Physics 541 Exam 2
Due May 12, 2008

Explain the physics for each topic below in your own words.

You do not have to write a perfect essay on each topic, but do write enough to convince me that you really understand the topic. You will only need to write a paragraph to convince me. However, do not forget that equations, pictures, and graphs are more effective than words. If a picture is worth a thousand words, and an equation is worth a thousand pictures, then ..... In other words, include the relevant equations and pictures.

1	Prove the variational principle
2	The art of picking a good trial wavefunction
3	Compare the variational method for the ground state with that for the first excited state
4	Self-consistent field methods
5	The Hartree method and the Hartree trial wavefunction
6	The Hartree-Fock method and the Hartree-Fock trial wavefunction
7	Direct integrals versus exchange integrals
8	The quantum mechanics of identical particles
9	Spin and statistics
10	Feynman's view of understanding the physics versus giving a freshman physics lecture on it
11	The classical turning points and their dragons
12	The action integral
13	The application of the WKB method to tunneling problems
14	Alpha decay and the Gamow factor
15	The application of the WKB method to bound state problems: <T>, <V>, and <H>
16	The WKB integral for two hard walls, two soft walls, and one hard and one soft wall
17	Airy's equation and Airy Functions
18	The Airy functions and the rainbow
19	The QM bouncing ball: the energy eigenvalues and eigenfunctions (the QM statics)
20	The QM bouncing ball: the time evolution of a Gaussian packet (the QM dynamics)

The Variational Method

Consider the normalized Gaussian variational wavefunction

$$\psi(x) = (2b/\pi)^{\frac{1}{4}}\ e^{-bx^2}$$

and the eighth-order potential

$$V(x) = A\ x^8$$

where $A > 0$ and $b > 0$.

(a) Calculate the expectation value of the kinetic energy $\langle T \rangle$.

(b) Calculate the expectation value of the potential energy $\langle V \rangle$.

(c) Calculate the variational upper-bound on the ground state energy $\langle H \rangle$.

(d) Plot $\langle T \rangle$, $\langle V \rangle$, and $\langle H \rangle$ versus b. Explain how the mathematical competition between $\langle T(b) \rangle$ and $\langle V(b) \rangle$ determines the variational upper-bound on the ground state energy.

(e) Explain how $\langle T \rangle$ depends on the shape of the wavefunction. Explain how $\langle V \rangle$ depends on the shape of the wavefunction and on the shape of the potential. Explain how the variational parameter b modulates the compromise between these two energies.

You might find the following Gaussian integrals useful.

$$\int_0^\infty x^{2n+1} e^{-px^2}\, dx = \frac{n!}{2p^{n+1}} \quad \text{for} \quad p > 0 \quad \text{and} \quad n = 0, 1, 2, 3, \ldots$$

$$\int_0^\infty x^{2n} e^{-px^2}\, dx = \frac{(2n-1)\cdots 5 \cdot 3 \cdot 1}{2(2p)^n}\sqrt{\frac{\pi}{p}} \quad \text{for} \quad p > 0 \quad \text{and} \quad n = 0, 1, 2, 3, \ldots$$

The WKB Method for Bound States

Consider the eighth-order potential

$$V(x) = A\, x^8$$

where $A > 0$.

(a) Calculate the classical turning points.

(b) Calculate the WKB integral between the classical turning points.

(c) Calculate the WKB energy eigenvalues.

(d) Compare your WKB ground-state energy with your variational upper-bound on the ground state energy.

The WKB Method for Barrier Penetration

Consider the eighth-order potential

$$V(x) = C - A\, x^8$$

where $A > 0$, $C > 0$, and $0 < E < C$.

(a) Calculate the classical turning points.

(b) Calculate the WKB integral between the classical turning points.

(c) Calculate the WKB transmission coefficient versus E, A, and C.