

The TIPT results for the energy corrections are given by:

$$E'_n = E_n^{(0)} + E_n^{(1)} + E_n^{(2)} + E_n^{(3)} + \dots$$

where

$$E_n^{(1)} = \langle n | \mathcal{H}_1 | n \rangle$$

$$E_n^{(2)} = \sum_1' \frac{\langle n | \mathcal{H}_1 | l \rangle \langle l | \mathcal{H}_1 | n \rangle}{E_n - E_l}$$

$$E_n^{(3)} = \sum_1' \sum_k' \frac{\langle n | \mathcal{H}_1 | l \rangle \langle l | \mathcal{H}_1 | k \rangle \langle k | \mathcal{H}_1 | n \rangle}{(E_n - E_l)(E_n - E_k)} - \sum_1' \frac{\langle n | \mathcal{H}_1 | l \rangle \langle l | \mathcal{H}_1 | n \rangle \langle n | \mathcal{H}_1 | n \rangle}{(E_n - E_l)^2}$$

And the TIPT results for the wavefunction corrections are given by:

$$|n'\rangle = |n^{(0)}\rangle + |n^{(1)}\rangle + |n^{(2)}\rangle + \dots$$

where

$$|n^{(1)}\rangle = \sum_1' \frac{|l\rangle \langle l|}{E_n - E_l} \mathcal{H}_1 |n\rangle$$

$$|n^{(2)}\rangle = \sum_1' \sum_k' \frac{|l\rangle \langle l|}{E_n - E_l} \mathcal{H}_1 \frac{|k\rangle \langle k|}{E_n - E_k} \mathcal{H}_1 |n\rangle - \frac{\langle n | \mathcal{H}_1 | n \rangle}{E_n - E_l} \sum_1' \frac{|l\rangle \langle l|}{E_n - E_l} \mathcal{H}_1 |n\rangle$$

Notice that the perturbation Hamiltonian  $\mathcal{H}_1$  appears in each correction term a total of m-times where m is the order of the correction.