# A Few Words About Spin One Half
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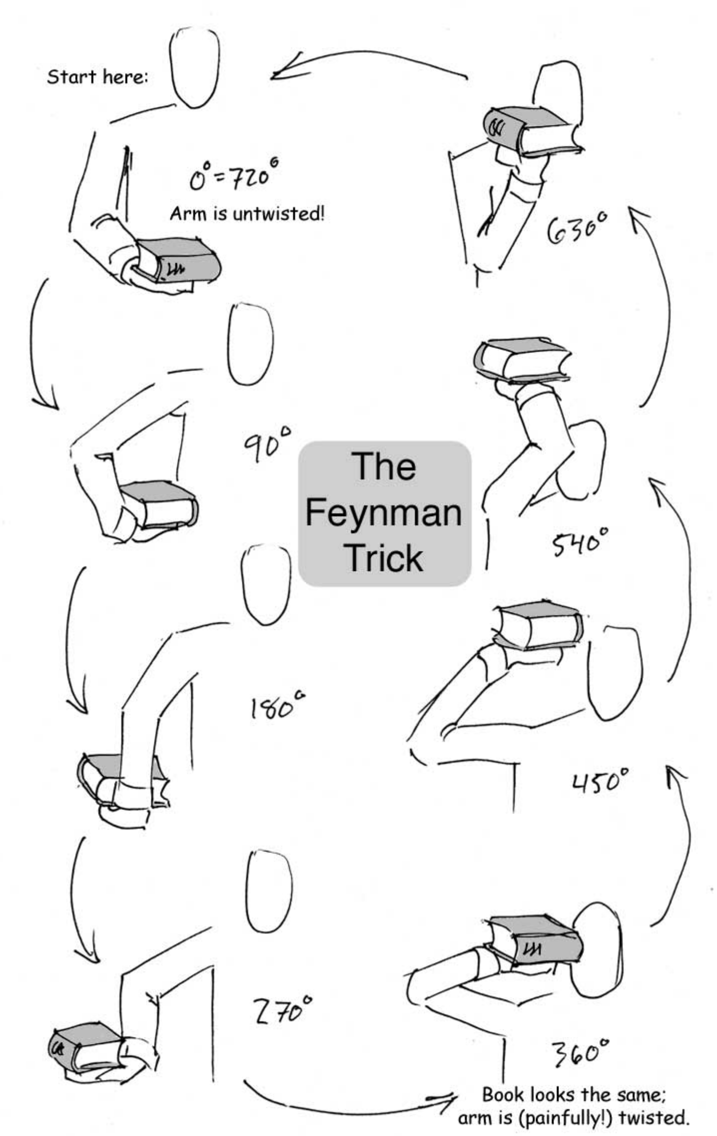
Gregory Pope prize essay

Here's a puzzle: take an object (say a nearby book), rotate it through three hundred and sixty degrees (not 180!) yet find that it's changed. Rotate it *another* three hundred sixty degrees, and it looks the same. What's going on?

The answer to this riddle is simple: the book was tied to something that didn't move. An amazing demonstration of this bizarre fact was popularized by the Nobel laureate Richard Feynman, who was inspired by the contortions of a waitress at a bar:



When you rotate the book through the first 360 degrees, it appears the same but your arm is twisted. If you keep rotating *in the same direction*, you can also untwist your arm. If you think this has more to do with physiology than physics, the same manipulation can be done with a length of ribbon instead.

What does this trick have to do with physics? Plenty. The system of (book + arm) we just described has the rotational properties of a mathematical object called a *spinor* that describes a very unusual yet very fundamental property of subatomic particles called *spin*. The discovery and explanation of spin was one of the major elements in the revolution produced by quantum mechanics, and the list of people who participated in its explanation reads like a veritable who's who of twentieth century physics.

The story begins back at the start of the last century, when the physics community was troubled by an increasing body of spectroscopic experiments. If you take a bare atom and put energy into it (using an apparatus almost identical to the modern neon light), you get energy out in the form of light. The troubling aspect was over what *kind* of light: the classical physics of the time asserted that atoms should be able to emit light of any color, while instead only a few specific colors were observed for each type of atom.

Physicists set out to understand this mystery by first answering the 'what' and only then attacking the more interesting question of 'how'. Starting with Niels Bohr's paper on the hydrogen atom in 1913, they were able to write down mathematical equations describing the frequencies of light that were being observed, but no one knew what these equations *meant*. What became clear was that the atom had a discrete internal structure: instead of having a range of continuous configurations, atoms could only occupy a specific set of states. Different states had different energy, and as an atom switched from one state to another it got rid of the energy difference in the form of light.

This idea – that, at its most fundamental level, nature is discrete rather than continuous – put the "quantum" in "quantum mechanics". The new theory that emerged explained all the observed spectroscopic data in terms of electrons in an atom jumping between different discrete energy states. Everything fit together – almost. When atoms were placed in a strong magnetic field, it was found that one color of light became two – the spectral lines split.

Obviously, these new colors meant that there were more states accessible to the electron than previously thought – but where were the new states coming from? Early theories ascribed them to unclearly stated properties of the "atomic core", but the correct idea started with a twenty-year-old graduate student named Ralph de Kronig. He found that the extra states could be explained if the electron was spinning, like a miniature top. According to previously established rules of quantum mechanics, the direction of spin had to be quantized – if the electron had what we would now call a spin of one-half (times the fundamental unit of quantum angular momentum), it would have two different ways of spinning and the observed splitting could be explained.

When Kronig explained this to Wolfgang Pauli, he responded that "that is really a very amusing idea". Pauli knew that according to classical physics, Kronig's proposal was impossible. It was known that if an electron had any spatial extent at all a catastrophe resulted: the electric charge on one side of the electron would repel that on the other side, meaning that if such a particle were given a push it would go faster and faster … without limit. The electron had to be a mathematical point, and how could a point spin – what was left to do the spinning? Under Pauli's criticism, Kronig didn't publish his results.

Six months later two other physicists, George Uhlenbeck and Samuel Goudsmit, had the same idea independently of Kronig. They sent their results away for publication but came under the same criticism Kronig faced. When they asked their advisor, Paul Ehrenfest, to retract the paper, he replied "I have already sent off your paper; besides, you are both young enough that you are permitted to make dumbheads of yourselves." The paper appeared in the journal *Nature* in 1926.

Later it was realized that the spinning top picture got more right than it did wrong, and ironically Uhlenbeck and Goudsmit were credited with the discovery of the electron's spin. The full explanation came a few years later with Paul Dirac's unification of quantum mechanics and special relativity, using new methods developed by the mathematicians David Hilbert and John von Neumann. Dirac was able to derive a fully quantum-mechanical equation governing the motion of electrons, and he found that an extra degree of freedom that behaved exactly as Uhlenbeck and Goudsmit claimed emerged naturally from the mathematics. According to Dirac's equation, the spin of the electron was indeed identical to that of a spinning top even though the physical circumstances had to be different. The spin of an electron is an essentially quantum mechanical phenomenon with no exact counterpart in the world of our everyday experience. As modern textbooks manage to say nonchalantly, spin is a "non-spatial degree of freedom".

The fact that the electron has a spin of one-half is significant in itself. A mathematical theorem known as the Pauli exclusion principle states that no two electrons in an atom can be in the same quantum state, which means that if we try to add an electron to an atom it needs to occupy a state that's higher in energy than that occupied by the electrons already there. The fact that new states are filled up in this orderly manner as we look at atoms with increasing number of electrons is responsible for the periodicity of the periodic table, a fact first realized by E. C. Stoner. If the electron didn't have spin one-half, all the electrons in an atom would enter the lowest energy state. The periodic table – chemistry – indeed, the material world as we know it would be impossible.

Even today it's premature to claim that we understand everything about the nature of spin. The prime example of this is the subtle phenomenon of entanglement, whose importance went overlooked for a long time. When two spins are in an entangled state, their behavior becomes correlated in a way which, again, has no classical counterpart. The details are too technical to describe in this amount of space, but perhaps the best qualitative description is that the information describing the entangled spins is *nonlocal*. In other words, the two spins together carry more information than the sum of that carried by the two spins in isolation. Many details of this picture remain unanswered questions as of this writing.

Once we have this quantum *correlation*, it's a comparatively small step onwards to quantum *communication* and then quantum *computation*. If we generate many pairs of entangled spins and give one member of each pair to two people (invariably called Alice and Bob; A and B for short) then A can send messages to B by performing manipulations on her spins which, thanks to entanglement, automatically influence the state of his spins. More interestingly, there's no way for a third party to eavesdrop on the conversation without breaking the entanglement in a detectable way.

Quantum computation is a field which has witnessed explosive growth in the past five years, although its conceptual roots go back much further (most credit the original idea to Feynman – his influence is inescapable!) If one makes use of entanglement in clever ways, one can effectively perform the same calculation for many different numbers all at once. A quantum computer wouldn't be able to do anything today's computers couldn't do, but certain types of programs would run much faster on a quantum computer than they could on any classical computer.

Throughout his career, Dirac was known for his mathematical abstraction and esoteric writing style, but he began his undergraduate study as an electrical engineer. It's one of science's many small ironies that, thanks to his work on spin, he has unintentionally left a profound influence on that field as electrical engineers (and physicists) race to build quantum computers and communications devices. Then again, we should expect the story of spin to have many surprising *turns* of events.

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