**Physics 541 Exam 1**
**Due April 28, 2008**

Explain the physics for each topic below in your own words.

You do not have to write a perfect essay on each topic, but do write enough to convince me that you really understand the topic. You will only need to write a paragraph to convince me. However, do not forget that equations, pictures, and graphs are more effective than words. If a picture is worth a thousand words, and an equation is worth a thousand pictures, then ..... In other words, include the relevant equations and pictures.

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|---|---|
| 1 | The orbital angular momentum L and the eigenvectors and eigenvalues of L^2 and Lz |
| 2 | The spin angular momentum S and the eigenvectors and eigenvalues of S^2 and Sz |
| 3 | The total angular momentum J and the eigenvectors and eigenvalues of J^2 and Jz |
| 4 | The commutation relations for L, S, and J |
| 5 | The uncertainty relations for L, S, and J |
| 6 | The ladder operators for L, S, and J |
| 7 | The Pauli spin matrices and spinors |
| 8 | The magnetic dipole moment of the electron and of the proton |
| 9 | The gyromagnetic ratio of the electron and of the proton |
| 10 | The Larmor precession frequency of the electron and of the proton |
| 11 | The Stern-Gerlach experiment |
| 12 | Spin up and spin down |
| 13 | The addition of spin 3/2 and spin 2 |
| 14 | The addition of spin 2 and spin 3 |
| 15 | The ladder of Jz angular momentum states for integer J and for half-integer J |
| 16 | Singlets, doublets, triplets, quartets, quintets, sextets, septets, octets, nonets, decets, ...... |
| 17 | The Clebsch-Gordan coefficients |
| 18 | The semiclassical vector model |
| 19 | The individual particle angular momentum basis (aka, the L and S basis) |
| 20 | The total angular momentum basis (aka, the J basis) |

## Problem 1. The Quantum Mechanics of Spin

Consider the quantum mechanical behavior of a spin 1/2 particle that starts out with the zero time state vector

$$\mid \psi(0) > \; => \; N \begin{pmatrix} 3+2i \\ 2-3i \end{pmatrix}$$

(a) Calculate the normalization constant N.

(b) Calculate the zero-time expectation values $< S_x(0) >$, $< S_y(0) >$, and $< S_z(0) >$.

(c) If $S_x$, $S_y$, and $S_z$ were measured for the zero-time state vector $\mid \psi(0) >$, what would be the respective possibilities and probabilities that would be obtained? Show that your results here agree with your zero-time expectation values from part b.

(d) Now suppose that a static magnetic field $\vec{B} = B_0\hat{z}$ is turned on at $t = 0$. Calculate the time-dependent state vector $\mid \psi(t) >$. The Hamiltonian is $H = -\gamma\vec{S}\cdot\vec{B}$.

(e) Calculate the time-dependent expectation values $< S_x(t) >$, $< S_y(t) >$, and $< S_z(t) >$ of the spin in the magnetic field.

(f) Make a sketch that shows the behavior of the spin versus time using the semiclassical vector model. Explain what the semiclassical spin does: What axis does it rotate around? How fast does it rotate? What are the time averages of $S_x$, $S_y$, and $S_z$? Explain how the time evolution of the semiclassical vector model is related to the time-dependent expectation values that you calculated in part e.

## Problem 2. Angular Momentum Addition

Consider an electron in a hydrogen atom in the state

$$N \begin{pmatrix} R_{32}\ Y_{2-2} \\ R_{43}\ Y_{33} \end{pmatrix}$$

(a) Calculate the normalization constant N.

(b) Sketch the radial probability distribution for a spin up electron. Sketch the radial probability distribution for a spin down electron.

If you measure the following quantitites, what are the possible values that you could obtain, and with what probabilities would you obtain them?

(c) $L^2$ and $L_z$

(d) $S^2$ and $S_z$

(e) $J^2$ and $J_z$

(f) the energy