx}{b}$. All charge in such a ring is equidistant from a point on the axis, so the ring is a convenient charge element to use in computing the potential at axial points. Let's find potential at a general axial point, distance x_0 from mid-point. Locate origin at the point in question:



$$\phi = \int \frac{dQ}{r} = \int_{-\frac{b}{2}+x_0}^{\frac{b}{2}+x_0} \left(\frac{Q\,dx}{b}\right)\frac{1}{\sqrt{a^2+x^2}}$$

$$\phi = \frac{Q}{b}\left[\ln\left(\sqrt{a^2+x^2} + x\right)\right]_{-\frac{b}{2}+x_0}^{\frac{b}{2}+x_0}$$

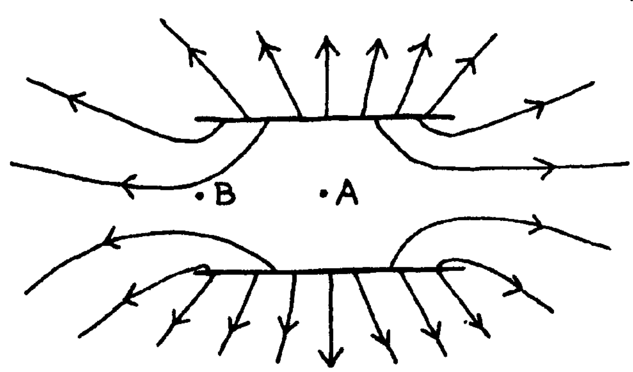
For potential at mid-point, set $x_0 = 0$:

$$\phi_A = \frac{Q}{b}\ln\frac{\sqrt{a^2+b^2/4} + b/2}{\sqrt{a^2+b^2/4} - b/2}$$

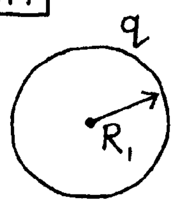
For potential at end, set $x_0 = \frac{b}{2}$:

$$\phi_B = \frac{Q}{b}\ln\frac{\sqrt{a^2+b^2} + b}{a}$$

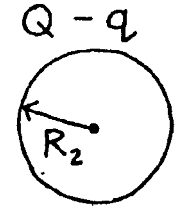
Then $$\phi_A - \phi_B = \frac{Q}{b}\ln\frac{a\left(\sqrt{a^2+b^2/4} + b/2\right)}{\left(\sqrt{a^2+b^2/4} - b/2\right)\left(\sqrt{a^2+b^2} + b\right)}$$



2.19



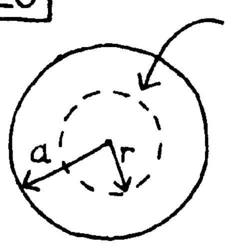
$$U = \frac{q^2}{2R_1} + \frac{(Q-q)^2}{2R_2}$$



$$\frac{dU}{dq} = \frac{q}{R_1} - \frac{(Q-q)}{R_2}$$

This must vanish for an extremum in U. But q/R_1 is just the <u>potential</u> ϕ_1 of that sphere and $(Q-q)/R_2$ is the potential ϕ_2 of the other sphere. So the condition can be expressed as equality of potential. It is easy to see that the extremum is a <u>minimum</u> in U, not a maximum: if $R_1 = R_2$, equal division of charge involves half as much energy as piling all of Q on one sphere.

2.20



total charge Q uniformly distributed.

charge inside radius $r = Q\dfrac{r^3}{a^3}$

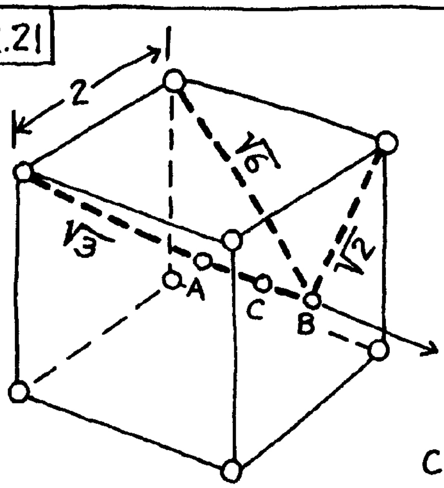
$E_r = Qr/a^3$, $r \leq a$ $\quad E_r = \dfrac{Q}{r^2}$, $r > a$

$$\left.\begin{aligned} \int_0^a E_r\,dr &= \phi(0) - \phi(a) = \frac{Q}{a^3}\int_0^a r\,dr = \frac{Q}{2a} \\ \int_a^\infty E_r\,dr &= \phi(a) - \phi(\infty) = Q\int_a^\infty \frac{dr}{r^2} = \frac{Q}{a} \end{aligned}\right\} \phi(0) - \phi(\infty) = \frac{3Q}{2a}$$

$$\frac{3Q}{2a} = \frac{3 \times 79 \times 4.8 \times 10^{-10}}{2 \times 6 \times 10^{-13}} = 9.5 \times 10^4 \text{ statvolts}$$

$$= 28.5 \text{ megavolts}$$

2.21



$$\phi_A = \frac{8}{\sqrt{3}} = 4.6188$$

$$\phi_B = \frac{4}{\sqrt{2}} + \frac{4}{\sqrt{6}} = 4.4614$$

Clearly B is below A and it is certainly downhill from there on out, for the fields of all 8 charges will be pushing our proton to the right. But is it all downhill on the path from A to B? To check that we might calculate the potential ϕ_C, halfway between A and B:

$$\phi_C = \frac{4}{\sqrt{1^2+1^2+1.5^2}} + \frac{4}{\sqrt{1^2+1^2+0.5^2}} = 4.6069$$

2.22

$$-0.15 \text{ volt} = \frac{Q}{4\pi\epsilon_0 r} \; ; \quad 4\pi\epsilon_0 = 1.11 \times 10^{-10} \quad r = 3 \times 10^{-7} \text{ m}$$

$Q = -0.15 \times 1.11 \times 10^{-10} \times 3 \times 10^{-7} = 0.5 \times 10^{-17}$ coulomb

$n = 0.5 \times 10^{-17} / 1.6 \times 10^{-19} = 30$ electrons

$E = \phi / r = .015$ volt$/ 3 \times 10^{-7}$m $= 5 \times 10^5$ volt/meter

2.23

$$5 \times 10^6 q_1 = \frac{q_1 q_2}{4\pi\epsilon_0 r}$$

proton charge

charge on Ag nucleus = 47e

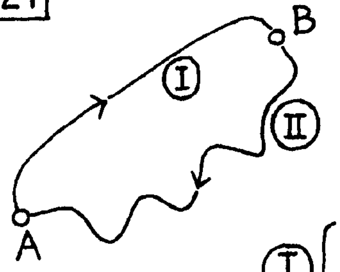
initial K.E. of proton

potential energy at closest head-on approach

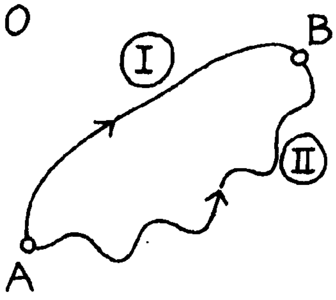
$$r = \frac{47 \times 1.6 \times 10^{-19}}{1.11 \times 10^{-10} \times 5 \times 10^6} = 1.35 \times 10^{-14} \text{ meter}$$

This is somewhat larger than the radius of the silver nucleus, which is about 5×10^{-15} meter, so it was reasonable to consider coulomb repulsion only.

$\boxed{2.24}$



Given that $\oint_A^A \mathbf{E}\cdot d\mathbf{s} = 0$ for any path, consider the path I + II:



$$(\mathrm{I})\int_A^B \mathbf{E}\cdot d\mathbf{s} + (\mathrm{II})\int_B^A \mathbf{E}\cdot d\mathbf{s} = 0$$

It follows that $(\mathrm{II})\int_A^B \mathbf{E}\cdot d\mathbf{s} = -(\mathrm{II})\int_B^A \mathbf{E}\cdot d\mathbf{s} = (\mathrm{I})\int_A^B \mathbf{E}\cdot d\mathbf{s}$ QED

$\boxed{2.25}$ The equipotential curve A crosses the y axis at $y = 1$. At that point the potential is

$$\phi_A = \frac{2}{1} + \frac{2}{3} - \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} = 1.253$$

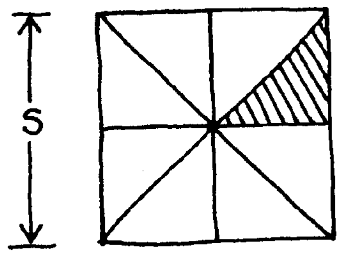
Curve B crosses the x axis, and itself, close to $x = 3.5$. At that point the potential is

$$\phi_B = \frac{4}{\sqrt{2^2 + 3.5^2}} - \frac{1}{2.5} - \frac{1}{4.5} = 0.370$$

Curve C crosses itself at the origin, where the potential is $\phi_C = 4/2 - 2/1 = 0$

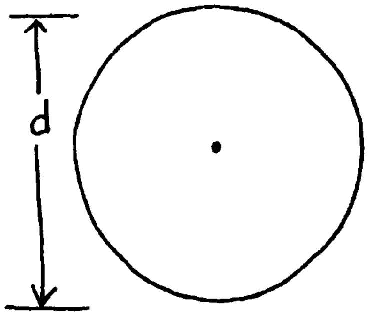
Equipotentials cross only at saddle points, where $\mathbf{E} = 0$. There is here a saddle point at the origin, where $\mathbf{E}$ is zero because of symmetry. The saddle point near $x = 3.5$, traversed by curve B, is more precisely located at $x = 3.44$. There is, of course, another saddle point at $x = -3.44$.

2.26 Apply the result of Problem 2.12 to each of the 8 triangles, with $\theta = 45°$ and $b = s/2$:



$$\phi_{\square} = 8\,\sigma\,\frac{s}{2}\ln\left(\frac{1+\sin 45°}{\cos 45°}\right)$$

$$= 3.5255\,\sigma s$$



$$\phi_{\odot} = \int_0^{d/2} \frac{2\pi r\sigma\,dr}{r} = \pi\sigma d$$

$$\pi\sigma d = 3.5255\,\sigma s \quad \text{if } \frac{d}{s} = 1.1222$$

As was to be expected, the disk is larger than the inscribed circle, but smaller than the circumscribed circle.

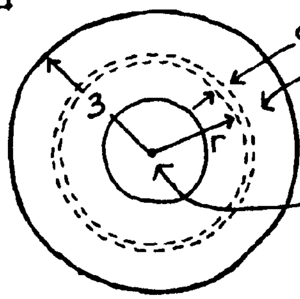
2.27 The potential at the rim of a charged disk is $4\sigma r$. Adding a ring of charge $\sigma 2\pi r dr$ costs, in energy, $\phi\, dq$ or

$$\sigma \times 2\pi r dr \times 4\sigma r : \qquad dE = 8\pi\sigma^2 r^2 dr$$

$$\int_{r=0}^{r=a} dE = \frac{8\pi}{3}\sigma^2 a^3 \qquad Q^2 = (\pi a^2\sigma)^2 \quad \text{so}$$

$$E = \frac{8Q^2}{3\pi a}$$ is the total energy required to assemble the disk of charge.

2.28



$\sigma = -4\ \text{esu/cm}^2$

$dq = \sigma \times 2\pi r\,dr$

$\phi = \int \frac{dq}{r} = 2\pi\sigma \int_1^3 dr = -16\pi$ statvolt

Electron's final K.E. $= e\phi = 4.8 \times 10^{-10} \times 16\pi = 2.41 \times 10^{-8}$ erg
Electron rest energy $mc^2 = 81 \times 10^{-8}$ erg. Since K.E.$/mc^2 \approx 0.03$ a non-relativistic calculation should be good enough :

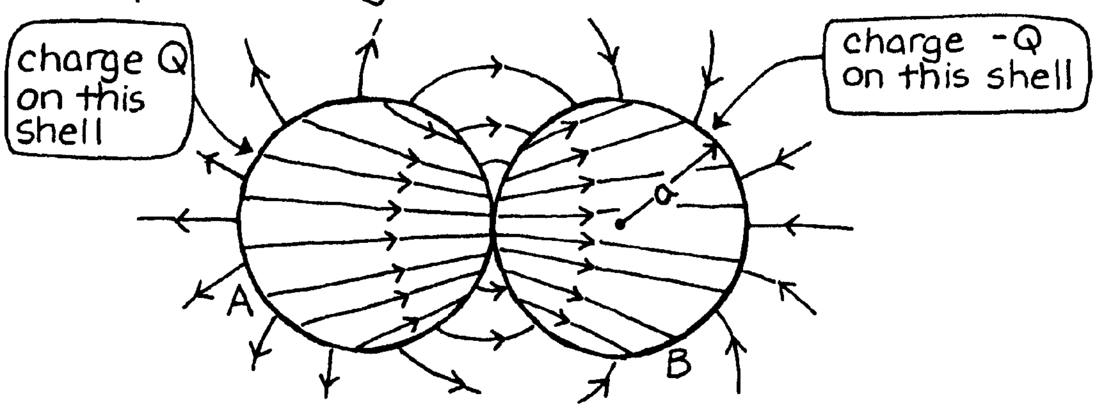
$$U = \left(\frac{2\ \text{K.E.}}{m}\right)^{1/2} = \frac{2 \times 2.41 \times 10^{-8}}{9 \times 10^{-28}} = 7.32 \times 10^9\ \text{cm/sec}$$

[A relativistic calculation using the same constants :

$$\gamma = 1 + \frac{\text{K.E.}}{mc^2} = 1 + \frac{2.41 \times 10^{-8}}{8.1 \times 10^{-7}} = 1.0298$$

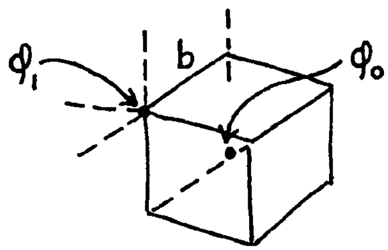
$\beta = (1 - 1/\gamma^2)^{1/2} = 0.2388 \qquad \gamma = \beta c = 7.16 \times 10^9$ cm/sec]

2.29 Outside both shells the electric field is that of two point charges. Inside each shell the field is that of a point charge at the center of the other shell.



The external field of A alone is that of point charge Q. To move shell B to infinity takes the same amount of work as moving the point charge Q to infinity with B stationary. But that takes just $Q^2/2a$ for that point charge Q is initially a distance of 2a from the center of shell B.

$\boxed{2.30}$ For given charge density ρ the potential at the center of a cube of side s must be proportional to Q/s where Q is the total charge ρs^3. Hence ϕ is proportional to $\rho s^3/s$, or to s^2 for fixed ρ. If we assemble 8 cubes of side b we make a cube of side $2b$. The potential at its center is $8\phi_1$, the sum of 8 corner potentials of the side b cube. But this must be 4 times the center potential of the side b cube. So we have $8\phi_1 = 4\phi_0$ or: $\phi_0 = 2\phi_1$.



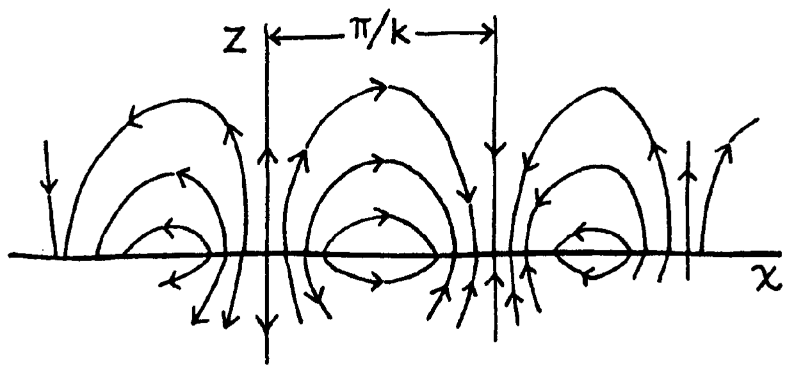
$\boxed{2.31}$

$$\phi = \phi_0 \cos kx\, e^{-kz} \qquad \frac{\partial\phi}{\partial x} = -k\phi_0 \sin kx\, e^{-kz} = -E_x$$

$$\frac{\partial^2\phi}{\partial x^2} = -k^2\phi_0 \cos kx\, e^{-kz} \qquad \frac{\partial\phi}{\partial z} = -k\phi_0 \cos kx\, e^{-kz} = -E_z$$

$$\frac{\partial^2\phi}{\partial z^2} = k^2\phi_0 \cos kx\, e^{-kz}$$

$$\nabla^2\phi = \frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} + \frac{\partial^2\phi}{\partial z^2} = 0$$



$$\sigma = \frac{1}{2\pi} E_z \text{ at } z = 0 \qquad \sigma = \frac{k}{2\pi}\phi_0 \cos kx$$

$\boxed{2.32}$ If the direction of $\hat{n}$ is reversed the left hand side must change sign. But the right hand side can only be positive.