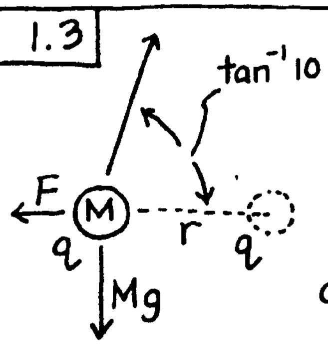
1.1 The ratio of the gravitational force Gm^2/r^2 to the electrical repulsion is Gm^2/e^2, which is

$$6.7 \times 10^{-8} \times (1.6 \times 10^{-24})^2/(4.8 \times 10^{-10})^2 = 7.4 \times 10^{-37}$$

For $r = 10^{-13}$ cm the electrical force e^2/r^2 is 2.3×10^6 dynes, equivalent to 23 newtons – about 5 pounds!

1.2 If $e^2/h^2 = mg$ then $h = e/(mg)^{\frac{1}{2}} = 480$ cm

1.3



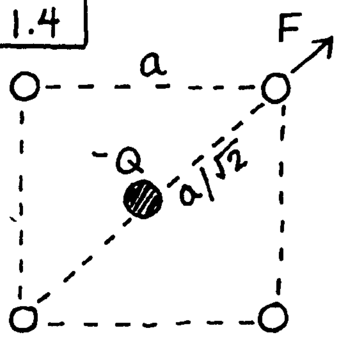
$M = 0.3$ kg $\quad g = 9.8$ ms^{-2} $\quad r = 0.5$ m

$F = 0.1\,Mg \qquad F = q^2/4\pi\epsilon_0 r^2$

$$q = r\,(4\pi\epsilon_0 F)^{\frac{1}{2}}$$

$$q = 0.5(1.11 \times 10^{-10} \times 0.1 \times 0.3 \times 9.8)^{\frac{1}{2}} = 2.86 \times 10^{-6}\,\mathrm{C}$$

1.4

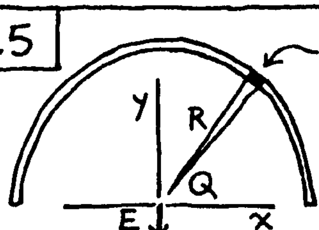


$$F = 2\left(\frac{\sqrt{2}}{2}\right)\frac{q^2}{a^2} + \frac{q^2}{(\sqrt{2}a)^2} - \frac{Qq}{(a/\sqrt{2})^2}$$

$$F = 0 \text{ for } Q = \left(\frac{1}{4} + \frac{1}{\sqrt{2}}\right)q = 0.957q$$

Equilibrium is unstable: inward displacement of a corner charge results in inward force on it.

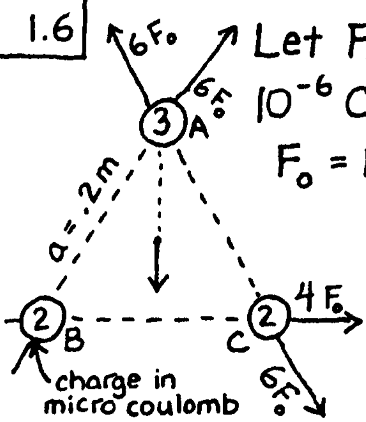
1.5



$$dQ = Q\frac{d\theta}{\pi} \qquad -dE_y = \frac{dQ}{R^2}\sin\theta$$

$$E = \frac{Q}{\pi R^2}\int_0^{\pi}\sin\theta\, d\theta = \frac{2Q}{\pi R^2}$$

1.6



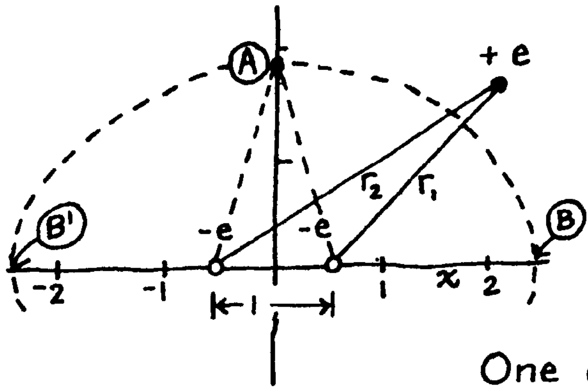
Let F_0 be the force between two charges of 10^{-6} C each, at the distance a of 0.2 m.

$F_0 = 10^{-12}/4\pi\epsilon_0 \times .04 = 0.225$ N. The force on A $= 2 \times 6F_0 \times \cos 30° = 2.34$ N. The force on B $= [(6+2)^2 + (2\sqrt{3})^2]^{\frac{1}{2}}\, F_0 = 1.96$ N

Field E is due only to excess charge at A

$$E = 10^{-6}/4\pi\epsilon_0 (a/\sqrt{3})^2 = 6.74 \times 10^5\ \mathrm{NC^{-1}}.$$

1.7 Without loss of generality we can put the two electrons on the x axis unit distance apart. For any location of the proton the potential energy of the system is:



$$U = \frac{e^2}{1} - \frac{e^2}{r_1} - \frac{e^2}{r_2}.$$ Thus for $U = 0$ we require:

$$\boxed{\frac{1}{r_1} + \frac{1}{r_2} = 1} \quad (1)$$

One obvious location meeting the requirement is A, with $r_1 = r_2 = 2$. To locate a position on the positive x axis, with $r_1 = x - \frac{1}{2}$, $r_2 = x + \frac{1}{2}$:
$\frac{1}{x-\frac{1}{2}} + \frac{1}{x+\frac{1}{2}} - 1 = 0$. This gives the quadratic equation

$2x - x^2 + \frac{1}{4} = 0$ with roots $x = 2.118$ and $x = -.118$

The latter root must be thrown out, for with $x < \frac{1}{2}$, r_1 and r_2 are not represented by $x - \frac{1}{2}$ and $x + \frac{1}{2}$. We are left with the point B at $x = 2.118$. Of course B', at $x = -2.118$ is equally good. Eq. 1 defines a curve in the x-y plane, and a surface of revolution around the x-axis, in space. This surface is the set of all points where the proton can be placed to give $U = 0$. The surface looks something like a prolate ellipsoid, but it isn't.

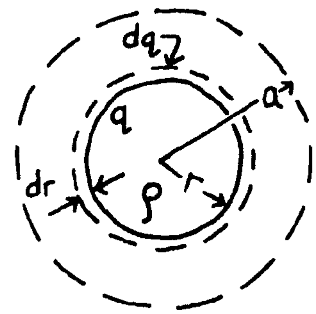
1.8

Suppose the array has been built from the left as far as the negative ion A. To add the next ion B, a positive ion, requires an amount of work:

$$-\frac{e^2}{s} + \frac{e^2}{2s} - \frac{e^2}{3s} + \cdots = -\frac{e^2}{s}\left[1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \cdots\right]$$

The expansion of $\ln(1+x)$ is $1 - \frac{x}{2} + \frac{x^2}{3} - \cdots$, converging for $-1 < x \leq 1$. Evidently the sum in brackets above is just $\ln 2$, or 0.693. Thus the energy of the chain, <u>per</u> <u>ion</u>, is $-0.693\frac{e^2}{s}$.

1.9 The charge in the sphere of radius r is: $q = \frac{4\pi}{3} r^3 \rho$. Its external field is the same as if q were at the center. The next shell to be added contains charge dq:



$dq = 4\pi r^2 dr \rho$. The work done in bringing up dq is therefore:

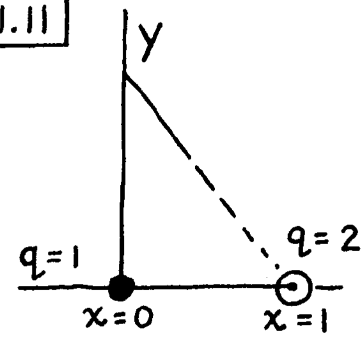
$dW = \frac{q \cdot dq}{r} = \frac{(4\pi\rho)^2}{3} r^4 dr$. Building up the whole sphere this way requires the work:

$W = \int_0^a \frac{(4\pi\rho)^2}{3} r^4 dr = \frac{(4\pi\rho)^2}{3} \frac{a^5}{5}$ The charge in the complete sphere is: $Q = \frac{4\pi}{3} a^3 \rho$. Thus the potential energy U, which is the same as W, can be written: $U = \frac{3}{5} Q^2/a$. Notice that Q^2/a has the proper dimensions $(\text{charge})^2/\text{distance}$. Indeed we could have predicted that much of our result without calculation.

1.10 Setting the potential energy $\frac{3}{5}\frac{e^2}{r_0}$ equal to mc^2 and solving for r_0:

$$r_0 = \frac{3}{5}\frac{e^2}{mc^2} = \frac{3}{5}\frac{(4.80 \times 10^{-10})^2}{(.91 \times 10^{-27})(3 \times 10^{10})^2} = 1.69 \times 10^{-13} \text{ cm}$$

1.11

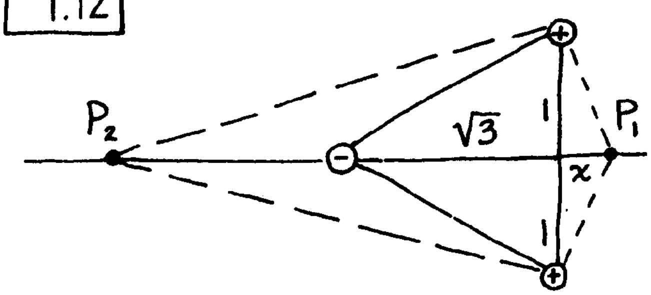


(a) The field E cannot be zero anywhere <u>between</u> two charges of opposite sign, or at any point closer to the greater than to the lesser charge. Hence the point we seek must lie on the negative x-axis. It is well to be clear about this before plunging into algebra. Let the point lie at $x = -s$. The field will vanish there if $1/s^2 = 2/(1+s)^2$, giving us the quadratic equation $s^2 - 2s - 1 = 0$, with roots $s = 1 \pm \sqrt{2}$. The positive root locates the point of vanishing field at $x = -2.414$. What is wrong with the other root? (b) At $(0, y)$ the field component E_y has the value $\frac{1}{y^2} - \frac{2y}{(1+y^2)^{3/2}}$. This vanishes

if $2y^3 = (1+y^2)^{3/2}$ which can be written

$2^{2/3} y^2 = 1 + y^2$, giving $y = 1/(2^{2/3} - 1)^{\frac{1}{2}} = 1.305$

1.12



Any point where the electric field of the three ions is zero must lie on the axis of symmetry. Let the side of the triangle be two units long and let x be the distance of the point P_1 from the line connecting the two positive ions. For $E = 0$ at P_1 we require that:

$$\frac{1}{(x+\sqrt{3})^2} = \frac{2}{(1+x^2)} \cdot \frac{x}{\sqrt{1+x^2}} \quad \text{, or} \quad \boxed{x = \frac{(1+x^2)^{3/2}}{2(x+\sqrt{3})^2}}$$

This is most readily solved by iteration: Evaluate the right side for some guessed initial x; then replace x

with that calculated value. For this equation the process converges rapidly on $x = 0.1463$ --- A second null point P_2 must lie somewhere to the left of the negative ion. To locate it, let positive x now run to the left from its previous origin. We get the same equation except that $+\sqrt{3}$ is changed to $-\sqrt{3}$. Iteration now leads to $x = 6.2045$. P_2 lies $(6.2045 - \sqrt{3})$ or 4.472 units to the left of the negative ion (much farther than in our diagram above).

$\boxed{1.13}$ (a) $\sigma = E/4\pi \quad 0.1/4\pi = 8.0 \times 10^{-3}$ esu/cm².

(b) Let h = rainfall depth = 0.25 cm, r = drop radius.

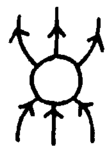
Number of drops above 1 cm² $= h/(4\pi r^3/3)$.

Charge q on each drop $= \sigma(4\pi r^3/3h)$. This charge causes radial field of strength q/r^2 at surface of drop:

$$q/r^2 = \sigma(4\pi r/3h) = E(r/3h)$$

$$= 0.1\,(.05/.75) = 6.7 \times 10^{-3} \text{ statvolt/cm}.$$

This is <u>only</u> the field caused by the charge on that drop. Even an uncharged drop has, in the field E, charges on its surface, of opposite sign on top and bottom. The associated field, which looks something

like this:  is described in Section 10.10 .

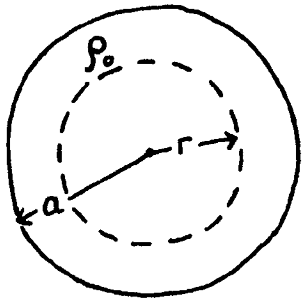
$\boxed{1.14}$ At a point $(0,0,z)$ on the z axis the field strength E_z is $Qz/(b^2+z^2)^{3/2}$. Differentiating:

$$\frac{1}{Q}\frac{dE_z}{dz} = \frac{b^2 - 2z^2}{(b^2+z^2)^{5/2}}.$$

$|E_z|$ is maximum at

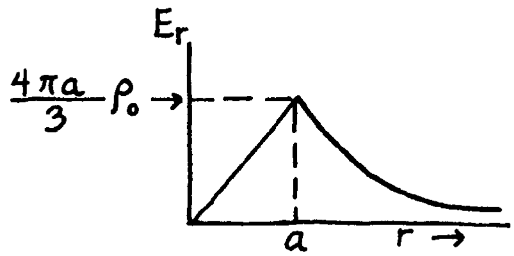
$$z = \pm b/\sqrt{2}.$$

1.15



On the sphere of radius r, the field is the same as if all charge inside that sphere, which amounts to $\frac{4\pi r^3 \rho_0}{3}$, were at the center :

$$E_r = \left(\tfrac{4}{3}\pi r^3 \rho_0\right)/r^2 = \frac{4\pi\rho_0 r}{3} \quad (r \leqslant a)$$

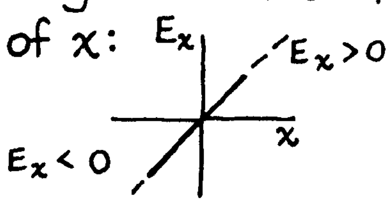


Outside the whole charge distribution the field is

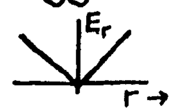
$$E_r = \frac{4}{3}\frac{\pi a^3 \rho_0}{r^2} \quad (r \geqslant a)$$

E_r is <u>continuous</u> at $r = a$

To discuss the continuity of the electric field in the neighborhood of the origin, look at E_x as a function of x:



clearly there is no discontinuity; there is no singularity whatever. The singularity suggested by plotting E_r like this :

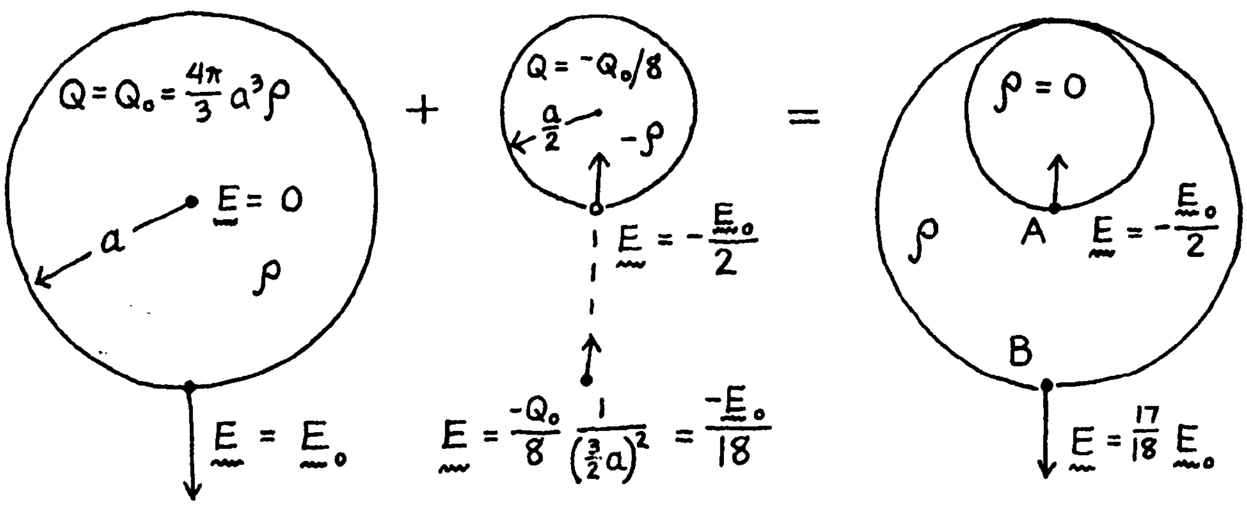


is spurious.

There is, properly, no negative range of r.

1.16

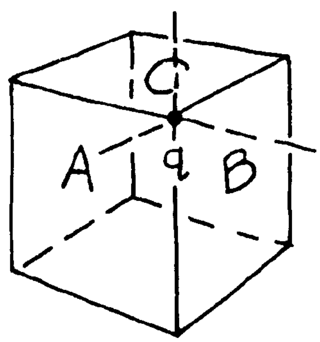
Use superposition :



A more remarkable fact, not hard to prove, is that within the cavity on the right the field $\mathbf{E}$ is perfectly uniform in magnitude and direction. It is $-\mathbf{E}_0/2$ at <u>every</u> point in the cavity.

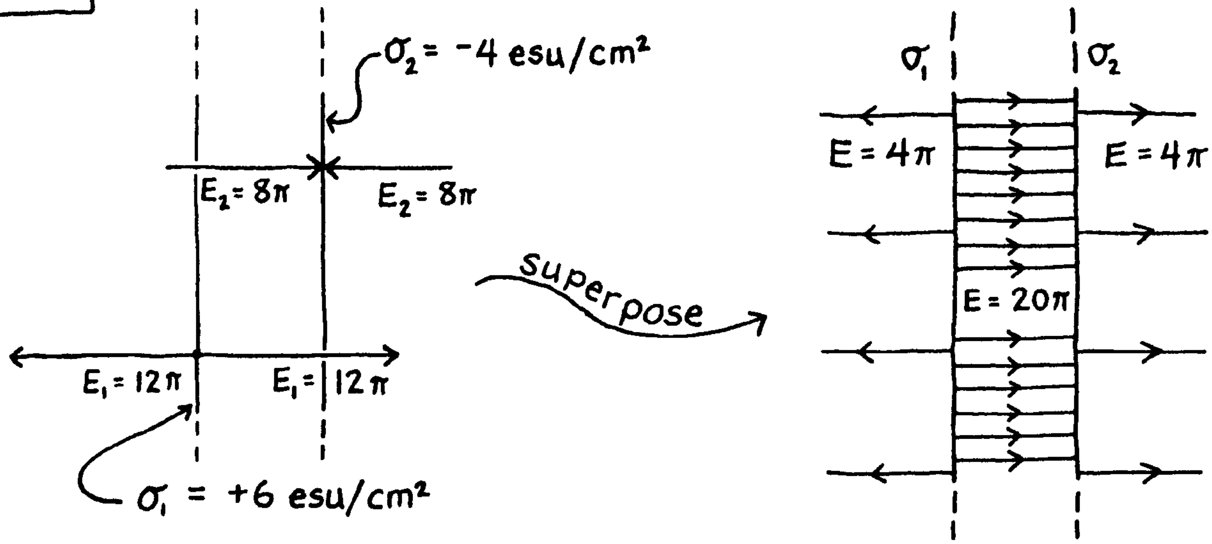
1.17 (a) The total flux is $4\pi q$ (Gauss's law). The flux through every face of the cube must be the same, because of symmetry. Hence, over any one of the six faces : $\int \mathbf{E} \cdot d\mathbf{a} = 4\pi q/6 = 2\pi q/3$

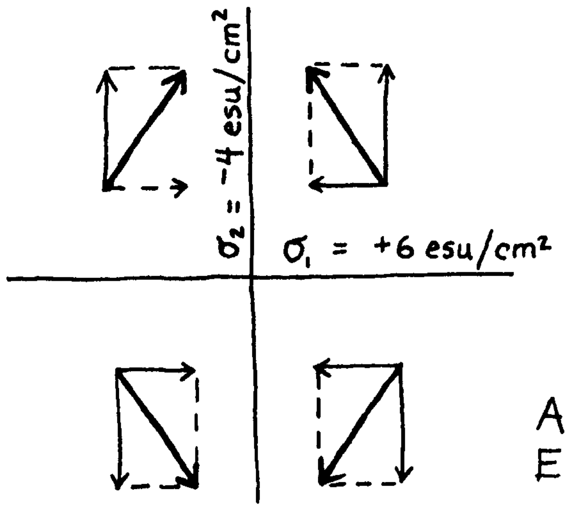
(b)



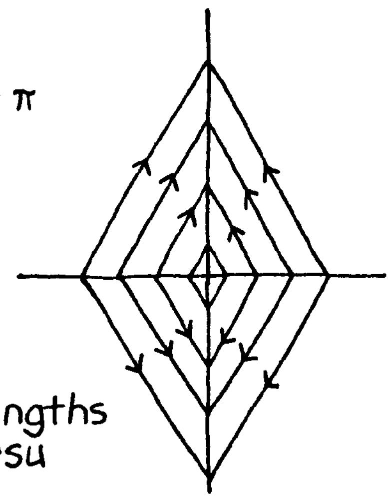
Because the field of q is parallel to the surface on each of the three faces A, B and C, the flux through these faces is zero. The flux through the other three faces must therefore add up to $4\pi q/8$, because our cube is one of eight such surrounding q. The three faces being symmetrically located with respect to q, the flux through each must be $\frac{1}{3} \cdot \frac{4\pi q}{8} = \frac{\pi q}{6}$.

1.18



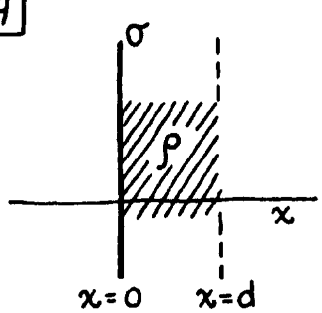


$E = 14.4\pi$



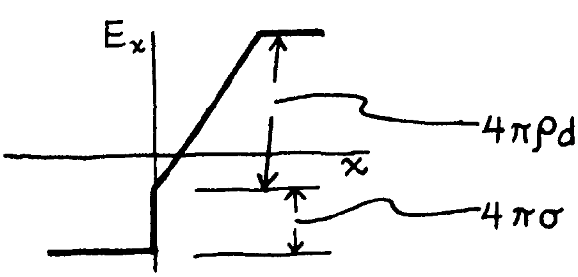
All field strengths
E in dyne/esu

1.19

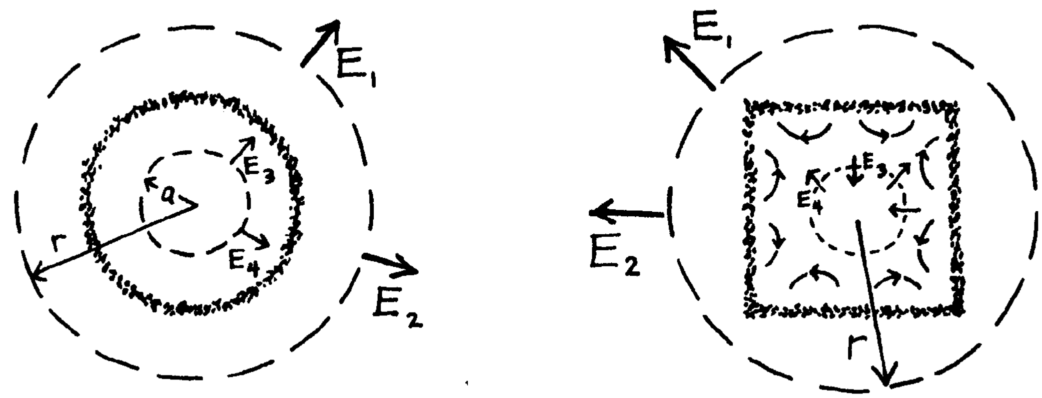


The total charge per cm² is $\sigma + \rho d$. For $x < 0$, $E_x = -2\pi(\sigma + \rho d)$ and for $x > d$ $E_x = 2\pi(\sigma + \rho d)$. More generally, at any point x, $E_x = -2\pi$ (charge per cm² to right of x) $+ 2\pi$ (charge per cm² to left of x) or $E_x = -2\pi(d-x)\rho + 2\pi(\sigma + x\rho)$ for $0 < x < d$.

$$E_x = 2\pi\sigma - 2\pi\rho d + 4\pi\rho x$$



1.20



The cylindrical tube of charge on the left has perfect axial symmetry. Hence $\mathbf{E}_1$ and $\mathbf{E}_2$ must be radial and equal in magnitude. Applying Gauss's Law, $2\pi r E_1 = 4\pi\lambda$, where λ is the charge per cm length of tube, and $E_1 = 2\lambda/r$, just as if the charge were concentrated on the axis. Inside the tube symmetry also demands that $E_3 = E_4$. But Gauss's Law requires that the surface integral over the cylinder of radius a be zero. Hence $E_3 = E_4 = 0$.

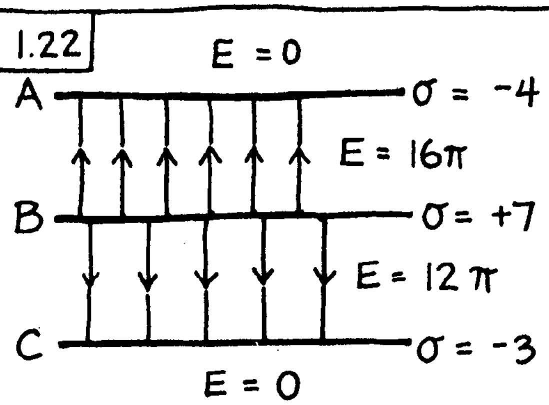
For the square tube of charge the integral over the cylinder of radius r must equal $4\pi \times$ charge enclosed, but nothing requires that $E_1 = E_2$. The integral of $\mathbf{E}$ over the small inner cylinder vanishes, but it can do so with $\mathbf{E}_3 \neq \mathbf{E}_4$ if, as is the case, they point in opposite directions. By comparing this tube with a square charged conducting tube, within which the field is indeed zero, you can deduce that E_4 must point inward (if the charge is positive).

1.21 The fraction of the negative charge which lies within a sphere of radius r_1 will be given by
$$\int_0^{r_1}\rho\,dv\Big/\int_0^{\infty}\rho\,dv.$$ With $\rho = Ce^{-2r/a_0}$ and $dv = 4\pi r^2 dr$

all we need is the integral
$$\int_0^{x_1} x^2 e^{-x}dx = -(x^2+2x+2)e^{-x}\Big]_0^{x_1} = 2-(x_1^2+2x_1+2)e^{-x_1}$$
For $x=\infty$ the integral is 2. For $r_1 = a_0$, $x_1 = 2$ and the integral is $2-10e^{-2}$. The fraction of electron charge inside $r = a_0$ is $(2-10e^{-2})/2 = 1-\frac{5}{(2.178)^2} = 0.323$

The net positive charge inside $r = a_0$ is therefore $0.677\times 4.8\times 10^{-10}$ esu and the field E_r at that radius is
$$\frac{0.677\times 4.8\times 10^{-10}}{(0.53\times 10^{-8})^2} = 1.15\times 10^{7}\ \text{dyne/esu}$$

1.22



Force on A in dynes/cm^2:

$$\sigma_A\frac{(0+16\pi)}{2} = 32\pi\ \text{(downward)}$$

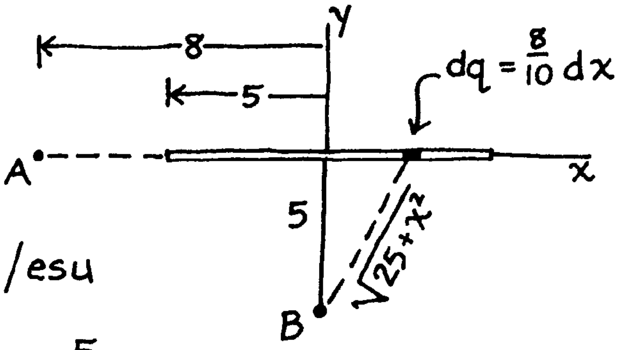
on B: $\sigma_B\frac{(16\pi-12\pi)}{2} = 14\pi$ (upward)

on C: $\sigma_C\frac{(0+12\pi)}{2} = 18\pi$ (upward)

1.23 $E^2/8\pi = Q^2/8\pi r^4$ $\quad dV = 4\pi r^2 dr$ $\quad$ Energy within sphere of radius $R_1 = \frac{Q^2}{2}\int_R^{R_1}\frac{dr}{r^2} = \frac{Q^2}{2R}\left(1-\frac{R}{R_1}\right)$

$= 0.9\,Q^2/2R$ for $R_1 = 10R$.

1.24



At A: $-E_x = \int_{-5}^{5} \frac{0.8\,dx}{(8+x)^2}$

$= 0.8\left(\frac{1}{3} - \frac{1}{13}\right) = 0.205$ dyne/esu

At B: $-E_y = \int_{-5}^{5} \frac{0.8\,dx}{(5^2+x^2)} \cdot \frac{5}{\sqrt{5^2+x^2}}$

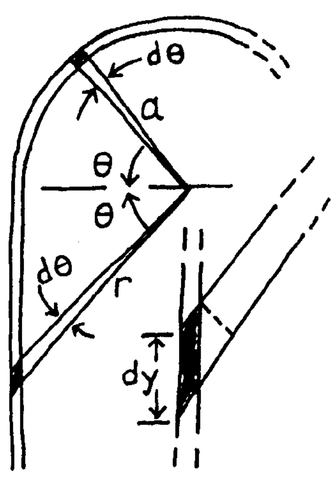
$$= \int_{-5}^{5} \frac{4\,dx}{(25+x^2)^{3/2}} = \frac{40}{125\sqrt{2}} = 0.226 \text{ dyne/esu}$$

1.25

$$15000 = 2 \times \frac{2\lambda}{4\pi\epsilon_0 r} \quad r = 1.5\text{ m} \quad 4\pi\epsilon_0 = 1.11 \times 10^{-10}$$

$$\lambda = 15000 \times 1.11 \times 10^{-10} \times 1.5/4 = 6.3 \times 10^{-7}\text{ C/m} = 6.3 \times 10^{-4}\text{ C/km}$$

1.26

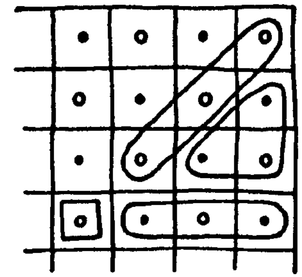


$$dq = \lambda a\,d\theta \quad \frac{dq}{r^2} = \frac{\lambda\,d\theta}{a}$$

$$dq = \lambda\,dy = \lambda r\,d\theta\left(\frac{r}{a}\right)$$

$$\frac{dq}{r^2} = \frac{\lambda\,d\theta}{a}$$

$\boxed{1.27}$ For the 7×7 board the work done in removing the central charge, expressed in units of e^2/s, is :



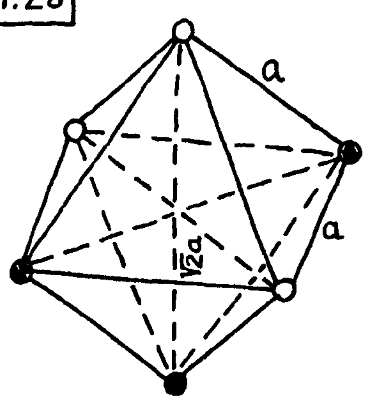
$$W = 4\left(\frac{1}{1} - \frac{1}{2} + \frac{1}{3}\right) + 4\left(-\frac{1}{\sqrt{2}} - \frac{1}{2\sqrt{2}} - \frac{1}{3\sqrt{2}}\right) + 8\left(\frac{1}{\sqrt{5}} - \frac{1}{\sqrt{10}} + \frac{1}{\sqrt{13}}\right)$$

$W = 1.414$ (It looks like $\sqrt{2}$, but it isn't!) W is positive, which is not surprising since the four nearest neighbors are all of the opposite sign. The path along which the charge is carried away doesn't matter and ought not to have been specified in the statement of the problem. For larger arrays we used a computer, with the following results :

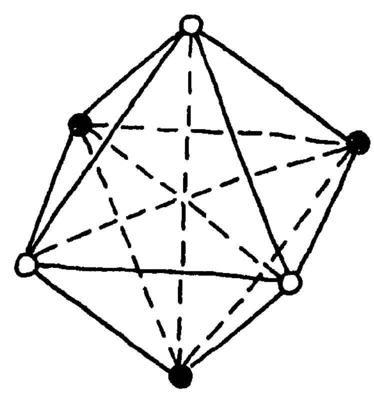
$N \times N$	21×21	99×99	101×101	103×103
$W/\left(\frac{e^2}{s}\right)$	1.54824	1.60120	1.60148	1.60175

For the infinite board W must be close to 1.614

$\boxed{1.28}$



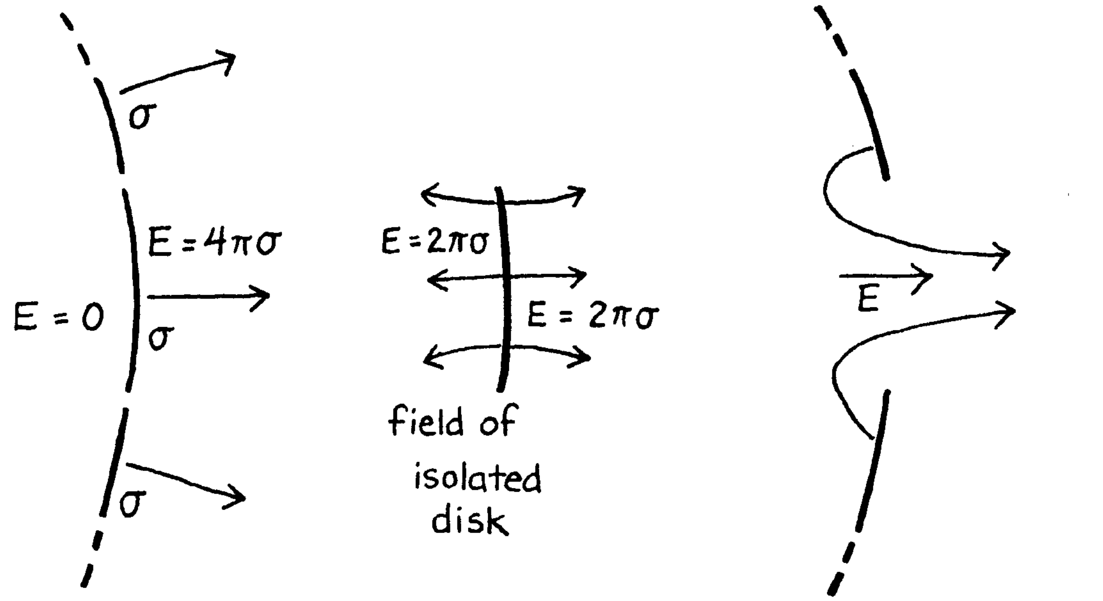
There are 15 pairs, the 12 edges and the 3 diagonals. Sum $\frac{\pm e^2}{r}$ over all pairs :



$$U = e^2\left[-\frac{8}{a} + \frac{4}{a} + \frac{2}{\sqrt{2}a} - \frac{1}{\sqrt{2}a}\right] \qquad U = e^2\left[-\frac{6}{a} + \frac{6}{a} - \frac{3}{\sqrt{2}a}\right]$$

$$= -3.293\ e^2/a \qquad\qquad = -2.121\ e^2/a$$

1.29



E at center of hole $= 2\pi\sigma$

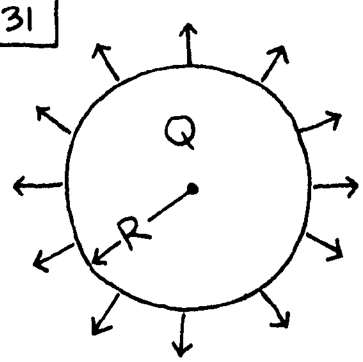
The field of the complete charged spherical shell is a superposition of the field of the charged disk and the field of the shell with a hole.

1.30

$$U = \frac{1}{8\pi}\int E^2 dv = \frac{1}{8\pi}\int_a^b \frac{Q^2}{r^4}\,4\pi r^2 dr = \frac{Q^2}{2}\int_a^b \frac{dr}{r^2}$$

$$= \frac{Q^2}{2}\left(\frac{1}{a} - \frac{1}{b}\right)$$

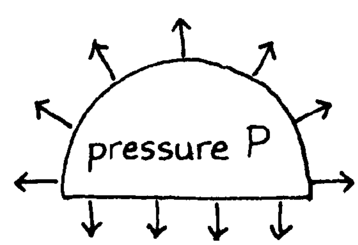
1.31



$$\sigma = Q/4\pi R^2$$

$$E = Q/R^2$$

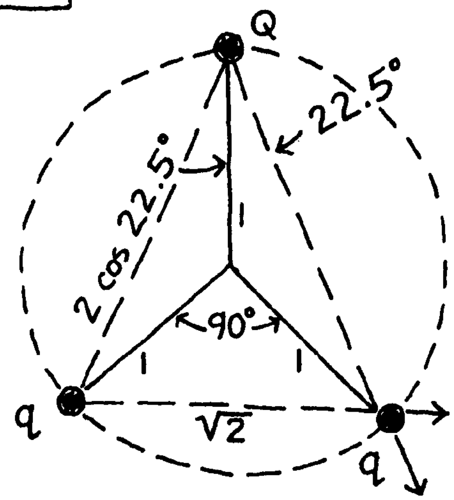
$$P = \frac{\sigma E}{2} = Q^2/8\pi R^4$$



Force on bottom of tank $= \pi R^2 P = \dfrac{Q^2}{8R^2}$

Equal force required to hold hemisphere down.

1.32

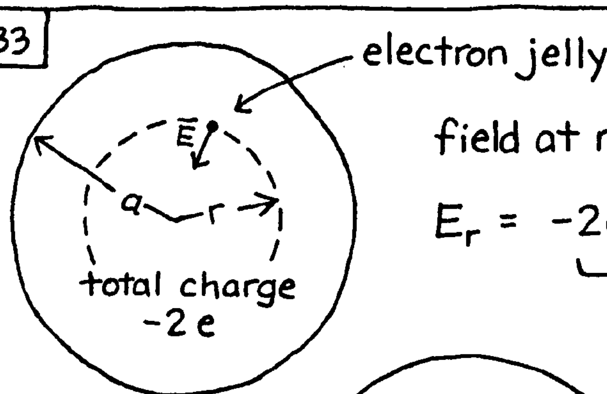


tangential field at q due to Q $= \dfrac{Q \sin 22.5°}{(2 \cos 22.5°)^2} = .1121\,Q$

tangential field at q due to the other $q = \dfrac{q}{(\sqrt{2})^2} \sin 45° = .3536\,q$

Total tangential field zero if $Q = 3.154\,q$

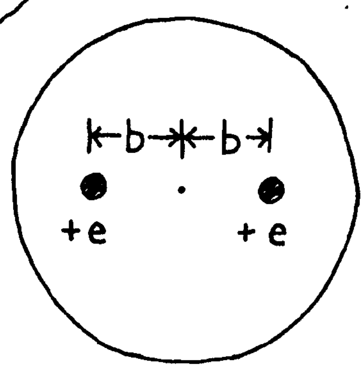
1.33



field at radius r:

$$E_r = \underbrace{-2e\left(\frac{r^3}{a^3}\right)}_{\text{charge inside radius } r}\frac{1}{r^2} = -\frac{2er}{a^3}$$

imbed protons in electron jelly:

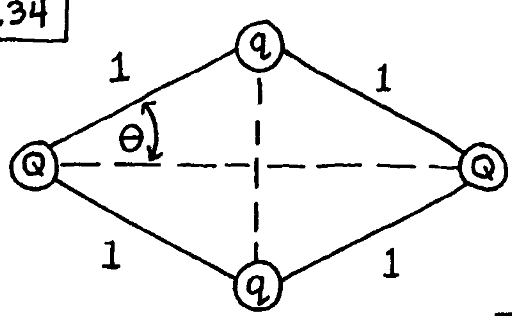


total force on proton is zero if:

$$\frac{2e^2 b}{a^3} = \frac{e^2}{4b^2}, \quad \text{or}$$

$$b = a/2$$

1.34



Tension must be same in all strings. Call it T. q is in equilibrium if $q^2/(2\sin\theta)^2 = 2T\sin\theta$. Similarly, force on Q is zero if $Q^2/(2\cos\theta)^2 = 2T\cos\theta$.

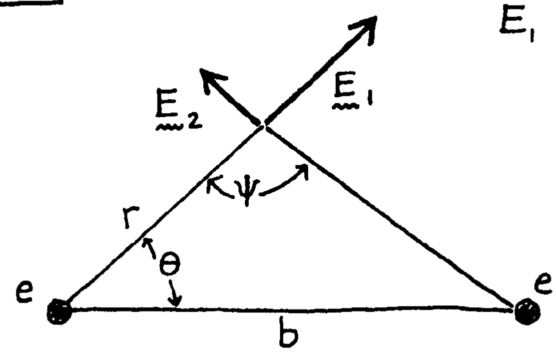
Together these equations give $Q^2\sin^3\theta = q^2\cos^3\theta$ or $q^2/Q^2 = \tan^3\theta$.

To solve by minimizing the energy, note that the only variable terms in the energy sum-over-pairs are $Q^2/2\cos\theta$ and $q^2/2\sin\theta$. Condition for minimum is:

$$0 = \frac{d}{d\theta}\left(\frac{Q^2}{\cos\theta} + \frac{q^2}{\sin\theta}\right) = Q^2\frac{\sin\theta}{\cos^2\theta} - q^2\frac{\cos\theta}{\sin^2\theta} \quad \text{or}$$

$$q^2/Q^2 = \tan^3\theta$$

$\boxed{1.35}$



$$E_1 = \frac{e}{r^2} \qquad E_2 = \frac{e}{r^2 + b^2 - 2rb\cos\theta}$$

$$\cos\psi = \frac{r - b\cos\theta}{(r^2+b^2-2rb\cos\theta)^{1/2}}$$

$$\underset{\sim}{E}_1 \cdot \underset{\sim}{E}_2 = E_1 E_2 \cos\psi$$

$$\int_{\substack{\text{all}\\\text{space}}} \underset{\sim}{E}_1 \cdot \underset{\sim}{E}_2 \, dv = \int_0^{2\pi} d\varphi \int_0^{\pi} \sin\theta \, d\theta \int_0^{\infty} \frac{e^2(r - b\cos\theta)\cancel{r^2}\,dr}{\cancel{r^2}(r^2+b^2-2rb\cos\theta)^{3/2}}$$

$$\int_0^{\infty} \frac{(r - b\cos\theta)\,dr}{(r^2+b^2-2rb\cos\theta)^{3/2}} = -(r^2+b^2-2rb\cos\theta)^{-1/2}\Big]_0^{\infty} = \frac{1}{b}$$

$$\int \underset{\sim}{E}_1 \cdot \underset{\sim}{E}_2 \, dv = \frac{e^2}{b}\int_0^{2\pi} d\varphi \int_0^{\pi} \sin\theta \, d\theta = \frac{4\pi e^2}{b} \qquad \text{Q.E.D.}$$