Time-Dependent Perturbation Theory
Physics 541 Homework Set 6
Due May 28, 2003

Three "Simple Square Well" Problems

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1. Consider an electron that is trapped in an infinite-wall square well potential with width $a = 1 \times 10^{-8}$ centimeters. At $t = 0^-$, the electron is in the ground state ($n = 1$) of this well. At $t = 0^+$, a rectangular potential dip is suddenly introduced at the center of the well for 5×10^{-18} seconds, and is then is removed. The depth of the dip is $V_0 = -10^4$ eV, the width of the dip is 10^{-12} centimeters, and the position of the center of the dip is $a/2$.

(a) Calculate the energies of the first four energy eigenstates.

(b) Calculate the probability that the particle is in the second ($n = 2$), third ($n = 3$), and fourth ($n = 4$) energy eigenstate immediately after the pulse is over.

(c) Calculate and plot the probability versus pulse duration that the particle is left in the $n = 3$ energy eigenstate for pulse durations of 5, 10, 15, 20, 25, 30, 35, 40 $\times 10^{-18}$ seconds. From your graph, what is the probability that the particle will be in the $n = 3$ state at 27×10^{-18} seconds?

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2. Reconsider the electron that is trapped in the infinite-wall square well potential of the previous problem. Now starting at $t = 0$ and ending at $t = t_1$, the harmonic perturbation $H_1 = A \sin(\omega_0 t)$ is applied. Here A is a constant (independent of time and space) with magnitude 1 eV. This perturbation causes the entire bottom of the well to be raised and lowered sinusoidally with the frequency $f_0 = \omega_0/2\pi$. Assume that the frequency $f_0 = 2.8 \times 10^{16}$ Hertz, so that the corresponding energy is $hf_0 = 114$ eV, which is precisely the energy required to go from the $n = 1$ energy eigenstate to the $n = 2$ energy eigenstate. Show that no excitation will occur into the $n = 2$ or any other level.

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3. Now change the position-independent perturbation of the previous problem to a position-dependent perturbation with

$$A(x) = -1 \text{ eV from x} = 0 \text{ to x} = \text{a}/2$$

$$A(x) = +1 \text{ eV from x} = \text{a}/2 \text{ to x} = \text{a}$$

and let this perturbation continue for 3.56×10^{-16} seconds—which is 10 complete cycles—and then stop.

(a) Calculate the probability that the $n = 2$ energy eigenstate is excited.

(b) Calculate the probability that the $n = 3$ energy eigenstate is excited.

(c) Calculate the probability that the $n = 4$ energy eigenstate is excited.

These three problems are Sherwin's problems 10.1, 10.3, and 10.4; see the "Sherwin TDPT" section of the virtual book for the complete details.

Three "Exotic Hydrogen Atom" Problems

The muonic hydrogen atom consists of a negative muon electrostatically bound to a positive proton. The mass of the muon is 206.77 times the mass of the electron. The charge and the g-factor of the muon are identical to the electron. Before you start using TDPT, calculate the new reduced mass, the new Bohr radius, and the new energy scale for muonic hydrogen.

4. Consider a muonic hydrogen atom which is in its ground state at $t = -\infty$. If an electric field $\vec{E}(t) = E_0 \exp(-t^2/\tau^2)\hat{z}$ is applied until $t = +\infty$, show that the probability that the muonic atom ends up in any of its $n = 2$ energy eigenstates is given to first order by

$$P(n = 2) = \left[\frac{eE_o}{\hbar}\right]^2 \left[\frac{2^{15} a_0^2}{3^{10}}\right] \pi\, \tau^2\; exp\left(-\omega^2\tau^2/2\right)$$

where $\omega = (E_{2lm} - E_{100})/\hbar$. This problem is also Shankar's exercise 18.2.2.

5. Redo "Solved Problem 1" in the virtual book for muonic hydrogen. This problem is also Griffith's problem 9.1.

6. Redo "Solved Problem 5" in the virtual book for muonic hydrogen: i.e., consider the possible muon transitions between the $n = 3, l = 0, m = 0$ energy eigenstate and the ground state. This problem is also Griffith's problem 9.13.