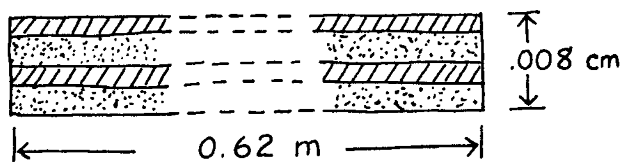
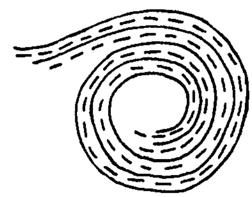
$\boxed{10.1}$ $C = \epsilon \frac{A}{s}$ farad $\quad \epsilon = 2.3\epsilon_0 = 2.3 \times 8.85 \times 10^{-12}$
$= 2.04 \times 10^{-11}$ farad/meter

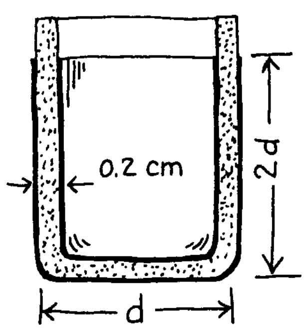
$s = .001$ inch $= 2.54 \times 10^{-5}$ m $\quad C = 5 \times 10^{-8}$ farad
$A = sC/\epsilon = .062\ m^2$ ribbon width $= .05$ m
Length of dielectric tape needed $= .062/.05 = 1.24$ m
You need an equal length of aluminum tape, not twice as much. Roll up a sandwich 62 cm long :





Area of cross-section $= 0.5\ cm^2$; overall diameter about 1 cm.

$\boxed{10.2}$ Assume height $= 2d$ (result will depend somewhat on proportions assumed).

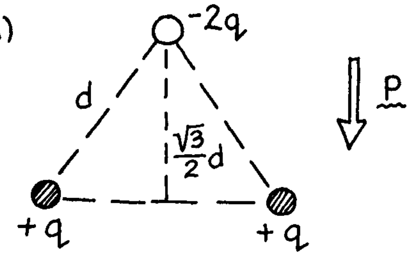


1 liter $= 10^3 cm^3 = 2d \times \frac{\pi}{4}d^2$, or $d = 8.6$ cm
area of capacitor $= \pi d \times 2d + \frac{\pi}{4}d^2 = \frac{9}{4}\pi d^2$
$= 522\ cm^2$

$$C = \frac{522 \times 4}{4\pi \times 0.2} = 830 \text{ cm}$$

This is the capacitance of a sphere of 830 cm radius, or about 54 feet diameter.

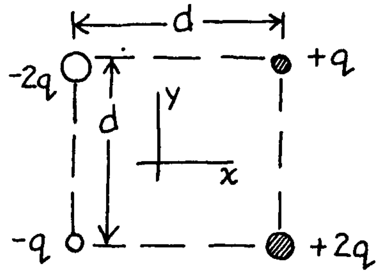
10.3 (a)



$$p = 2q\frac{\sqrt{3}}{2}d = \sqrt{3}qd$$

(b) $p = 0$

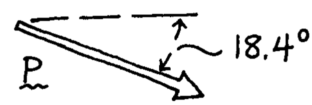
(c)



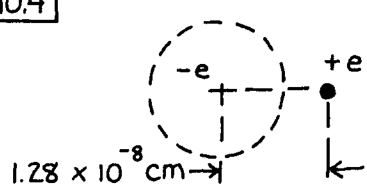
$$p_x = +3qd$$

$$p_y = -qd$$

$$p = \sqrt{10}\,qd = 3.16\,qd$$

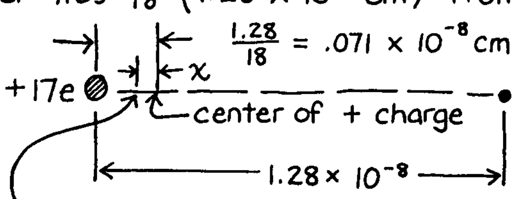


10.4



$p = 1.28 \times 10^{-8}\text{cm} \times 4.8 \times 10^{-10}\text{esu}$
$= 6.15 \times 10^{-18}$ esu - cm,
which is about six times the actual dipole moment, 1.03×10^{-18}.

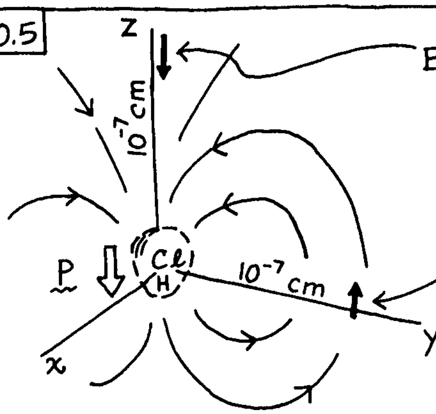
By the "center" of a charge distribution we mean the point with respect to which, as origin, $\int \underset{\sim}{r}\,\rho\, dv = 0$. Considering the positive charges by themselves, the center lies $\frac{1}{18}$ $(1.28 \times 10^{-8}\,\text{cm})$ from the $C\ell$ nucleus:



center of negative charge distribution, total charge $-18e$, must lie to left of center of + charge by distance x such that $18ex = 1.03 \times 10^{-18}$, or

$$x = \frac{1.03 \times 10^{-18}}{18 \times 4.8 \times 10^{-10}} = .0119 \times 10^{-8}\ \text{cm}$$

10.5



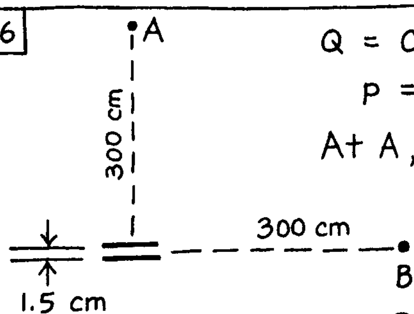
$$E = \frac{2p}{z^3} = \frac{2 \times 1.03 \times 10^{-18}}{(10^{-7})^3}$$

$$= 2.06 \times 10^3 \ \frac{\text{statvolts}}{\text{cm}}$$

$$E = \frac{p}{y^3}$$

$$= 1.03 \times 10^3 \ \frac{\text{statvolts}}{\text{cm}}$$

10.6



$$Q = CV = 250 \times 6 = 1500 \text{ esu}$$

$$p = QS = 1500 \times 1.5 = 2250 \text{ esu-cm}$$

$$\text{At A, } E = \frac{2p}{r^3} = \frac{2 \times 2250}{(300)^3}$$

$$= 1.67 \times 10^{-4} \text{ statvolts/cm}$$

$$\text{At B, } E = \frac{p}{r^3} = 0.833 \times 10^{-4} \text{ statvolts/cm}$$

10.7 The time constant was $\tau = RC = \frac{\rho s}{A} \cdot \frac{A}{4\pi s} = \frac{\rho}{4\pi}$

With dielectric constant ϵ, C is increased by that factor, so $\tau = \frac{\rho\epsilon}{4\pi}$

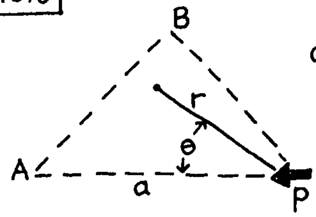
If C, per cm², is 10^{-6} farad, this implies a dielectric thickness s such that

$$10^{-6} \times 9 \times 10^{11} = \frac{\epsilon}{4\pi s} \quad \text{or} \quad s = \frac{3}{4\pi \times 9 \times 10^5} = 2.6 \times 10^{-7} \text{cm}$$

$\tau = RC = 1000$ ohms $\times 10^{-6}$ farads $= 10^{-3}$ sec. (independent of area, since $C \sim$ area, $R \sim \text{area}^{-1}$).

$$\rho = \frac{1000}{2.6 \times 10^{-7}} \approx 4 \times 10^9 \text{ ohm-cm}.$$

10.8

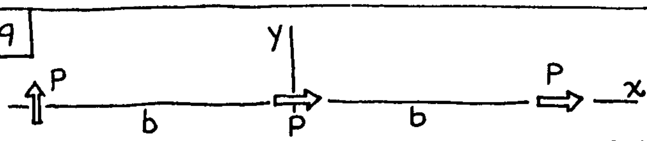


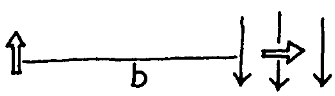
$$\varphi = \frac{p\cos\theta}{r^2} \qquad \varphi_A = \frac{p}{a^2}$$

$$\varphi_B = p \times \frac{.707}{(a^2/2)} = \frac{1.414\,p}{a^2}$$

$$\text{work done} = \varphi_B - \varphi_A = \frac{0.414\,p}{a^2}$$

10.9

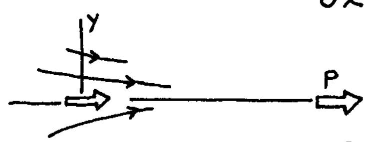




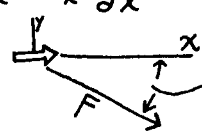
$$E_y = \frac{-p}{(b+x)^3} \qquad \left.\frac{\partial E_y}{\partial x}\right)_{x=0} = \frac{3p}{b^4}$$

$$F_y = p_x \frac{\partial E_y}{\partial x} = \frac{3p^2}{b^4} \;;\; F_x = 0$$

$$E_x = \frac{2p}{(p-x)^3}$$



$$\left.\frac{\partial E_x}{\partial x}\right)_{x=0} = \frac{6p}{b^4} \qquad F_x = p_x \frac{\partial E_x}{\partial x} = \frac{6p^2}{b^4} \;;\; F_y = 0$$

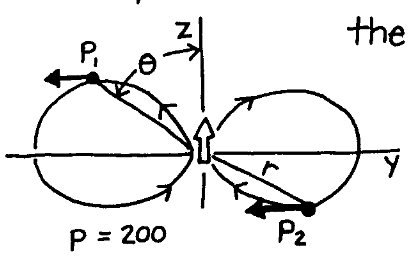


$$\tan^{-1} 0.5 = 26.6°$$

$$F = \frac{p^2}{b^4}\sqrt{3^2 + b^2} = \frac{6.71\,p^2}{b^4}$$

10.10

A point where the dipole's field $= -5\hat{y}$ must lie in the y-z plane. There are only 2 such points, P_1 and P_2, where $E_z = 0$ and $E_y = -5$.



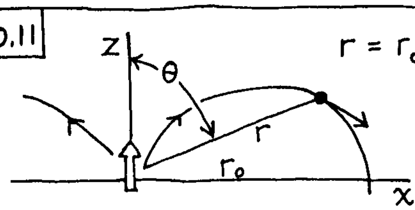
If $E_z = 0$, $3\cos^2\theta - 1 = 0$, or $\cos^2\theta = \frac{1}{3}$ $\sin^2\theta = \frac{2}{3}$

$$E_y = \frac{3p\sin\theta\cos\theta}{r^3} = 5 \quad \text{with } p = 200$$

$$r^3 = \frac{3 \times 200}{5}\sqrt{\frac{1}{3} \times \frac{2}{3}} = 56.57 \qquad r = 3.84 \text{ cm}$$

At P_1 $y = -r\sin\theta = -3.135$ cm ; $z = r\cos\theta = 2.216$ cm

$\boxed{10.11}$



$r = r_0 \sin^2\theta \qquad x = r_0 \sin^3\theta \qquad z = r_0 \sin^2\theta \cos\theta$

$$\frac{dx}{d\theta} = 3 r_0 \sin^2\theta \cos\theta$$

$$\frac{dz}{d\theta} = r_0 (2\sin\theta \cos^2\theta - \sin^3\theta) = r_0 \sin\theta (3\cos^2\theta - 1)$$

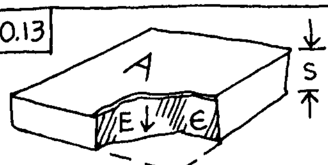
$$\frac{dz}{dx} = \frac{3\cos^2\theta - 1}{3\sin\theta\cos\theta} = \frac{E_z}{E_x} \text{ in field of dipole } p\hat{\underset{\sim}{z}}.$$

$\boxed{10.12}$ Refer to Sect. 10.10. Eq. 46 gives the polarization of the dielectric sphere: $p = \frac{3}{4\pi} \frac{\epsilon - 1}{\epsilon + 2} E_0$. In the limit $\epsilon \to \infty$, this becomes $P = \frac{3}{4\pi} E_0$. The field strength inside the dielectric sphere, $E = \frac{3}{2+\epsilon} E_0$ (Eq. 45) goes to zero as $\epsilon \to \infty$. In the limit the sphere becomes an equipotential, which is correct for a conducting sphere.

$$p = \text{volume} \times P = \frac{4\pi}{3} a^3 \times \frac{3}{4\pi} E_0 = a^3 E_0$$

Hence the polarizability α, defined by $p = \alpha E_0$, is just a^3 for the conducting sphere of radius a.
$\alpha_H = 0.66 \times 10^{-24}\,\text{cm}^3$ (Table 10.2) A conducting sphere of equal polarizability has a radius
$(0.66 \times 10^{-24})^{1/3} = 0.87 \times 10^{-8}$ cm .

$\boxed{10.13}$



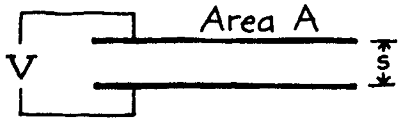
$C = \frac{\epsilon A}{4\pi s}$ Work done in charging capacitor $= \frac{1}{2} C \varphi^2$

$\varphi = Es \qquad \text{volume} = As$

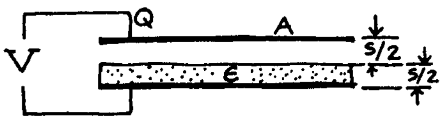
$$U = \frac{\text{energy stored}}{\text{volume}} = \frac{1}{2}\left(\frac{\epsilon A}{4\pi s}\right) s^2 E^2 \left(\frac{1}{As}\right) = \epsilon E^2/8\pi$$

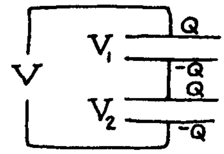
In the electromagnetic wave in the dielectric, if the amplitude of the electric field is E_0, that of the magnetic field is $B_0 = \sqrt{\epsilon}\, E_0$. Hence the energy density in the magnetic field, $B^2/8\pi$, is equal to that in the electric field, as in a wave in vacuum.

10.14



$$C_0 = \frac{A}{4\pi s}$$





$C_1 = 2C_0$

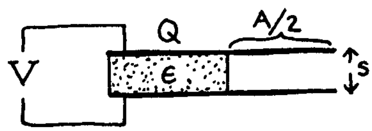
$C_2 = 2\epsilon C_0$

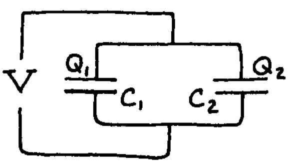
This is equivalent to two capacitors, C_1 and C_2 in series.

$$V_1 = \frac{Q}{C_1} = \frac{Q}{2C_0} \qquad V_2 = \frac{Q}{C_2} = \frac{Q}{2\epsilon C_0} \qquad V = V_1 + V_2 = \frac{Q}{2C_0}\left(1+\frac{1}{\epsilon}\right)$$

The capacitance of the combination is:

$$C = \frac{Q}{V} = \frac{2C_0}{1+\frac{1}{\epsilon}} = \frac{2\epsilon}{\epsilon+1}C_0$$





This is equivalent to two capacitors, C_1 and C_2 in parallel.

$$C_1 = \frac{\epsilon C_0}{2} \qquad C_2 = \frac{C_0}{2} \qquad Q_1 = C_1V = \frac{\epsilon}{2}C_0V \qquad Q_2 = C_2V = \frac{C_0}{2}V$$

The capacitance of the combination is:

$$C = \frac{Q}{V} = \frac{Q_1+Q_2}{V} = \frac{\epsilon+1}{2}C_0$$

10.15 The number of water molecules in 1 cm^3 is

$n = 6 \times 10^{23}/18 = 3.3 \times 10^{22}\ cm^{-3}$

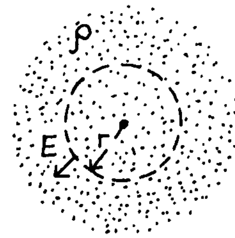
$P = np = 3.3 \times 10^{22}\ cm^{-3} \times 1.84 \times 10^{-18}$ esu – cm

$= 6.1 \times 10^4$ esu cm^{-2} This is equivalent

to $6.1 \times 10^4/4.8 \times 10^{-10}$, or 1.3×10^4 electrons per cm^2.

[Note that this is somewhat smaller than the number of surface molecules per cm^2, which must be something like $n^{2/3} \approx 1.0 \times 10^{15}\ cm^{-2}$.]

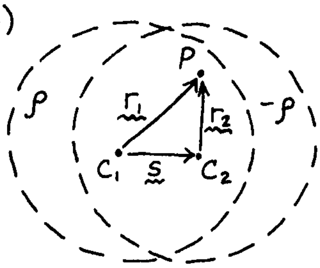
10.16 (a) Apply Gauss's law to sphere of radius r: charge enclosed $= \frac{4\pi}{3} r^3 \rho$;



$$4\pi r^2 E = 4\pi q = 4\pi \left(\frac{4\pi}{3} r^3 \rho\right)$$

$$E = \frac{4\pi}{3} \rho r$$

(b)



Let $\mathbf{s}$ be the vector displacement from C_1 to C_2. At the point P,

$\mathbf{E}_1 = \frac{4\pi}{3} \rho \mathbf{r}_1$ and $\mathbf{E}_2 = \frac{4\pi}{3}(-\rho)\mathbf{r}_2$

The total field is thus

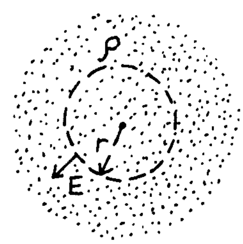
$$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2 = \frac{4\pi}{3} \rho (\mathbf{r}_1 - \mathbf{r}_2)$$

But $\mathbf{r}_1 - \mathbf{r}_2 = \mathbf{s}$ Hence $\mathbf{E} = \frac{4\pi\rho}{3} \mathbf{s}$

This holds for any point inside both distributions. Note that $-\mathbf{s}$ is the displacement of the positive charge distribution with respect to the negative, so that the volume polarization is $\mathbf{P} = -\rho \mathbf{s}$. Therefore,

$$\mathbf{E} = \frac{4\pi\rho \mathbf{s}}{3} = -\frac{4\pi}{3}\mathbf{P}$$ in agreement with Eq. 43.

(c) For the cylindrical distribution, Gauss's law applied to unit length of cylinder is:



$$2\pi r E = 4\pi(\pi r^2 \rho), \text{ or } E = 2\pi r \rho$$

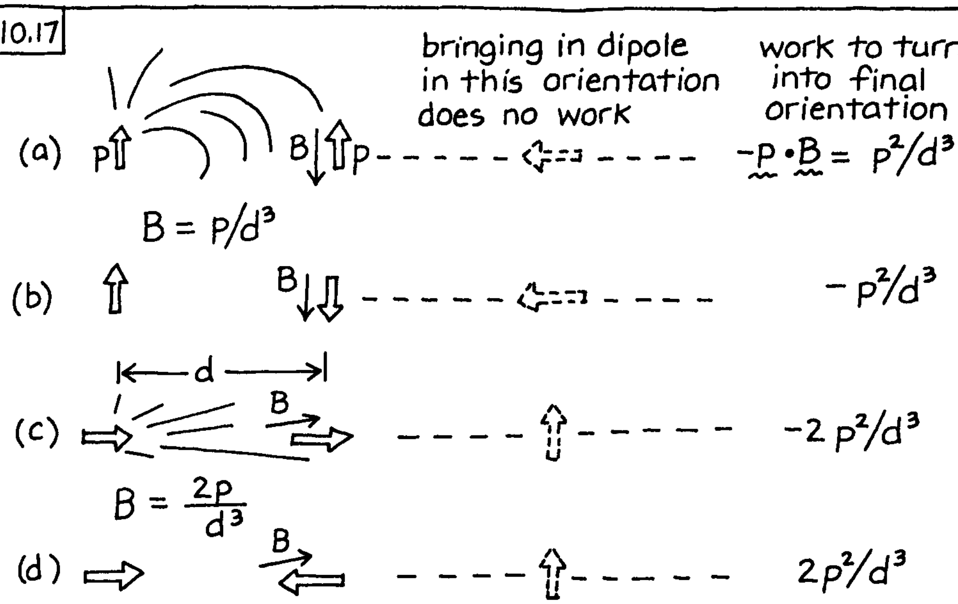
As in (b): $\mathbf{E}_1 = 2\pi\rho \mathbf{r}_1$

$\mathbf{E}_2 = 2\pi(-\rho)\mathbf{r}_2$

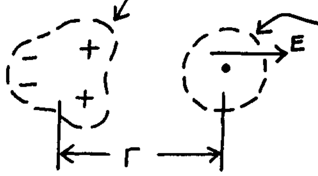
Thus $\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2 = 2\pi\rho(\mathbf{r}_1 - \mathbf{r}_2) = 2\pi\rho\mathbf{s}$. In this case also, $\mathbf{P} = -\rho\mathbf{s}$ so we obtain, for the field inside a cylinder with uniform transverse polarization:

$$\mathbf{E} = -2\pi\mathbf{P}$$

10.17



10.18



H_2O molecule $p = 1.84 \times 10^{-18}$ esu - cm

negative ion $q = -e$

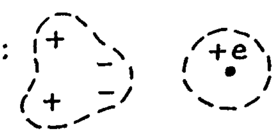
$E = \frac{2p}{r^3}$ $F = eE$

work to separate $= \int_{r_1}^{\infty} \frac{2pe}{r^3}\,dr = \frac{ep}{r_1^2}$

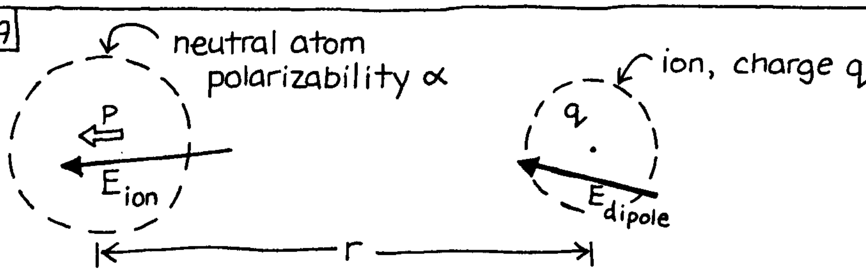
For $r_1 = 1.5 \times 10^{-8}$ cm

$$\text{work} = \frac{4.8 \times 10^{-10} \times 1.84 \times 10^{-18}}{(1.5 \times 10^{-8})^2} = 3.9 \times 10^{-12}\,\text{erg} = 2.4\ \text{e-v}$$

A positive ion would be bound like this :



10.19



Field of ion, $E_{ion} = \frac{q}{r^2}$, induces dipole $p = \alpha E_{ion}$ in neutral atom. Field of induced dipole, $E_{dipole} = \frac{2p}{r^3}$, causes force $F = q E_{dipole}$ on ion :

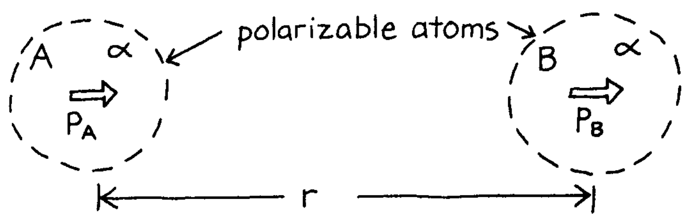
$F = q\left(\frac{2p}{r^3}\right) = \frac{2q}{r^3} \times \frac{\alpha q}{r^2} = \frac{2\alpha q^2}{r^5}$. This force is attractive for either sign of q.

Work to separate from distance $r_1 = \int_{r_1}^{\infty} F\,dr = \frac{\alpha q^2}{2 r_1^4}$

If $q = e$ and $\alpha = 27 \times 10^{-24}\,\mathrm{cm}^3$ this is 4×10^{-14} erg for

$$r_1 = \left[\frac{27 \times 10^{-24} \times (4.8 \times 10^{-10})^2}{2 \times 10^{-14}}\right] = 9 \times 10^{-8}\,\mathrm{cm}$$

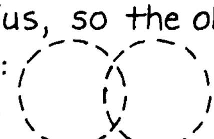
10.20



p_B is induced by the field of dipole p_A : $p_B = \alpha \frac{2p_A}{r^3}$

p_A is induced by the field of dipole p_B : $p_A = \alpha \frac{2p_B}{r^3}$

Then $p_A = \frac{4\alpha^2}{r^6} p_A$, which is satisfied by $p_A = 0$

or by any value of p_A if $r^6 = 4\alpha^2$. If $r^6 < 4\alpha^2$, p_A (and p_B) would increase until limited by nonlinearity of polarizability. The critical distance r_c is $(2\alpha)^{1/3}$. Atomic polarizability α is typically, in order of magnitude, an atomic volume. (Section 10.5). Thus r_c is not much larger than an atomic radius, so the object we are concerned with looks like this:  . Whether the lowest state of this system is a spontaneously polarized structure cannot be decided by considering only the interactions of dipoles. Ordinarily the lowest state of two similar atoms would be symmetrical with $\mathbf{p}_A + \mathbf{p}_B = 0$. But we cannot exclude the possibility that the symmetry is "spontaneously broken".

$\boxed{10.21}$

In SI : $E_{max} = \dfrac{1.4 \times 10^4 \text{ volt}}{2.54 \times 10^{-5} \text{ m}} = 5.5 \times 10^8 \text{ volt/m}$

stored energy $= \dfrac{3.25\, \epsilon_0 E^2}{2} = 4.3 \times 10^6 \text{ J/m}^3$

stored energy per kg $= \dfrac{4.3 \times 10^6 \text{ J m}^{-3}}{1400 \text{ kg m}^{-3}} = 3100 \text{ J/kg}$

$gh = .75 \times 3100 \qquad h = \dfrac{.75 \times 3100}{9.8} = 240 \text{ meters}$

The "D" cell in Prob. 4.29 stored 1.8×10^5 J/kg, 60 times as much as this mylar capacitor. But the capacitor can deliver all the stored energy in less than a microsecond!

$\boxed{10.22}$

$$\chi = \frac{\epsilon - 1}{4\pi} = \frac{CNp^2}{kT} \qquad kT = 4 \times 10^{-14} \text{ erg}$$

	ϵ	N	p	C
H_2O	80	3.3×10^{22} cm^{-3}	1.84×10^{-18} esu cm	2.2
NH_3	23	2.9×10^{22}	1.43×10^{-18}	1.2
CH_3OH	34	2.5×10^{22}	1.70×10^{-18}	1.4

10.23 Energy stored in unit volume $= \dfrac{\epsilon E_0^2}{8\pi}$

Power dissipated in ohmic resistance, per unit volume,

$= E^2\sigma$ Time average dissipation $= \frac{1}{2} E_0^2 \sigma$

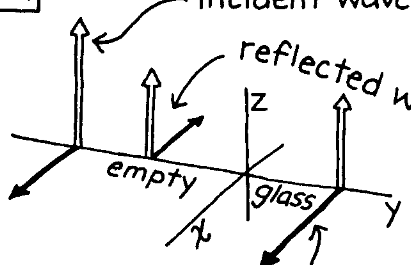
$$Q = \frac{\omega \times \text{energy stored}}{\text{power loss}} = \frac{\omega \epsilon}{4\pi\sigma}$$

For sea water $\sigma = 0.04\,(\text{ohm-cm})^{-1} = 3.6 \times 10^{10}\ \text{sec}^{-1}$

For $10^9\ H_z$ $\quad Q = \dfrac{2\pi \times 10^9 \times 80}{4\pi \times 3.6 \times 10^{10}} = 1.1$

[Energy is lost before wave has time to travel more than a fraction of a wavelength. Microwave radar won't find submarines!]

10.24



incident wave $\begin{cases} \mathbf{E} = \hat{z} E_i \sin(ky - \omega t) \\ \mathbf{B} = \hat{x} B_i \sin(ky - \omega t) \end{cases}$

reflected wave $\begin{cases} \mathbf{E} = \hat{z} E_r \sin(ky - \omega t) \\ \mathbf{B} = -\hat{x} B_r \sin(ky - \omega t) \end{cases}$

wave in glass $\begin{cases} \mathbf{E} = \hat{z} E_0 \sin(k'y - \omega t) \\ \mathbf{B} = \hat{x} B_0 \sin(k'y - \omega t) \end{cases}$

At $y = 0$, $\mathbf{E}$ and $\mathbf{B}$ must be continuous (no jump in either at the surface of the glass). :

$$E_i + E_r = E_0 \qquad B_i - B_r = B_0$$

$$B_i = E_i \qquad B_r = E_r \qquad B_0 = nE_0$$

$$\left.\begin{aligned} E_i + E_r &= E_0 \\ E_i - E_r &= nE_0 \end{aligned}\right\} \longrightarrow \boxed{\begin{aligned} E_0 &= \frac{2}{1+n} E_i \\ E_r &= \frac{1-n}{1+n} E_i \end{aligned}}$$

If $n = 1.6$, $\dfrac{E_r^2}{E_0^2} = \left(\dfrac{1-n}{1+n}\right)^2 = 0.053$