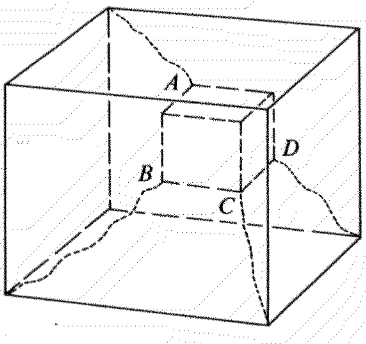

Figure 41.5.
"Orientation-entanglement relation" between a cube and the walls of a room. A 360° rotation of the cube entangles the threads. A 720° rotation might be thought to entangle them still more—but instead makes it possible completely to disentangle them.

§41.5. SPINORS

Paint each face of a cube a different color. Then connect each corner of the cube to the corresponding corner of the room with an elastic thread (Figure 41.5). Now rotate the cube through $2\pi = 360°$. The threads become tangled. Nothing one can do will untangle them. It is impossible for every thread to proceed on its way in a straight line. Now rotate the cube about the same axis by a further 2π. The threads become still more tangled. However, a little work now completely straightens out the tangle (Figure 41.6). Every thread runs as it did in the beginning in a straight line from its corner of the cube to the corresponding corner of the room. More generally, rotations by $0, \pm 4\pi, \pm 8\pi, \ldots$, leave the cube in its standard "orientation-entanglement relation" with its surroundings, whereas rotations by $\pm 2\pi, \pm 6\pi, \pm 10\pi, \ldots$, restore to the cube only its orientation, not its orientation-entanglement relation with its surroundings. Evidently there is something about the geometry of orientation that is not fully taken into account in the usual concept of orientation; hence the concept of "orientation-entanglement relation" or (briefer term!) "version" (Latin *versor,* turn). Whether there is also a detectable difference in the physics (contact potential between a metallic object and its metallic surroundings, for example) for two inequivalent versions of an object is not known [Aharonov and Susskind (1967)].

In keeping with the distinction between the two inequivalent versions of an object, the spin matrix associated with a rotation,

$$R = \cos(\theta/2) - i(\boldsymbol{n}\cdot\boldsymbol{\sigma})\sin(\theta/2), \tag{41.48}$$

reverses sign on a rotation through an odd multiple of 2π. This sign change never shows up in the law of transformation of a vector, as summarized in the formula

$$X \longrightarrow X' = RXR^* \tag{41.49}$$

(two factors R; sign change in each!). The sign change does show up when one turns from a vector to a 2-component quantity that transforms according to the law

$$\xi \longrightarrow \xi' = R\xi. \tag{41.50}$$

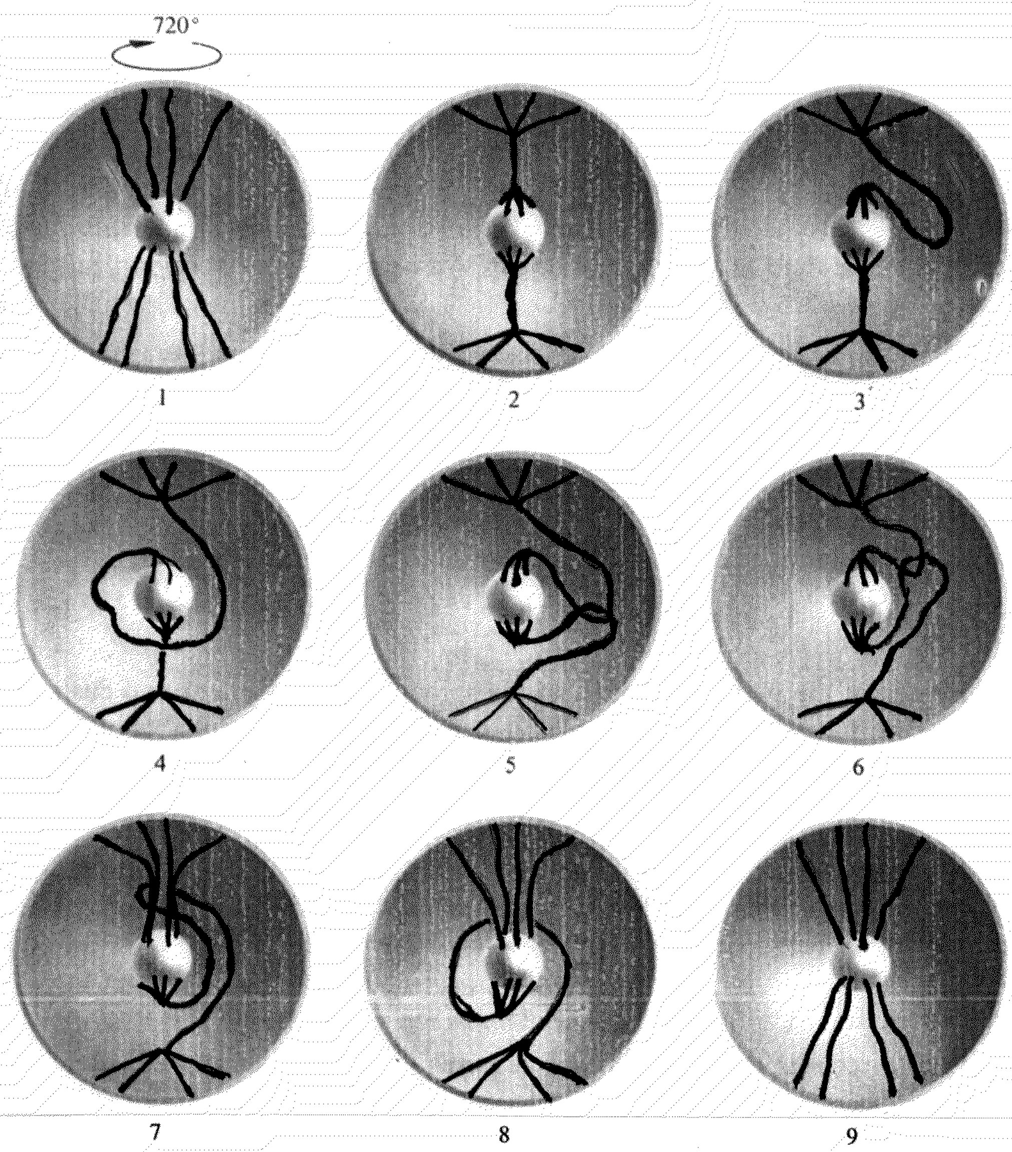

Figure 41.6.
An object is connected to its surroundings by elastic threads as in Figure 41.5. (Eight are shown here; any number could be used.) Rotating the object through 720° and then following the procedure outlined (Edward McDonald) in frames 2–8 (with the object remaining fixed), one finds that the connecting threads are left disentangled, as in frame 9 (lower right).

Such a quantity is known as a spinor. A spinor reverses sign on a 360° rotation. It therefore provides a reasonable means to keep track of the difference between the two inequivalent versions of the cube. More generally, with each orientation-entanglement relation between the cube and its surroundings one can associate a different value of the spinor ξ. Moreover, there is nothing that limits the usefulness of the spinor concept to rotations. Also, for the general combination of boost and rotation, one can write

Lorentz transformation of a spinor

$$\xi \longrightarrow \xi' = L\xi. \tag{41.51}$$