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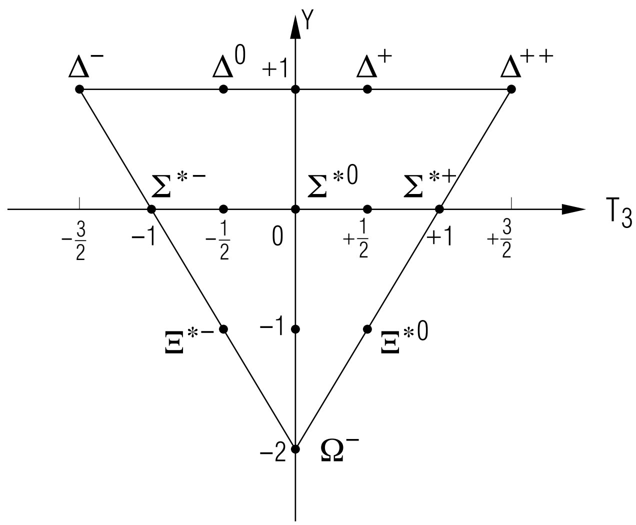
MISN-0-282

SU(3) AND THE QUARK MODEL



Project PHYSNET Physics Bldg. Michigan State University East Lansing, MI

SU(3) AND THE QUARK MODEL

by

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Input Skills:

1. Explain briefly the isospin concept and calculate the total isotopic spin for a system of particles (MISN-0-278).
2. Explain how hadrons interact via the strong interaction (MISN-0-280).
3. List the basic couplings of the weak interaction and explain the function of the W particle (MISN-0-281).

Output Skills (Knowledge):

K1. Give the three forms of the Y vs. T plots for hadrons, and list the values of Y and T for the particles in each one.

K2. State the properties that all particles in any supermultiplet have in common.

K3. Give the essential quark characteristics for each of the three quarks: baryon number, spin, strangeness and charge.

K4. Write the symbols for the three quarks and the three antiquarks.

K5. State how many quarks and/or antiquarks make up mesons, baryons, and antibaryons.

K6. Discuss the model which accounts for the difference in particle masses within a supermultiplet.

K7. Discuss the basics of particle decays and interactions, in terms of quarks.

Output Skills (Problem Solving):

S1. Given the quark content of a particle, calculate B, Y, T, T_3, Q for that particle, and conversely.

S2. Given a particle decay, draw a quark diagram that represents it.

External Resources (Required):

1. M. J. Longo, *Fund. of Elem. Part. Physics*, McGraw-Hill (1973).
2. G. F. Chew, et al, *Scientific American*, Feb. 1964.

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1. Abstract

We here deal deal with a way of classifying the hadrons: according to SU(3) symmetry operations. This system is known as the quark model or the eight-fold way. The quark model of high energy physics is analogous to the periodic table of the elements in that it provides an ordering of the particles.

2. Readings

Longo, Chapter 8

G. F. Chew, M. Gell-Mann, and A. H. Rosenfeld, "Strongly Interacting Particles," *Scientific American* (Feb. 1964).

3. The Quark Concept

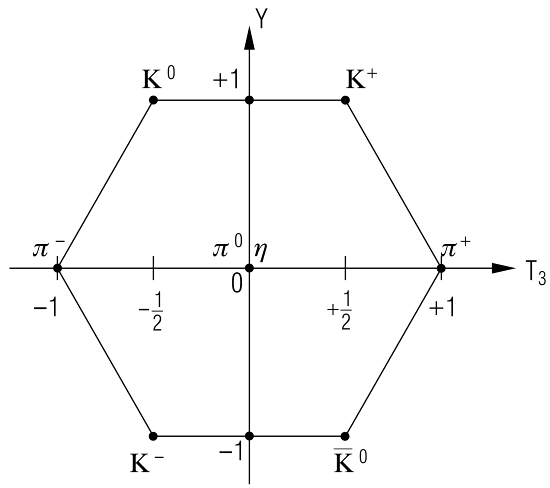
3a. Quarks as Elementary Particles. One of the goals of high energy physics is to explain the properties (mass, spin, charge, isotopic spin, strangeness) of the hadrons in terms of something more fundamental. The idea of the quark model is to invent a small set of particles, imbue them with appropriate properties, and use them to construct the hadrons much as neutrons and protons are used to construct the various atomic nuclei. The hypothetical fundamental building blocks are called quarks. They have never been isolated and have never been observed.

3b. Observed Symmetry Patterns in Hypercharge and Isospin. The quark model deals with the isotopic spin (both magnitude and 3-component), hypercharge, strangeness, and baryon number of the hadrons.

We begin with an observation that, if the particle properties are plotted a certain way, then there are only three forms to the plots. All of the plots have hypercharge on the vertical axis and 3-component of isotopic spin on the horizontal axis. We plot on the same graph only those particles which have the same baryon number, spin, and intrinsic

parity.[1] The particles on any one diagram have nearly the same mass.

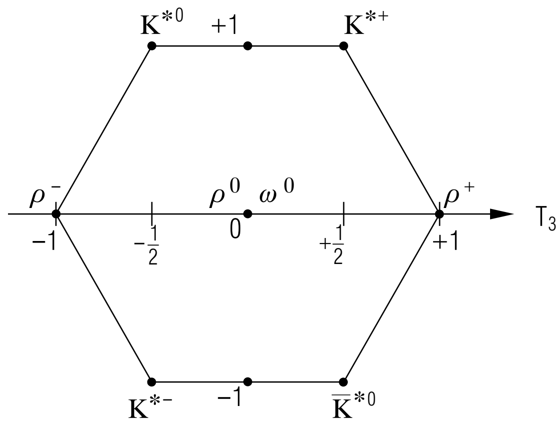
We start with the 0^- mesons (spin 0, odd parity):



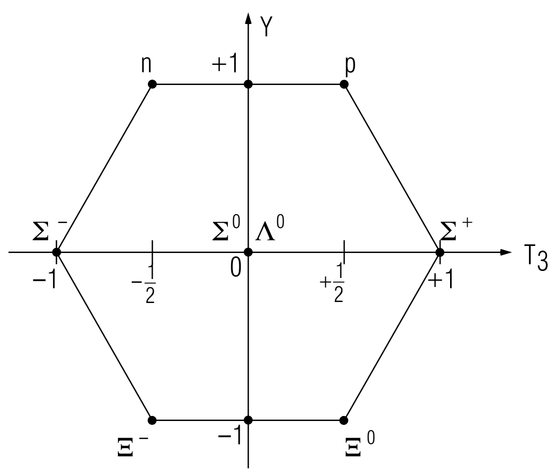
Both the π^0 and the η have $T_3 = 0$. However the π^0 belongs to an isospin triplet and has $T = 1$ while the η is an isospin singlet and has $T = 0$. This plot contains 8 particles and they are jointly called an octet.

There is an octet of 1^- mesons:

[1] A hadron can be assigned an intrinsic parity (+ or −) depending on whether or not the wave function of the particle, when the particle has zero orbital angular momentum, changes sign with operation by the parity operator. Intrinsic parity is denoted by a + or − superscript on the spin. This is a minor technical point for this discussion. You should realize that all particles in a given diagram have the same spin but there may be more than one diagram corresponding to a given spin.



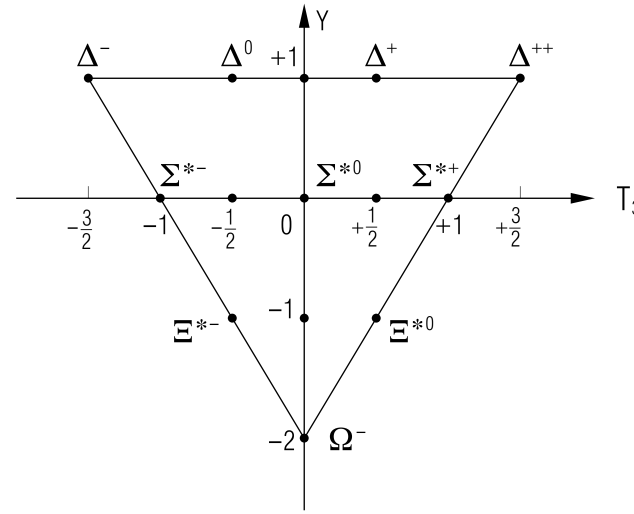
There is an octet of $1/2^+$ baryons:



The octet always consists of an isospin doublet with $Y = +1$, an isospin triplet with $Y = 0$, an isospin singlet with $Y = 0$, and an isospin doublet with $Y = -1$.

The second type pattern to be considered is the decimet, composed of ten particles: an isospin quartet with $Y = +1$, an isospin triplet with $Y = 0$, an isospin doublet with $Y = -1$, and an isospin singlet with $Y = -2$. Only baryons have been found to form decimets and, as we

shall see, the quark model provides a reason why. Here is a decimet of $3/2^+$ baryons:



This is the only complete decimet known but there are undoubtedly others at higher masses (yet to be discovered "resonances").

It is important to note that if a particle belongs to a given multiplet, all of its isospin partners belong to the same multiplet. The patterns shown here combine several sets of isospin partners to form a larger pattern than that provided by isospin alone.

We have now described two of the three forms, the octet and the decimet. The third form is the simplest. It consists of a single particle with $Y = 0$, $T = 0$, and $T_3 = 0$ and is called a singlet. It is easy to confuse one of these particles with the isotopic spin singlet which occurs in the octet of the same spin and parity. For example, the $\phi(1019)$ meson may be a 1^- singlet meson. If differs from the ω^0 only in mass. Which belongs to the octet and which to the singlet? We shall see that the quark model assigns different quark content to the $Y = 0$, $T = 0$ singlet and to the $Y = 0$, $T = 0$ particle in the octet. But quarks are not observable, so this distinction cannot be used. The point is that some assignments of particles to octets or singlets are arbitrary at present and, in fact, the physical particle may be some superposition of the two states.

If all particles (including those undetected at present) are put into singlets, octets, and decimets, the patterns have some predictive power. For example:

a. There are no $T > 3/2$ particles.

b. There can be no $Y > +1$ particles.

c. There can be no $Y < -2$ particles.

d. $Y = 1$ particles have either $T = 3/2$ or $T = 1/2$.

e. $Y = 0$ particles have either $T = 1$ or $T = 0$.

f. $Y = -1$ particles have $T = 1/2$.

g. $Y = -2$ particles have $T = 0$.

3c. Overview of the Quark Model of Elementary Particles. The job of the quark model is to explain these patterns. The basic ideas are:

a. There are 3 quarks and 3 antiquarks. (Recent developments have caused physicists to postulate the existence of a fourth quark but we shall discuss the three quark model now and introduce the fourth quark later.)

b. A meson consists of a quark and an antiquark.

c. A baryon consists of 3 quarks; an antibaryon of 3 antiquarks.

d. The quarks are assigned values of Y, T, and T_3.

e. By combining quarks in all possible ways which are consistent with (b) and (c), it is possible to construct those particles that appear in the singlets, octets, and decimets, and no others. In this regard, two different combinations of quark states may actually have the same quark content but they are combined to give two different values of the magnitude of the total isotopic spin. For example, a $T_3 = 1/2$ and a $T_3 = -1/2$ particle may combine to give either a $T = 1$, $T_3 = 0$ state or a $T = 0$, $T_3 = 0$ state.

f. The quark content of a particle in a certain multiplet is the same as the quark content of the analogous particle in other multiplets of the same type. For example, the quark content of the ρ^- is the same as the quark content of the π^-. The *dynamics* of the quarks are different and this explains the different mass and spin of the ρ^- and π^-.

g. Quarks are also assigned spin and baryon number. The total angular momentum of the quarks is the spin of the composite particle and the net baryon number of the quarks is the baryon number of the composite particle.

4. Quark Content

4a. Quark Constituents of Hadrons. It is possible to build all of the hadrons we have been discussing from the following three quarks (and their antiquarks):

Quark	B	T	T_3	σ	S	Y	Q
u	1/3	1/2	1/2	1/2	0	1/3	2/3
d	1/3	1/2	−1/2	1/2	0	1/3	−1/3
s	1/3	0	0	1/2	−1	−2/3	−1/3
$\overline{\text{u}}$	−1/3	1/2	−1/2	1/2	0	−1/3	−2/3
$\overline{\text{d}}$	−1/3	1/2	1/2	1/2	0	−1/3	1/3
$\overline{\text{s}}$	−1/3	0	0	1/2	1	2/3	1/3

Here B = baryon number, T = magnitude of isospin, T_3 = 3-component of isospin, σ = spin in units of $\hbar$, Y = hypercharge, and Q = charge in units of e. The first 5 entries (B, T, T_3, σ, and S) are assigned. Y and Q are calculated from B, S, and T_3: $Y = B + S$, $Q = Y/2 + T_3$.

The s quark is the only quark with non-vanishing strangeness. In fact the number of s quarks in the particle defines its strangeness. Conservation of strangeness then stems from the conservation of the number of s quarks. Conservation of baryon number also stems from conservation of quarks.

4b. Quark Constituents of Mesons. Mesons are constructed by combining a quark with an antiquark. There are 6 different combinations of a quark with a different antiquark: $d\bar{s}$, $u\bar{s}$, $\overline{\text{d}}$s, $\overline{\text{u}}$s, u$\overline{\text{d}}$, and $\overline{\text{u}}$d. Each of these has a definite baryon number (0), strangeness, charge, isotopic spin magnitude, and 3-component of isotopic spin. These combinations can be identified with particles on the basis of the values of these quantities.

	S	Q	T_3	T	Particle
$d\bar{s}$	1	0	$-1/2$	$1/2$	K^0 , K^{*0}
$u\bar{s}$	1	1	$1/2$	$1/2$	K^+ , K^{*+}
$\bar{d}s$	-1	0	$1/2$	$1/2$	$\overline{K}^0$, $\overline{K}^{*0}$
$\bar{u}s$	-1	-1	$-1/2$	$1/2$	K^- , K^{*-}
$u\bar{d}$	0	1	1	1	π^+ , ρ^+
$\bar{u}d$	0	-1	-1	1	π^- , ρ^-

Given the values in the table of Sect. 4a, you should be able to generate this table.

It is of interest to check out the isotopic spin quantum numbers. The combination $d\bar{s}$ has $T_3 = 1/2$. The maximum T can be is $1/2$ since the combination consists of a $T = 1/2$ particle and a $T = 0$ particle.

Hence T must be $1/2$. The combination $u\bar{d}$ has $T_3 = 1$ and $T_{max} = 1$ so T must be 1. You should check the other combinations.

There are 3 combinations of a quark with its own antiquark: $u\bar{u}$, $d\bar{d}$, and $s\bar{s}$. None of these has a definite value of T. For example, $u\bar{u}$ has $T_3 = 0$ but can be either a $T = 1$ or a $T = 0$ state. Quantum mechanically, the state $u\bar{u}$ is a linear superposition of the $T = 1$ and $T = 0$ states. Similarly for the other combinations. It is, however, possible to form combinations of the states $u\bar{u}$, $d\bar{d}$ and $s\bar{s}$which are states of pure T. These are:

a. $(1/\sqrt{2})(u\bar{u} - d\bar{d})$: $T = 1$, $T_3 = 0$: π^0 meson

b. $(1/\sqrt{6})(u\bar{u} + d\bar{d} - 2s\bar{s})$: $T = 0$, $T_3 = 0$: ηmeson

c. $(1/\sqrt{3})(u\bar{u} + d\bar{d} + s\bar{s})$: $T = 0$, $T_3 = 0$: η'meson

The meaning of the linear combination in (a), for example, is this: if one could examine the quark content of the π^0, one would find $u\bar{u}$ half the time and $d\bar{d}$ half the time. The probability of finding a certain quark-antiquark combination is given by the square of the coefficient in front of that combination. For (b), it is not that there are 2 s and 2 $\bar{s}$ quarks in the η. Rather, the η has a 2/3 probability of being an $s\bar{s}$, a 1/6 probability of being a $u\bar{u}$, and a 1/6 probability of being a $d\bar{d}$.

Of the three $T_3 = 0$ combinations, one forms part of the isospin triplet of the octet, one forms the isospin singlet of the octet, and the last forms the SU(3) singlet.

4c. Meson Supermultiplets and Quarks. There evidently exist in nature many meson octets and singlets. They differ in spin, intrinsic parity, and mass. The particles of the lowest mass octet are stable under the strong interaction. Others are not: they decay to members of the lowest octet in times on the order of 10^{-24} sec and are the meson resonances.

Three meson octets and three meson singlets have been experimentally completed. They are given by the Particle Data Group following the meson table and the quark content is given in the following chart. You should consult the meson table of the Particle Data Group and observe that there are many other mesons which have been discovered and which do not form completed octets as yet. It is a good exercise to see where these particles might fit into as yet incomplete groups and then watch for discoveries of particles that fit the missing places.

Meson Supermultiplets with Approximate Masses

Note: The last particle listed in the octet and the particle listed as the corresponding singlet may be different mixtures of the same quark states.

Quark Content	Spin 0	Spin 1	Spin 2
Octet:			
$u\bar{d}$	$\pi^+(140)$	$\rho^+(770)$	$A_2{}^+(1310)$
$(1/2)(u\bar{u} - d\bar{d})$	$\pi^0(135)$	$\rho^0(770)$	$A_2{}^0(1310)$
$\bar{u}d$	$\pi^-(140)$	$\rho^-(770)$	$A_2{}^-(1310)$
$u\bar{s}$	$K^+(494)$	$K^{*+}(892)$	$K_N{}^+(1420)$
$d\bar{s}$	$K^0(498)$	$K^{*0}(892)$	$K_N{}^0(1420)$
$\bar{d}s$	$\overline{K}^0(498)$	$\overline{K}^{*0}(892)$	$\overline{K}_N{}^0(1420)$
$\bar{u}s$	$K^-(494)$	$K^{*-}(892)$	$\overline{K}_N{}^-(1420)$
$(1/6)(u\bar{u} + d\bar{d} - 2s\bar{s})$	$\eta(549)$	$\phi(1019)$	$f'(1514)$
Singlet:			
$(1/3)(u\bar{u} + d\bar{d} + s\bar{s})$	$\eta'(958)$	$\omega(784)$	f(1270)

4d. Spin and Quark Components. Presumably the energy (and hence the mass) increases rather drastically with orbital angular momentum. Spin 0 states can be constructed if the quark and antiquark have antiparallel spins and zero orbital angular momentum. This is the lowest mass set of particles. Spin 0 particles also result if the quark spins are parallel and the quarks have 1 unit of orbital angular momentum directed opposite to the spins. These particles evidently have masses on the order of 800 MeV larger than the first set of particles.

Spin 1 mesons result from any one of the following combinations:

a. Quark spins parallel and zero orbital angular momentum. This is evidently the lowest mass set of spin 1 mesons.

b. Quark spins antiparallel and 1 unit of orbital angular momentum.

c. Quark spins parallel and 2 units of orbital angular momentum directed opposite to the spins.

4e. Large Masses as Excited States. Quantum mechanically, one can consider the mesons of a large mass set (the spin 1 mesons, for example) to be excited states of mesons in the lowest mass set. All the K^-'s, for example, have the same properties except spin and mass and can be thought of as having the same quark content.

4f. Baryon Supermultiplets and Quarks. The baryons are constructed from 3 quarks, antibaryons from 3 antiquarks. Note that this prescription automatically satisfies the rules for assignment of baryon number and automatically makes the baryons fermions: they must have spin of half a positive odd integer. There are 10 ways to combine 3 quarks 3 at a time. They are:

	Y	T_3	T
uuu	1	3/2	3/2
uud	1	1/2	3/2 or 1/2
udd	1	−1/2	3/2 or 1/2
ddd	1	−3/2	3/2
uus	0	1	1
uds	0	0	1 or 0
dds	0	−1	1
uss	−1	1/2	1/2
dss	−1	−1/2	1/2
sss	−2	0	0

These form into groups of eight spin 1/2 baryons and ten spin 3/2 baryons.

The baryon octet:

quarks:	T	particle:
uud	1/2	p
udd	1/2	n
uds	0	Λ^0
uus	1	Σ^+
uds	1	Σ^0
dds	1	Σ^-
uss	1/2	Ξ^0
dss	1/2	Ξ^-

The baryon decimet:

quarks:	T	particle:
uuu	3/2	Δ^{++}
uud	3/2	Δ^+
udd	3/2	Δ^0
ddd	3/2	Δ^-
uus	1	Σ^{*+}
uds	1	Σ^{*0}
dds	1	Σ^{*-}
uss	1/2	Ξ^{*0}
dss	1/2	Ξ^{*-}
sss	0	Ω^-

Note that uud and udd can each form two different states, one with $T = 3/2$ and one with $T = 1/2$. The $T = 1/2$ states (uud with $T_3 = 1/2$ and udd with $T_3 = -1/2$) occur in the spin 1/2 octet while the $T = 3/2$ states (uud with $T_3 = 1/2$ and udd with $T_3 = -1/2$) are augmented with the other two states in the isospin multiplet (uuu with $T = 3/2$ and ddd with $T = 3/2$ and $T_3 = -3/2$) and occur in the spin 3/2 decimet.

Similarly the combination uds can form two different states, one with $T = 1$, $T_3 = 0$ and one with $T = 0$, $T_3 = 0$. The $T = 1$ state occurs in both groups while the $T = 0$ state occurs only in the octet.

5. Su(3) Operators

5a. The Quark-Quark Interaction and SU(3). There is more to the quark model than just the construction of particles from quarks. The chief idea behind the model is that the interaction which gives rise to the particles has a high degree of symmetry. The symmetry we are talking about is very much analogous to the ideas of invariance under parity or time reversal. That is, there is a group of operators (eight in number) which do not change the interaction when they operate on it (similar in nature to the fact that the electromagnetic interaction does not change when operated on by the parity operator i.e. when ($\vec{r}$) is replaced by (-$\vec{r}$) and ($\vec{p}$) by (-$\vec{p}$). The eight operators are collectively known as the SU(3) indicates that the basis of the group consists of 3 independent states (the 3 quarks).

5b. Hadron-Hadron Interaction: SU(3) from QQ Interaction. The abstract properties of the operators can be discussed and some general properties of the strong interaction derived. We shall not do this since the mathematics required (group theory) is generally not part of an undergraduate education and, although the ideas are not difficult, it would take too much time. Suffice it to say that the construction of hadrons from quarks, with properties as given, forces SU(3) symmetry on the strong interaction.

5c. Operations on Quarks: Quark Model Basic Postulates. Since we have already postulated the quarks, we can list the operators of SU(3) in terms of how they transform the quarks. The operators are denoted by λ_i:

	operator		causes:
$\lambda_1 u = 0$	$\lambda_1 d = u$	$\lambda_1 s = 0$	d → u
$\lambda_2 u = d$	$\lambda_2 d = 0$	$\lambda_2 s = 0$	u → d
$\lambda_3 u = u$	$\lambda_3 d = d$	$\lambda_3 s = 0$	
$\lambda_4 u = 0$	$\lambda_4 d = 0$	$\lambda_4 s = u$	s → u
$\lambda_5 u = s$	$\lambda_5 d = 0$	$\lambda_5 s = 0$	u → s
$\lambda_6 u = 0$	$\lambda_6 d = 0$	$\lambda_6 s = d$	s → d
$\lambda_7 u = 0$	$\lambda_7 d = s$	$\lambda_7 s = 0$	d → s
$\lambda_8 u = u/3$	$\lambda_8 d = d/3$	$\lambda_8 s = -s/3$	

One of the operators turns a u into a d quark, another turns a u into an s, etc., so that each quark is turned into each of the others by one of the operators. In addition, λ_3 and λ_8 are special in that they do not change the character of the quark. λ_3 produces the same quark state but multiplied by twice its isotopic spin. λ_8 produces the same quark state but multiplied by its hypercharge.

In the language of quantum mechanics the quark states are chosen to be eigenstates of the operators corresponding to the 3-component of isotopic spin and hypercharge. Another way of saying the same thing is that each quark has a definite value for T_3 and Y, and these values are constants of its motion; a u quark, for example, always has $T_3 = 1/2$ and $Y = 1/2$. The basic postulates of the quark model are:

a. the fundamental interaction which produces the hadrons is invariant under the SU(3) operators,

b. a meson is composed of a quark and an antiquark,

c. a baryon is composed of three quarks.

These postulates give rise to the grouping of hadrons into singlets, octets, and decimets. Operating on one of the particles of an octet, for example, turns it into one or more of the other particles in the octet. SU(3) provides the rationale for grouping the hadrons in to supermultiplets, as the octets, decimets and singlets are collectively called.

5d. SU(3) Operators, States, Insufficiency. The eight operators are not arbitrarily chosen. Every operator which operates in a space which is specified by 3 basis states can be written as a linear combination of these 8, augmented by the identity operator ($\lambda_0 u = u$, $\lambda_0 d = d$, $\lambda_0 s = s$). Several conclusions can be drawn from this statement. First, the 8 operators ($\lambda_1, \ldots, \lambda_8$) are not unique. One can form many other sets of 8 independent operators but these will always be linear combinations of those we have written down. Different authors, in fact, use different sets but all sets will lead to the same physical conclusions. Second, if strong interaction physics can be described in terms of what happens to 3 independent states (i.e. 3 quarks) then the theory can be written in terms of the 8 SU(3) operators and the identity operator. There is nothing that can be done to a quark which is not describable by some combination of these operators. There is however evidence that things do happen in nature which are not describable by the SU(3) operators and physicists no are forced to postulate a fourth quark and deal with the operators of su(4). You should also realize that the SU(3) operators deal with isotopic spin and hypercharge. There are two other important quantities, namely spin and baryon number, which are used to describe quarks and which are outside the domain of SU(3).

6. Su(3) and Interactions

6a. Interaction Invariance under Selected SU(3) Operations. The postulates and mathematical reasoning behind the quark model and SU(3) symmetry seem to be invalid physically. If the strong interaction is invariant under the SU(3) operators, then all the hadrons of a given octet or decimet should have the same mass. This follows because the operators change one quark into another, or what is the same thing, change one member particle of a supermultiplet (as the singlets, octets, and decimets are called) into another member of the same supermultiplet without changing any of the interactions. Since the interactions are presumably responsible for the masses, all the particles of a supermultiplet must have the same mass.

If the quark model is to be valid, it must be that the interaction responsible for the particles of the supermultiplet is not the strong interaction but some other interaction which is invariant under the operations of SU(3).

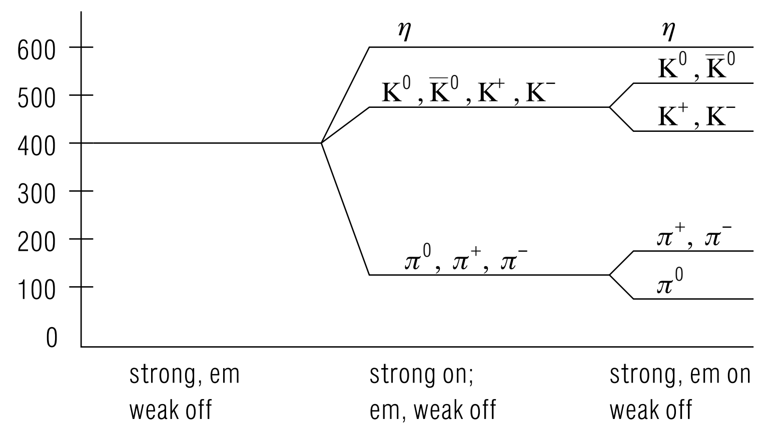
It is presumed that if all interactions would be turned off except this interaction, all particles in a supermultiplet would be identical. Particles in different supermultiplets would still be different (have different spins and different masses) because of internal quark dynamics. For example, the K^0, $\overline{K}^0$, K^+, K^-, π^+, π^-, π^0 and mesons would be experimentally indistinguishable from each other as would the K^{*0}, K^{-*0}, K^{*+}, K^{*-}, ρ^+, ρ^-, ρ^0 and ω mesons but the particles of the second octet would have mass and spin which would be different from the mass and spin respectively of the first octet.

The complete strong interaction is not invariant under all the SU(3) operators. Since the complete strong interaction conserves isotopic spin and hypercharge, it must be invariant under λ_1, λ_2, λ_3, and λ_8, the operators associated with isotopic spin and hypercharge. It is not invariant under λ_4, λ_5, λ_6, or λ_7, may now have different mass.

Looking at the chart of the operators (Sect. 5c), we see that particles which have the same quark content, except for the interchange of a u and d quark, will have the same mass and still be indistinguishable when the strong interaction is turned on. Particles which differ by more than this interchange will generally have different masses and this mass difference is associated with the strong interaction. In more detail, the strong interaction is not invariant under the operators λ_4, λ_5, λ_6, λ_7 of SU(3). These operators interchange u and s quarks or d and s quarks. We conclude that particles which differ in quark content by the substitutions $u \to s$, $d \to s$, $s \to u$, or $s \to d$ differ in mass by virtue of the energy associated with the strong interaction. Particles which differ by the substitutions $u \to d$ or $d \to u$ do not differ in mass by virtue of the strong interaction. For example, K^0, $\overline{K}^0$, K^+, K^- all have the same mass and π^+, π^-, π^0 all have the same mass but these two masses are different and differ from that of the η.

When the electromagnetic interaction is turned on, invariance with respect to isotopic spin is violated and a mass difference between the K^+ and K^0 is generated. Similarly, a mass difference between the π^+ and π^0 appears. The π^+ and π^- have the same mass because one is formed from the other by charge conjugation, and both the electromagnetic and strong interactions are invariant under this operation.

Presumably, the weak interaction produces a mass difference between particle and antiparticle, if charge conjugation invariance is violated. Mass effects of the weak interaction are too small to be observed at this time. The influence on mass of the various parts of the total interaction can be diagrammed as follows (for the lowest mass meson octet):



The mass of the original particle, plotted here at 400 MeV, is of course unknown since the interactions can not be turned off in practice.

6b. Comparison to Atomic Magnetic Splittings. The situation here is very similar to the magnetic states of an electron in an atom. For an electron with a specified principal quantum number corresponding to the values of m_ℓ are degenerate; they all have the same energy. When a magnetic field is turned on, the degeneracy is lifted and states with different m_ℓ have different energy. The splitting is given by $\Delta E = (e/mc)Bm_\ell$. For the spin 0 mesons, the original particle (with all interactions turned off) can be considered a quantum mechanical energy level that is 8-fold degenerate. The strong interaction splits the degeneracy to form 3 states, two of which are still degenerate (one 4-fold and one 3-fold) and the electromagnetic interaction further splits the degeneracy.

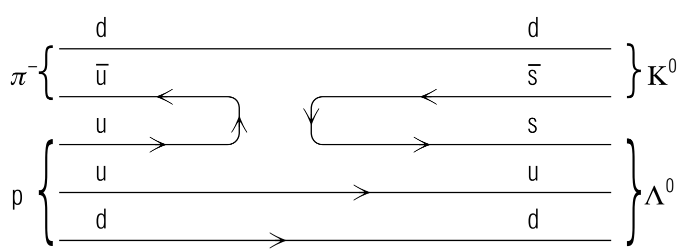
6c. Strong Interactions; Decays. Strong decays and interactions preserve strangeness and isotopic spin. In terms of quarks, the net number of u quarks is preserved (counting +1 for u and −1 for $\overline{\rm u}$); similarly for s and d quarks. Some examples of strong decays are:

a.
$$\begin{array}{lcccccc} & \rho^+ & \rightarrow & \pi^+ & + & \pi^0 \\ \text{or:} & \rm u\overline{d} & \rightarrow & \rm u\overline{d} & + & (1/\sqrt{2})(\rm u\overline{u} - d\overline{d}). \end{array}$$
The spin of one quark flipped to change the particle from spin 1 to spin 0 and the energy was used to form the $\rm u\overline{u} - d\overline{d}$ combination. The $\pi^+ - \pi^0$ system has one unit of orbital angular momentum.

b.
$$\begin{array}{lcccccc} & \rm K^{*+} & \rightarrow & \rm K^0 & + & \pi^+ \\ \text{or:} & \rm u\overline{s} & \rightarrow & \rm d\overline{s} & + & \rm u\overline{d} \end{array}$$
Energy from the $\rm u\overline{s}$ bond is used to create a $\rm d\overline{d}$ pair. The d is then coupled to the $\overline{\rm s}$ to form the $\rm K^0$ and the $\overline{\rm d}$ is coupled to the u to form the π^+.

c.
$$\begin{array}{lcccccccc} & \pi^- & + & \rm p & \rightarrow & \rm K^0 & + & \Lambda^0 \\ \text{or:} & \rm \overline{u}d & + & \rm uud & \rightarrow & \rm d\overline{s} & + & \rm uds \end{array}$$
The $\overline{\rm u}$ and one of the u's annihilate and energy is used to create an $\rm s\overline{s}$ pair. The $\overline{\rm s}$ couples to the d to form $\rm K^0$ and the s couples to the ud to form Λ.

This process can be represented schematically by:



6d. Electromagnetic Interactions; Decays. Several particles, stable via the strong interaction, decay electromagnetically. The electromagnetic decay (or interaction) need not conserve the magnitude of isospin although it does conserve the 3-component. Again net quark content cannot change. Quark-antiquark pairs may annihilate to produce a photon, and quark-antiquark pairs may be produced, but these are the only changes allowed. Examples are:

a.
$$\begin{array}{lcccccc} & \Sigma^0 & \rightarrow & \Lambda^0 & + & \gamma \\ \text{or:} & \rm uds & \rightarrow & \rm uds & + & \gamma \end{array}$$
In the Σ^0 the isotopic spin vectors of the u and the d are aligned and $T = 1$. In the Λ^0 the isotopic spin vectors of the u and the d are antiparallel and $T = 0$. In both cases $T_3 = 0$. This is an example of a change in isospin without change in quark content.

b.
$$\begin{array}{lcccccc} & \lambda & \rightarrow & \gamma & + & \gamma \\ \text{or:} & 1/\sqrt{6}(\rm u\overline{u} + d\overline{d} - 2s\overline{s}) & \rightarrow & \gamma & + & \gamma \end{array}$$
This is an example of the annihilation of a quark antiquark pair to produce a pair of γ's.

c.
$$\begin{array}{lcccccc} & \pi^0 & \rightarrow & \gamma & + & \gamma \\ \text{or:} & (1/2)(\rm u\overline{u} - d\overline{d}) & \rightarrow & \gamma & + & \gamma \end{array}$$
This is also an example of quark-antiquark annihilation.

d.
$$\begin{array}{lcccccc} & \gamma & + & \rm p & \rightarrow & \pi^0 + \rm p \\ \text{or:} & \gamma & + & \rm duu & \rightarrow & (1/2)(\rm u\overline{u} - d\overline{d}) + duu \end{array}$$
The energy of the γ is used to create a quark-antiquark pair. Actually, this interaction may occur via a virtual nucleon:

$$\gamma + \rm duu \rightarrow duu \rightarrow (1/2)(u\overline{u} - d\overline{d}) + duu$$

where the intermediate proton violates conservation of energy at both its inception and its decay.

6e. Weak Interactions; Decays. The weak interaction changes one type quark into another type quark. Hadrons enter the 6 basic couplings of the weak interaction in 3 ways:

$$\begin{array}{rcl} \rm p + \overline{n} & \leftrightarrow & \text{leptons,} \\ \rm p + \bar{\Lambda} & \leftrightarrow & \text{leptons,} \\ \rm p + \Lambda & \leftrightarrow & \rm p + n. \end{array}$$

In terms of quark content, the first can be written $\rm dud + \overline{dd\overline{u}} \rightarrow$ leptons. If the weak interaction changes a u quark to a d quark, all quarks can be paired with their antiquarks and total annihilation of all quarks can occur. Since leptons presumably do not contain quarks, either this or the other alternative ($\overline{\rm d}$ to $\overline{\rm u}$) must occur.

The second interaction requires the weak interaction to change u to s or $\overline{\rm s}$ to $\overline{\rm u}$ while the third requires the change $\overline{\rm d}$ to $\overline{\rm s}$.

Again, it is seen that the effect of the weak interaction is to change one type quark into another type quark.

If weak processes occur via the W particle, it must be that W's can decay into quark-antiquark pairs where the quark and antiquark are not necessarily of the same type. The W's can undergo the following two types of processes:

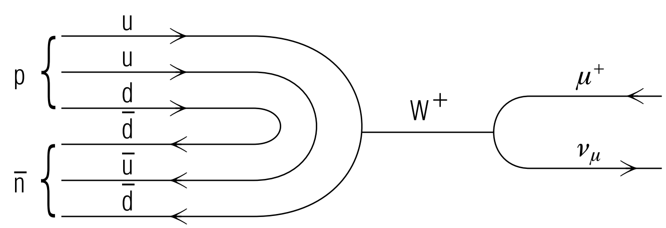
$$\mathrm{W} \leftrightarrow \text{ lepton } + \text{ antilepton}$$

and:

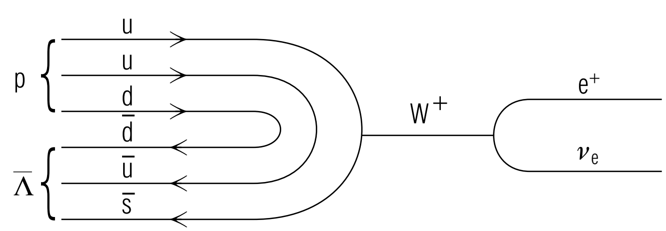
$$\mathrm{W} \leftrightarrow \text{ quark } + \text{ antiquark}\,.$$

Here are examples of each of the three types of weak couplings involving hadrons:

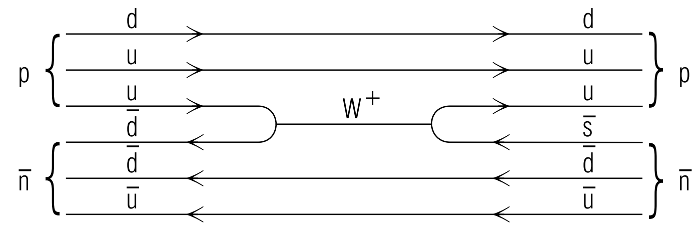
a. $\mathrm{p} + \overline{\mathrm{n}} \rightarrow$ leptons



b. $\mathrm{p} + \bar{\Lambda} \rightarrow$ leptons



c. $\mathrm{p} + \overline{\mathrm{n}} \rightarrow \mathrm{p} + \bar{\Lambda}$



7. Problems with the Quark Model

The quark model leads to some questions, as yet unresolved. The quarks are charged and interact electromagnetically; some other force is needed to bind the quarks in a particle. For example the $\bar{\Xi}$ is composed of 3 negatively charged quarks which electromagnetically repel each other.

A new set of particles, called gluons, has been proposed. The gluons act as exchange particles between quarks and produce the binding required to hold particles together. It may be that the gluons are indeed responsible for the basic strong interaction.

Another problem that arises has to do with the quark content itself. Why do 3 quarks bind together and not 4? Why do quarks and antiquarks form a bound state and not 2 quarks? If ought to be possible to construct a baryon of the form $\mathrm{qqqq\overline{q}}$ and a meson of the form $\mathrm{q\overline{q}q\overline{q}}$.

Beyond this, physicists want to know why there are no particles with fractional baryon number or fractional charges. That is, what prevents a particle of the form qqqq?

Some of these questions are partially answered by a model which is discussed elsewhere.[2]

Acknowledgments

Construction of this module was supported in part by the U. S. Coast Guard Academy for a Directed Studies Program. Preparation of this module was supported in part by the National Science Foundation, Division of Science Education Development and Research, through Grant #SED 74-20088 to Michigan State University.

[2]See "Color and Charm" (MISN-0-283).

PROBLEM SUPPLEMENT

1. Draw quark diagrams, similar to the ones at the end of Sect. 4b or 4d, for these decays:

   a. $\Delta^+ \rightarrow \mathrm{n} + \pi^+$

   b. $\Lambda^0 \rightarrow \mathrm{p} + \pi^-$

   c. $\pi^+ \rightarrow \mu^+ + \nu_\mu$

2. Give the quark content of a particle with baryon number +1, charge $+e$, and strangeness +1. This particle is called an "exotic" particle and is denoted by Z^*. Its quark content cannot be of the form qqq and it is being sought experimentally as a test of the rules for constructing baryons.

3. Find the appropriate characteristics of the following quark combinations and show that they match the characteristics of one or more of the particles. Identify the particle or particles.

   a. uud

   b. $\mathrm{u\bar{s}}$

   c. $\mathrm{\bar{d}\bar{d}\bar{s}}$

   d. $\mathrm{u\bar{d}}$

   e. uds

   f. $\mathrm{\bar{u}d}$

MODEL EXAM

Quark	B	T	T_3	σ	S	Y	Q
u	1/3	1/2	1/2	1/2	0	1/3	2/3
d	1/3	1/2	−1/2	1/2	0	1/3	−1/3
s	1/3	0	0	1/2	−1	−2/3	−1/3
$\bar{u}$	−1/3	1/2	−1/2	1/2	0	−1/3	−2/3
$\bar{d}$	−1/3	1/2	1/2	1/2	0	−1/3	1/3
$\bar{s}$	−1/3	0	0	1/2	1	2/3	1/3

1. See Output Skills K1-K7 in this module's *ID Sheet*. The actual exam may have one or more of these skills, or none.

2. Draw quark diagrams, similar to the ones at the end of Sect. 4b or 4d, for these decays:
   (a) $\Delta^+ \rightarrow \mathrm{n} + \pi^+$; (b) $\Lambda^0 \rightarrow \mathrm{p} + \pi^-$; (c) $\pi^+ \rightarrow \mu^+ + \nu_\mu$.

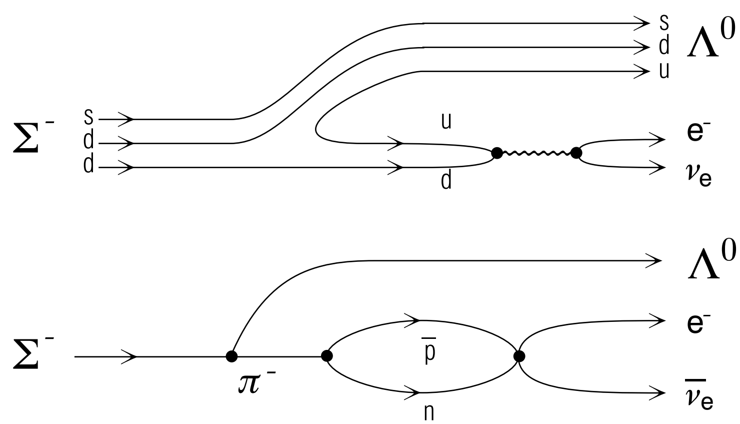
3. Give the quark content of a particle with baryon number +1, charge +e, and strangeness +1. This particle is called an "exotic" particle and is denoted by Z^*. Its quark content cannot be of the form qqq and it is being sought experimentally as a test of the rules for constructing baryons.

4. Find the appropriate characteristics of the following quark combinations and show that they match the characteristics of one or more of the particles. Identify the particle or particles: (a) uud; (b) $\mathrm{u\bar{s}}$; (c) $\mathrm{\bar{d}\bar{d}\bar{s}}$; (d) $\mathrm{u\bar{d}}$; (e) uds; (f) $\mathrm{\bar{u}d}$.

PHYSNET

MISN-0-281

THE WEAK INTERACTION



Project PHYSNET Physics Bldg. Michigan State University East Lansing, MI

THE WEAK INTERACTION

by

J. Christman

Title: **The Weak Interaction**

Author: J. R. Christman, Dept. of Physical Science, U.S. Coast Guard Academy, New London, CT

Version: 11/8/2001 Evaluation: Stage B1

Length: 2 hr; 16 pages

Input Skills:

1. Interpret Particle Diagrams and give the associated coupling constants (MISN-0-279).

Output Skills (Knowledge):

K1. List the three categories of weak interactions and give examples and possible Particle Diagrams for each.

K2. Give arguments that lead to predictions of the mass and spin of the W particles.

Output Skills (Problem Solving):

S1. Given a weak decay, devise a plausible Particle Diagram for it, showing the weak interaction as a four-fermion interaction.

S2. Given a weak decay, devise a plausible Particle Diagram for it, showing the weak interaction as the exchange of a (charged) W or a (neutral) Z^0.

External Resources (Required):

1. M. J. Longo, *Fundamentals of Elementary Particle Physics*, Prentice-Hall (1973).
2. *Scientific American*, March, 1959.

Post-Options:

1. "SU(3) and the Quark Model" (MISN-0-282).
2. "Current Work in Elementary Particles" (MISN-0-284).

THE WEAK INTERACTION

by

J. Christman

1. Overview

This module fills in some details of the weak interaction. In particular, it deals with the basic couplings of the interaction and with the W particles which are exchanged in it.

2. Assigned Readings

- Chapter 6, Longo
- S. B.,Treiman, "The Weak Interactions," *Scientific American*, March, 1959.

3. Characteristics of the Weak Interaction

3a. The Weak Force: Universal. All particles participate in the weak interaction in the sense that all known particles (except the resonances) have been observed to participate in an interaction or decay that involves the weak force (at one or more vertices in the associated Particle Diagram). Study of the weak force, however, is complicated by the strong interaction: some interactions may proceed in more than one step, the first step being a strong decay to other particles, some of which then interact weakly. The strong process takes place in such a short time and over such a small distance that it is impossible to observe and hence it is impossible (except by indirect evidence) to ascertain whether or not the strong interaction actually took place. This makes deduction of the weak part of the interaction uncertain.

3b. Huge Fluxes For Direct Neutrino Observation. Neutrinos are the only particles that interact via the weak force alone so they make ideal "bullets" to study the weak interaction. The weak interaction is so weak, however, that only about 1 neutrino in every 10^{12} undergoes an interaction with the nucleons in fluids used to detect neutrinos. So the experimental study of the weak interaction requires enormous neutrino fluxes and also detection chambers the size of large rooms.

The participation of a neutrino guarantees that the interaction is weak. However, it is extremely difficult, in most cases, to show that a neutrino is, in fact, even present.

3c. Change in S if One Weak Vertex. All first order weak interactions (i.e., decays with one weak vertex), either do not change strangeness or else change strangeness by ± 1. That is, $\Delta S = 0, \pm 1$ for first order weak decays. Second order weak interactions, decays with two weak vertices, are extremely rare and will not be considered here.

3d. Range of the Weak Interaction. Theories suggest that the range of the weak interaction is on the order of $10^{-16} - 10^{-17}$ m, which is shorter than the range of the strong interaction.

3e. A Four-Fermion Interaction. A weak interaction vertex in a Particle Diagram must have exactly four particle lines and the particles must all be fermions. The interaction strength at the vertex is denoted g_w.

4. Categories of Weak Interaction

4a. Categories Based on Particles Involved. Weak interactions are classified in three categories: "leptonic," in which only leptons are involved; "semi-leptonic," in which both leptons and hadrons are involved; and "hadronic," in which only hadrons are involved.

4b. Examples of Weak "Leptonic" Processes. Here are two scattering reactions and a decay involving only leptons in the initial and final states (note that the neutrino is chargeless so the scattering cannot be electromagnetic):

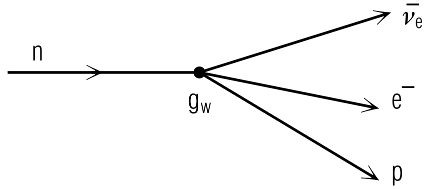
$$\nu_e + e^- \rightarrow \nu_e + e^- \qquad \text{(scattering)}$$

$$\nu_\mu + \mu^- \rightarrow \nu_\mu + \mu^- \qquad \text{(scattering)}$$

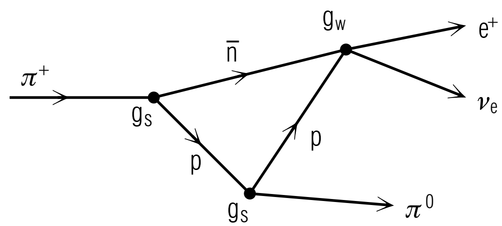
$$\mu^+ \rightarrow e^+ + \bar{\nu}_\mu + \nu_e$$

4c. Examples of Weak "Semi-Leptonic" Processes. Here are some diagrams for Weak "Semi-Leptonic" Processes:

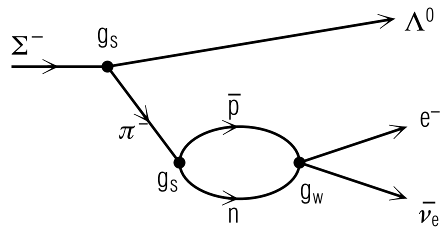
n → p + e$^-$ + $\bar{\nu}_e$ (nuclear β decay):



$\pi^+ \to \pi^0 + e^+ + \nu_e$:

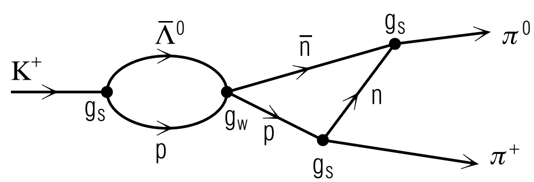


$\Sigma^- \to \Lambda^0 + e^- + \bar{\nu}_e$:

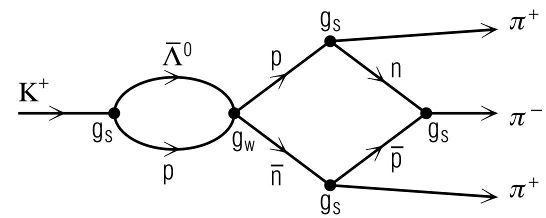


4d. Examples of Weak "Hadronic" Processes. Here are some diagrams for Weak "Hadronic" Processes:

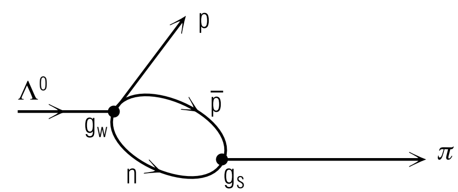
$K^+ \to \pi^+ + \pi^0$:



$K^+ \to \pi^+ + \pi^+ + \pi^-$:



$\Lambda^0 \to p + \pi^-$:



4e. Strong Decays Occur Before Weak Decays. The hadrons that enter the weak vertex of the above diagrams are the lowest mass baryons with $S = 0$ and $S = 1$. All hadrons couple strongly to at least one of these baryons, and since the strong interaction is so fast one expects an initial hadron to first interact strongly until one of these low mass baryons is produced.

5. The Intermediate Vector Boson

5a. The W Particle as the Weak-Force Meson. The electromagnetic interaction is carried by the photon and the strong interaction is carried by hadrons, in the sense that the interactions are "caused by" exchange of such intermediary particles. Similarly, the weak interaction is carried by the "intermediate vector boson" and its symbol is W. There is a positively charged W^+, a negatively charged W^-, and a neutral W^0. These particles can be produced by appropriate pairs of weakly interacting particles and they decay into appropriate pairs:

$$p + \bar{n} \leftrightarrow W^+ \qquad \bar{p} + n \leftrightarrow W^-$$
$$p + \bar{\Lambda}^0 \leftrightarrow W^+ \qquad \bar{p} + \Lambda^0 \leftrightarrow W^-$$
$$\mu^+ + \nu_\mu \leftrightarrow W^+ \qquad \mu^- + \bar{\nu}_\mu \leftrightarrow W^-$$

For example, β decay proceeds according to:

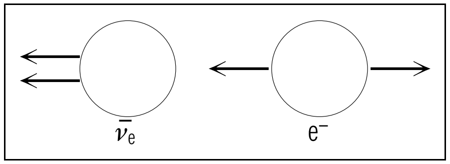
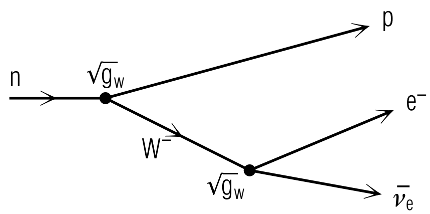


Figure 1. Spins and momenta from W^- decay.



The coupling constant at each vertex is $\sqrt{g_w}$ to make the overall coupling constant g_w.

5b. The Mass of the W Particle. Since the range of the weak interaction is less than 10^{-16} m, the mass of the W must be greater than an amount determined by the "uncertainty principle":

$$mc^2 > \frac{\hbar}{\Delta t} = \frac{\hbar c}{R} = \frac{(1.05 \times 10^{-}34) \times (3 \times 10^8)}{10^{-16}} = 3.15{\times}10^{-10}\,\mathrm{J} = 2.0\,\mathrm{GeV}\,.$$

(Note: $1\,\mathrm{GeV} = 10^3\,\mathrm{MeV}$.) Experimentally, W particles have been seen. The observed mass of the W is approximately 80 GeV.

5c. The Spin of the W Particle. The spin of the W can be deduced from observations of the spins of its decay products. Consider, for example, the β decay of the neutron (see the diagram in Sect. 5a and Fig. 1, this section). In the center of mass frame of the electron and antineutrino, the spin of the antineutrino is $\hbar/2$ in the direction of its momentum (this is true for the antineutrino in any frame) and the spin of the electron is observed to be $\hbar/2$ in the direction opposite to its momentum. The orbital angular momentum is zero. If the particles result from the decay of a W, the spins and momenta of the decay products look as in Fig. 1. Note that the total spin is $\hbar$, to the left. Since angular momentum is conserved, the spin of the W^- must have been $\hbar$. This is in fact the reason for its name "intermediate vector boson." An integer spin particle is a boson and a spin 1 particle has associated with it a vector field (another vector boson, the photon, is associated with the vector electromagnetic field). It is also easy to deduce that the W particles have electron family number 0, muon

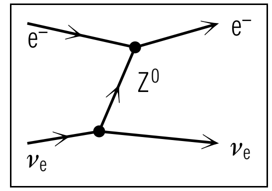


Figure 2. The weak-interaction exchange of a neutral particle (the Z).

family number 0, and the baryon family number 0.

5d. The Neutral Weak Boson. A neutral boson is not needed for exchange in the usual weak couplings of nuclear physics; all of them involve a transfer of charge and so involve the exchange of charged W's. However, the observed weak scattering of one lepton by another does require the exchange of a neutral boson (see Fig. 2.). Our current theoretical understanding is that the W^0 cannot itself be observed, but that it and another unobservable particle combine two different ways to form the observed weak-interaction Z^0 and the well-observed electromagnetic-interaction γ (the "photon"). Apart from lepton scattering, other reactions such as

$$\nu_\mu + \mathrm{p} \rightarrow \nu_\mu + \mathrm{p} + \pi^0$$

can occur via the exchange of the Z^0.

5e. A Problem with Neutral K Decay. An important example of weak neutral exchange should be the decay of the neutral kaon. The more usual decay products include at least one pion. The decay to leptons,

$$\mathrm{K}^0 \rightarrow \mu^+ + \mu^-\,,$$

is extremely rare. With a Z^0 existing, K^0 can decay that way via a first order weak decay and for some time the rarity of that decay mode was taken as evidence that the neutral "weakon" did not exist. With evidence for the Z^0 in neutrino scattering (see Sect. 5d), a new explanation was required for the rarity of the neutral kaon decay to muons. The solution is another quantum number, called "charm," which we shall discuss elsewhere.[1]

Acknowledgments

Preparation of this module was supported by the United States Coast Guard Academy for a Directed Studies Program. Preparation of this

[1] See "Color and Charm" (MISN-0-283).

module was supported in part by the National Science Foundation, Division of Science Education Development and Research, through Grant #SED 74-20088 to Michigan State University.

PROBLEM SUPPLEMENT

Note: If you do not understand how an answer in this supplement was arrived at, kindly go back to the text and work through it carefully. Make sure you understand all of the text examples before coming back to this supplement. The text is organized for learning, whereas this supplement is designed to help you *test* whether you learned the subject from the text.

Problems:

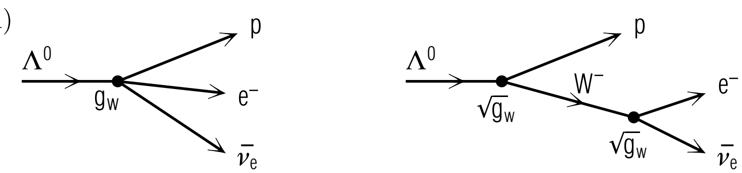
Devise plausible diagrams for the following weak decays, both without and with intermediate "weakons."

1. $\Xi^0 \to \Lambda^0 + \pi^0$ (C)
2. $\mathrm{K}^+ \to \pi^0 + \mathrm{e}^+ + \nu_\mathrm{e}$ (B)
3. $\pi^+ \to \mu^+ + \nu_\mu$ (E)
4. $\Lambda^0 \to \mathrm{p} + \mathrm{e}^- + \bar{\nu}_\mathrm{e}$ (A)
5. $\Sigma^- \to \Lambda^0 + \mathrm{e}^- + \bar{\nu}_\mathrm{e}$ (D)

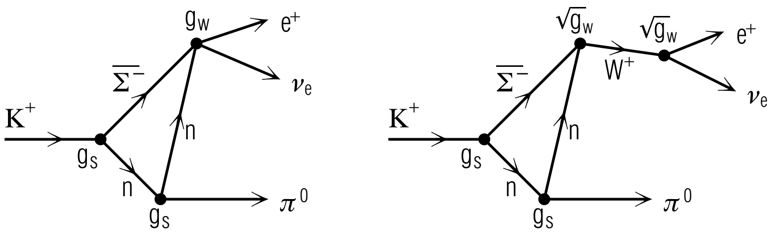
Note: In some cases there are a number of legitimate possibilities for intermediate states. For example, in Answer (C) the (p, $\overline{\mathrm{p}}$) intermediate state could equally well be (n, $\overline{\mathrm{n}}$).

Answers:

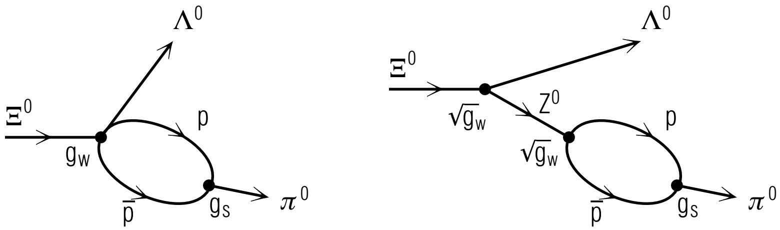
(A)



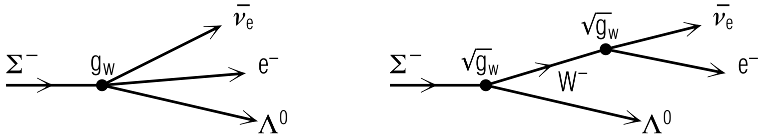
(B)


K^+
g_s
n
$\overline{\Sigma^-}$
g_w
e^+
ν_e
n
g_s
π^0
K^+
g_s
n
$\overline{\Sigma^-}$
$\sqrt{g_w}$
W^+
$\sqrt{g_w}$
e^+
ν_e
n
g_s
π^0

(C)


Λ^0
Ξ^0
g_w
p
$\bar{p}$
g_s
π^0
Λ^0
Ξ^0
$\sqrt{g_w}$
Z^0
$\sqrt{g_w}$
p
$\bar{p}$
g_s
π^0

(D)


Σ^-
g_w
$\bar{\nu}_e$
e^-
Λ^0
Σ^-
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W^-
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Λ^0

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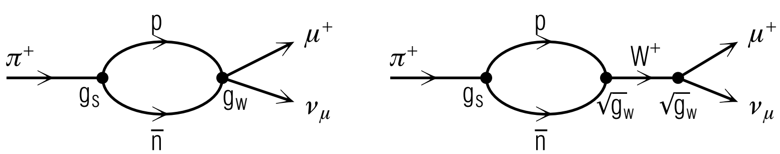
π^+
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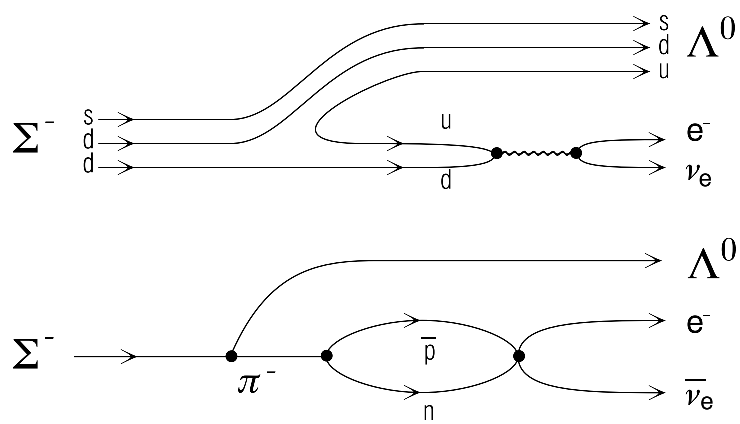
MISN-0-280

THE STRONG INTERACTION



Project PHYSNET Physics Bldg. Michigan State University East Lansing, MI

THE STRONG INTERACTION
by
J. R. Christman

Title: **The Strong Interaction**

Author: J. R. Christman, Dept. of Physical Science, U. S. Coast Guard Academy, New London, CT

Version: 11/8/2001 Evaluation: Stage B1

Length: 2 hr; 12 pages

Input Skills:

1. Explain how isospin is applied to strong interactions (MISN-0-278).
2. Apply the uncertainty principle to energy conservation violation by intermediate states (MISN-0-279).
3. Interpret Particle Diagrams and give the associated coupling constants (MISN-0-279).
4. Define "virtual particle" (MISN-0-279).

Output Skills (Knowledge):

K1. Relate the strong interaction to the exchange of hadrons.

K2. Explain what is required to produce real, observable (not virtual) particles.

K3. Describe how resonance particles are placed into four meson and six baryon categories.

K4. Discuss how particles in the same category can differ.

K5. Explain how categorization arises from differences in internal parts (quarks) and motion of quarks.

Output Skills (Rule Application):

R1. Given the mass of an exchanged particle, estimate the range of the corresponding interaction.

R2. Estimate the mass and lifetime of a resonance particle from a plot of cross-section vs. energy.

R3. Given a scattering process and a single exchanged hadron, draw the corresponding Particle Diagram.

External Resources (Required):

1. M. J. Longo, *Fundamentals of Elementary Particle Physics*, McGraw-Hill (1973); and *Scientific American*: R. E. Marshak, "Pions," (Jan. 1957); R. D. Hill, "Resonance Particles," (Jan. 1963).

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New authors, reviewers and field testers are welcome.

THE STRONG INTERACTION

by

J. R. Christman

1. Abstract

Ater a review of properties of the strong interaction, this module introduces two new ideas: (a) the cloud of virtual hadrons around each hadron; and (b) the resonance particles.

2. Readings

1. Longo, Chapter 3.
2. R. E. Marshak, "Pions," *Scientific American* (January, 1957).
3. R. D. Hill, "Resonance Particles," *Scientific American* (January, 1963).

3. Description

3a. General Effects, Range, Lifetimes, Conserved Quantities. Only hadrons interact via the strong interaction. Hadrons scatter from other hadrons, producing changes in their motion, and/or other particles, via this interaction mechanism. It is sufficiently strong to bind low energy hadrons together and this binding is responsible for the binding of neutrons and protons in the nucleus. It is of short range (10^{-15} m). Particles which decay via the strong interaction have lifetimes typically on the order of 10^{-24} sec. This is also roughly the time two particles travelling near the speed of light are within 10^{-15} m of each other. The strong interaction is charge independent: for example, it is the same for neutrons as for protons.

Strong-interaction scatterings and decays conserve energy, momentum, charge, baryon number, electron number, muon number, strangeness, and isotopic spin. The interaction is invariant under parity, charge conjugation and time reversal.

3b. Hadron Exchange: Exchanged Mass & Interaction Time. The nature of the interaction itself is believed to be associated with the exchange of hadrons between the interacting particles. The lightest hadron

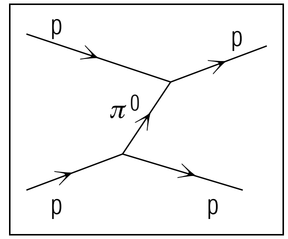


Figure 1.

is the pion with a mass of about 140 MeV, so one possibility for strong nucleon-nucleon scattering is shown in Fig. 1.

For some time Δt, the system is in violation of the law of energy conservation by at least 140 MeV. The maximum length of time this can occur is:

$$\Delta t \simeq \frac{\hbar}{\Delta E} = \frac{1.05 \times 10^{-34}}{140 \times 10^{6} \times 1.6 \times 10^{-19}} = 4.7 \times 10^{-24}\,\text{sec}$$

which is roughly the time of a typical strong interaction. An estimate for the range of the interaction can be obtained by assuming the pion moves at a speed close to the speed of light. It then will move about

$$4.7 \times 10^{-24} \times 3 \times 10^{8} = 1.4 \times 10^{-15}\,\text{m}$$

in time Δt. This is indeed the range of the strong interaction.

Hadrons with greater mass than the pion can be exchanged but the exchange must take place in shorter times and over smaller distances. The short range part of the strong interaction is thus complicated by the possibility of multiple pion exchange as well as kaon, eta and baryon exchange.

3c. Charge Exchange. The various charge states of the pion have roles to play, as the diagrams in Fig. 2 show.

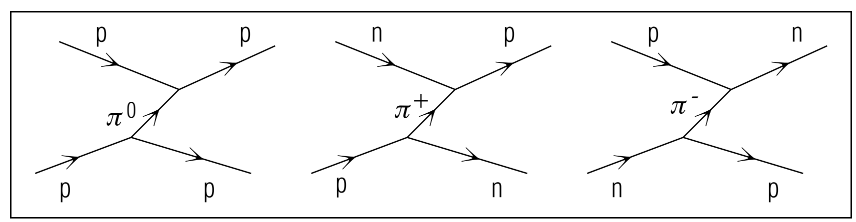


Figure 2.

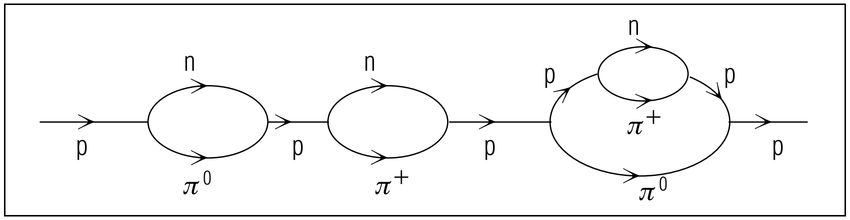


Figure 3.

Note that the proton and neutron can exchange their identities during the course of the interaction.

4. Hadron States

4a. Virtual Particles: Necessity, Examples. It is unreasonable to assume that a proton, for example, emits a pion only in the presence of another hadron for, in that event, there would be a mechanism which signals the presence of the second particle. It must be that the "proton" sometimes exists in a state which consists of a proton and a pion. In fact the trail of a proton may look as shown in Fig. 3 and any of a large number of diagrams.

Since the coupling constant $g_s \approx 1$, the vertices p $\to$ p+π, p $\to$ p+2π, ..., p $\to$ p + $n\pi$ all compete with each other. Of course the larger the number of pions, the greater is the violation of energy conservation and the shorter the time the particle can be in such a state.

The virtual particles accompanying the hadrons are not limited to pions: any hadron can be created. The only requirements are that the appropriate conservation laws for the strong interaction, except conservation of energy, hold at each vertex. Examples for the pion are shown in Fig. 4.

4b. Open- and Closed-Channel States. The proton has been used as an example above. The same statements are true for all hadrons. Given any hadron, there is a non-zero probability that the hadron will be in a state with any combination of other hadrons consistent with the conservation laws. These various possibilities are called channels. An open channel is any state which conserves all quantities, including energy, appropriate for the strong interaction. A closed channel is any state which conserves all quantities, except energy, appropriate for the strong interaction. Final, observed states must be open channels with respect to

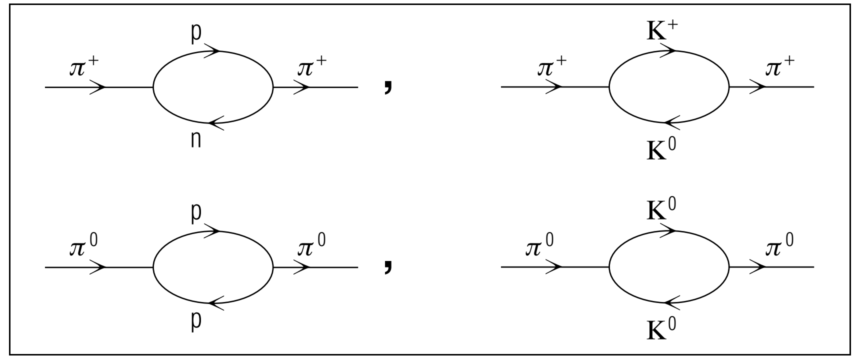


Figure 4.

the initial state. Intermediate states can be open or closed channels.

4c. Comparison of Virtual and Real Decays. Our picture of the hadron system is a collection of particles, each one of which continually produces the others. If the energy (mass) of a single particle is greater than the sum of the masses of the particles in another state, the particle decays to that state. Otherwise the particles when in the other state are virtual and unobservable, and the state is short lived. The interaction between two hadrons takes place via the exchange of such virtual particles. When the total energy is great enough for the virtual particles to become real, they do, and the interaction produces other hadrons as the final product.

5. Resonance Particles

5a. Particles as Resonances. Total cross sections for hadron-hadron scattering as functions of energy show a great deal of structure. The cross section is a measure of the probability that a particle will be scattered out of the original beam; that is, it measures the probability that an interaction takes place. As an example, the cross section of π^- incident on p as a function of center of mass total energy is shown in Fig. 5.

Fairly narrow, well defined peaks are interpreted as particles (called resonance particles). The interpretation is justified as follows. Certain interactions involving intermediate particles give rise to cross sections with peaks. An example of such an interaction is shown in the Fig. 6, where X is an intermediate particle. Energy need not be conserved at vertices but the peak in the cross section occurs at an incident energy

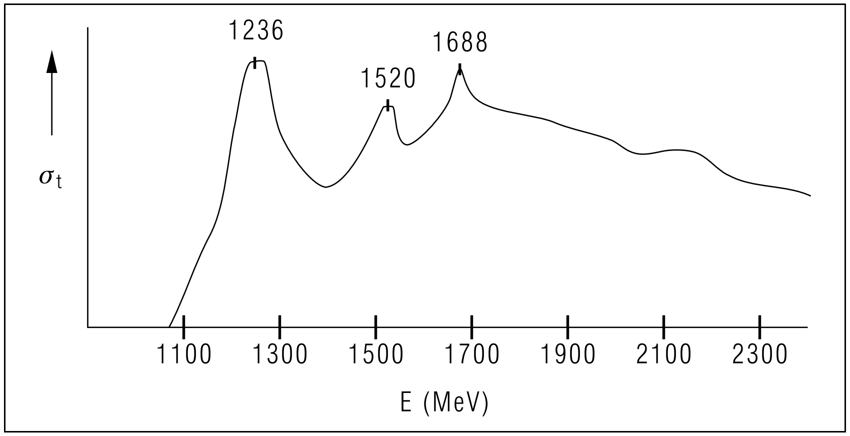


Figure 5.

where energy is in fact conserved. Since the energy plotted is the center of mass energy (i.e. in a frame for which the momentum of the X is zero) the total center of mass energy is just the mass (times c^2) of the X particle. Furthermore the width of the peak is related to the time of the interaction by $\Delta E \Delta t \simeq \hbar$. Here ΔE is the width of the peak and Δt is the lifetime of the X particle.

In addition, by appropriate analysis of data and application of conservation laws, the resonance particle X can be assigned other particle attributes: charge, spin, isotopic spin, strangeness, and baryon number. Confidence in the idea that a resonance is the signal for the appearance of a particle is increased when it is observed that the same resonance particles with the same particle properties appear in a variety of scattering processes. For example, the resonance at 1236 MeV occurs not only in π^--p scattering but also in γ-p scattering etc.

5b. Overview of Resonance Particles. Analysis of peaks in cross-section vs. energy curves has lead to the discovery of hundreds of res-

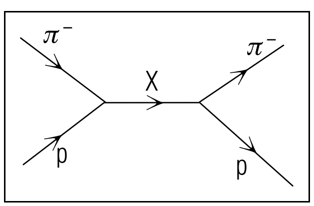


Figure 6.

onance particles, all of them hadrons and all of them decaying via the strong interaction. The resonance particles, according to accepted ideas, must be produced virtually by all hadrons. However, it is only when the interaction takes place at energies near the resonance particle mass that the influence of the resonance particle on the cross-section is seen.

5c. Resonance-Particle Symbols. With the proliferation of resonance particles, the same symbol has come to be used for several particles having similar properties, with the mass written in parentheses after the symbol. The π^--p resonances are denoted Δ(1236), N*(1520), N*(1688), etc. Sometimes the spin of the particle is also given inside the parenthesis: Δ(1236,3/2), N*(1520,3/2), N*(1688,5/2).

6. Particle Names

6a. Baryon Names; T, Y. Particle groups' names differentiate between the groups' hypercharge and isotopic spin. For baryons the group names are:

T	Y	Name
0	0	Λ
0	-2	Ω
1/2	+1	N
1/2	-1	Ξ
1	0	Σ
3/2	+1	Δ

For a particular member of a group, charge is indicated by superscript while the mass in MeV and the spin in units of $\hbar$ are placed in parenthesis after the name. For example, the proton is N$^+$(940,1/2).

6b. Meson Names; G-Parity, T, Y. For some mesons, an additional number is needed. This need arises because, for the non-strange mesons, both particle and antiparticle are in the same isospin multiplet. For example, the pion triplet contains both the π^+ and π^-. We state without proof that the operator

$$e^{+i\pi T_2},$$

where T_2 is the 2-component of isospin, changes one member of the multiplet into another. In fact, it just reverses the change caused by the charge conjugation operator. We define the new operator $\mathcal{G}$ by

$$\mathcal{G} = C e^{-i\pi T_2},$$

which does not change the particle. G does however change some wave functions into their negatives. For example:

$$\mathcal{G}\,\pi^+ = -\pi^+,$$

$$\mathcal{G}\,\pi^0 = -\pi^0.$$

Here the particle symbol is used to represent the wave function: If the wave function changes sign the particles is said to have odd G-parity ("odd" is denoted $-$) and, if the wave function does not change sign, the particle is said to have *even* G-parity (denoted $+$).

Names for mesons are

T	Y	G	Name
0	0	$+$	η
0	0	$-$	ω
1/2	+1	X	K
1/2	-1	X	$\overline{\text{K}}$
1	0	$+$	ρ
1	0	$-$	π

You need not know any details of G-parity. Just remember that the π and ρ are the same except for an internal quantum number G. Similarly for the η and ω.

6c. Evolution of Names. All known hadrons can be placed in one of the meson or baryon categories. With the exception of the Δ these names are the same as are used for the particles which are stable under the strong interaction (N = nucleon, not be confused with n = neutron).

Be warned that some older names are still in use and that a proliferation of names as well as of particles exists. When a new particle is discovered it is impossible to place it in the scheme before its properties have been identified and the temporary name given to it may become more or less permanent.

6d. The Berkeley Particle Data Group Hadron Tables. Tables (see Guide) give a fairly complete list of known particles as of 1973. These tables and other data are published annually by the Particle Data Group at Berkeley. This group maintains a complete, up-to-date file of high energy data. In the Berkeley tables, the symbol I is used for isospin.

7. Hadron Structure

7a. All Hadrons: Possible Exchange Particles. All of the particles participate in the strong interaction and have lifetimes on the order of 10^{-24} sec. These times are too short to be measured directly but are inferred from the widths of the resonances. All could conceivably act as exchange particles (and hence could be the strong interaction, in a sense). Of course most of the masses are so great that they play little role except at very short range or at very high energies.

7b. The Excited State Hypothesis. This enormous number of hadrons leads physicists to believe that many are really excited states of others. This in fact is the basis of the naming scheme. Perhaps the hadrons have some sort of internal machinery (they are composed of smaller parts). Then the $N(1470)$ particle could be composed of the same constitutents as the $N(940)$, the stable nucleon. They are the same particle except that the constitutents have different internal motions and the particle has a different energy by virtue of the internal motions. This extra energy manifests itself as extra mass. The internal constitutents may have their own spin angular momentum plus orbital angular momentum and these then contribute to the spin of the particle.

7c. Quarks as Hadron Constituents. The particles which nature uses to build the hadrons are called quarks. All N particles have the same number and type of quarks but the motions of the quarks within them are different for the different N particles. This scheme can account for all properties, except mass, of all hadrons. We study the quark model in more detail in "SU(3) and the Quark Model" (MISN-0-282).

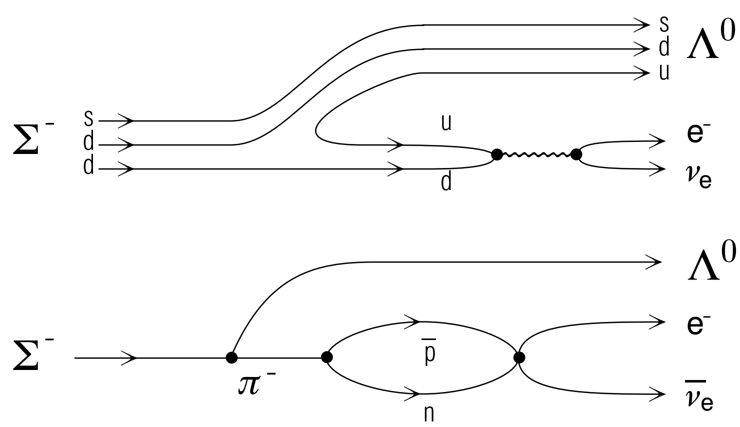
Acknowledgments

Preparation of this modules was supported by the United States Coast Guard Academy for a Directed Studies Program. Preparation of this module was supported in part by the National Science Foundation, Division of Science Education Development and Research, through Grant #SED 74-20088 to Michigan State University.

PHYSNET

MISN-0-279

STRENGTHS OF INTERACTIONS AND PARTICLE DIAGRAMS



Project PHYSNET Physics Bldg. Michigan State University East Lansing, MI

STRENGTHS OF INTERACTIONS AND PARTICLE DIAGRAMS

by

J. R. Christman

Title: **Strengths of Interactions and Particle Diagrams**

Author: J. R. Christman, Department of Physical Science, U.S. Coast Guard Academy, New London, Connecticut

Version: 11/12/2001 Evaluation: Stage 1

Length: 2 hr; 16 pages

Input Skills:

1. State the energy-time uncertainty principle (MISN-0-241).
2. Given a decay or interaction use the conservation laws to determine if it can occur and, if so, which force is responsible (MISN-0-278).

Output Skills (Knowledge):

K1. State rough values for these coupling constants: g_s, g_{em}, g_w.

K2. Define: virtual particle, real particle.

K3. Relate "coupling constant" to "intrinsic rate."

Output Skills (Rule Application):

R1. Given any two of exchanged mass, time, and range for an interaction, estimate the third.

R2. Given a Feynman Diagram, estimate the couplng constant for the process.

R3. Given a Feynam Diagram, describe the process in words.

Post-Options:

1. "The Strong Interaction" (MISN-0-280).
2. "SU(3) and the Quark Model" (MISN-0-282).

STRENGTHS OF INTERACTIONS AND PARTICLE DIAGRAMS

by

J. R. Christman

1. Diagrams

1a. Description of Particle Diagrams. An interaction is sometimes schematically represented on a diagram with position plotted vertically and time plotted horizontally. For example, β decay of the neutron might be represented by the diagram in Fig. 1. This shows a neutron which decays at point A into a proton, an electron, and an antineutrino. Such diagrams are called *time-ordered diagrams* but we will call them *particle diagrams* or simply *diagrams*.[1] Note that they are only schematic in that they show only one space dimension. Furthermore, the slopes of the lines are drawn for clarity of pictorial representation and are not meant to represent particle velocities as one might expect. The coordinate axes, x and t, are normally omitted from the diagram.

1b. Cataclysmic Nature of the Vertex. Particle diagrams emphasize the cataclysmic view the physicist has of the microscopic world. The diagrams always show the annihilation and creation of particles at a single point in space and time. As far as is known there is not a continuous change from one set of particles to another but rather a discontinuous change. Any point on the diagram at which this occurs is called a vertex.

[1] These diagrams are very simply related to the more modern Feynman diagrams: See "Feynman Diagrams: An Introduction" (MISN-0-364). Generally, an n-vertex Feynman diagram sums n! time-ordered diagrams. However, some of the nice developments in this module cannot be based upon Feynman diagrams.

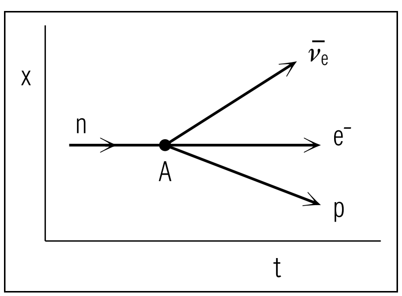


Figure 1.

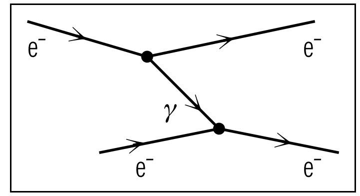


Figure 2.

The diagram above has one vertex, labeled A.

1c. Universality; "Continuous" Interactions. According to the current picture, all interactions in nature take place via cataclysmic vertices. Those interactions which appear to be continuous, in reality consist of many cataclysmic events separated by time intervals too small to be detected by the apparatus at hand. This is analogous to wind pressure, which gives the appearance of being continuous. On a small enough scale, however, the wind pressure you feel can be seen to be due to many small blows of air molecules as they strike you. Similarly, particle interactions may appear either continuous or cataclysmic, depending on the scale of the interaction. However, the cataclysmic interaction is the fundamental one since the other can be derived from it, just as in the case of wind pressure.

1d. Example: Electromagnetic Scattering. Scattering, in which the particle identities do not change, is believed to occur by the mechanism of the exchange of a particle between the two primary scattering particles. For example, charged particles do not respond to the electric field of other charged particles the way classical theory prescribes. Charge still creates electric and magnetic field intensities but these must be interpreted as a measure of the probability of finding a photon. It is the photon, transferred from one particle to another, that transfers energy and momentum and thus is the "cause" of the electromagnetic force. For example, the mutual deflection of two electrons is described by the diagram in Fig. 2. The photon is created at one vertex, annihilated at the other.

1e. Summing Diagrams, Diagram Interpretation. Diagrams are a pictorial representation of the mathematical theory used to predict outcomes of an interaction. In fact, the first step in such a calculation is to draw all possible diagrams which connect the desired initial and final states of the system being studied. Then rules tell you how to calculate the contribution of each diagram. They include an integration over all possible times for each vertex. Finally, the contributions of the diagrams

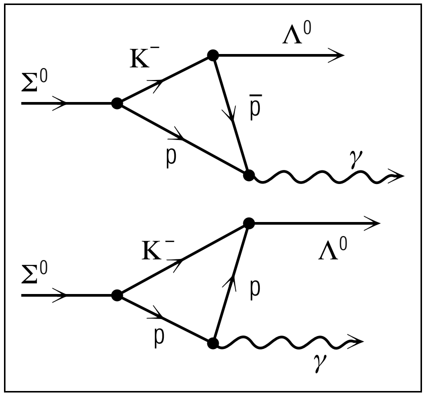


Figure 3.

are summed to obtain the total probability amplitude for the process. As a concrete example, in Fig. 3 we show two diagrams which contribute to the decay of the Σ^0 into a Λ and a γ. These diagrams' contributions must be summed, along with those of other diagrams not shown, in order to get the probability amplitude and hence the total probability transition rate.

1f. Creating Other Diagrams by Line Reversal. You can use a diagram to create other allowed diagrams by changing one or more of the particle lines to an antiparticle line. This is accomplished by merely changing a line's particle label, if necessary,[2] and reversing the line's directional arrow. It may also be convenient to alter the slope of the line on the diagram, as illustrated in the sequence in Fig. 4. There one starts off with bremmstrahlung (photon emission in electron-proton scattering) in the top diagram and winds up with photoelectric pair production on a photon in the bottom diagram. One major change is made in going from each diagram to the next. All of the diagrams shown are allowed ones.

2. Coupling Constants

2a. Decay Rate; Coupling Constant, Density of States. The transition rate for a decay or interaction is proportional to the product of the square of a *coupling constant* g and a *density of states* factor ρ:

$$R \propto g^2 \rho(E).$$

[2]No label change is necessary if the particle is a neutral boson (γ, π^0, ρ^0, etc.) hence is its own antiparticle.

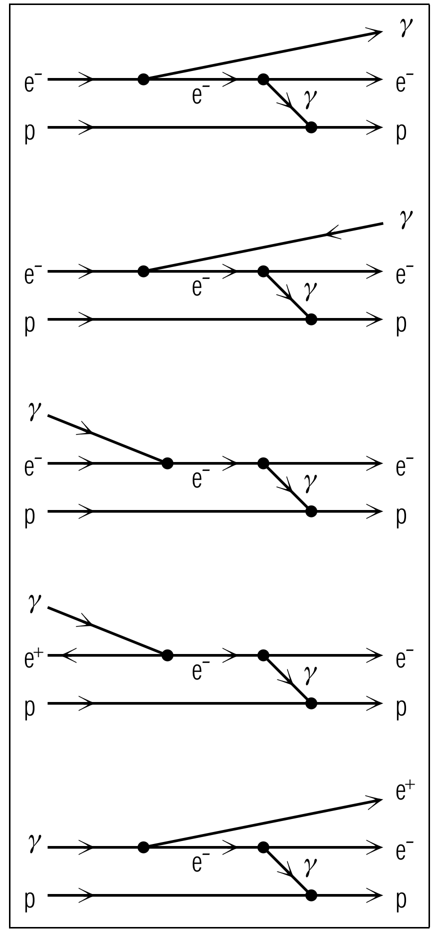


Figure 4.

The transition rate R is a measure of the probability per unit time that the interaction will occur. For decays, $R \propto 1/T$, where T is the lifetime of the particle.

The density of states ρ is a measure of the number of quantum mechanical states per unit energy range that are available for the final products. The more states that are available, the higher the transition rate to them.

The coupling constant g^2 can be interpreted as an *intrinsic rate*. For example, in β decay, g^2 measures the intrinsic rate that a neutron will

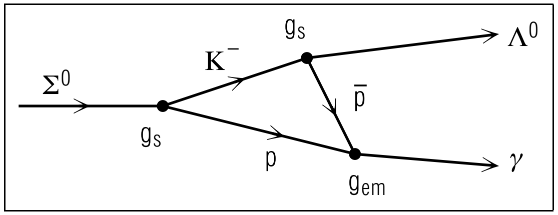


Figure 5.

decay into a proton, positron, and antineutrino. But the original energy can also be distributed in many different ways among the products, and $\rho(E)$ measures the number of different ways per unit energy interval.

2b. The Fundamental Coupling Constants. The coupling constant for a process, g, is the product of a number of *fundamental coupling constants*. The fundamental coupling constants are associated with the types of forces and are

$$\textit{strong}: \quad g \approx 1, \tag{1}$$
$$\textit{electromagnetic}: \quad g_{em} = e/\sqrt{\hbar c} \simeq 1/\sqrt{137} \approx 8.5 \times 10^{-2}, \tag{2}$$
$$\textit{weak}: \quad g_w \approx 10^{-7}. \tag{3}$$

2c. Coupling Constants and Vertices. One fundamental coupling constant is associated with each vertex of a diagram and g is the product of all the fundamental coupling constants which appear in the diagram.

2d. A Diagram Contributing to Neutral Sigma Decay. One contribution to the decay of a Σ^0 is shown in the three-vertex diagram of Fig. 5. Here Σ^0 decays strongly to a proton and a kaon; the kaon in turn decays strongly to a lambda and an antiproton. The latter annihilates the proton electromagnetically to produce a γ. The overall coupling constant is $g = g_s g_s g_{em} \simeq 1/\sqrt{137}$ where the last approximation holds since $g_s \simeq 1$.

A note: the decay $\Sigma^0 \to \Lambda^0 + \gamma$ cannot proceed according to the one-vertex diagram of Fig. 6. This is because uncharged particles cannot produce photons directly. There must be an intermediate decay to produce charged particles, one of which produces the γ. There are other diagrams for the decay of the Σ^0; these make use of other intermediate particles besides the kaon and the antiproton.

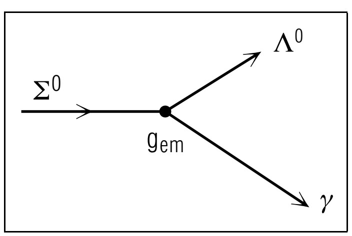


Figure 6.

2e. Other Diagrams Also Contribute. Often many different diagrams can lead to the same final state.[3] If there are in fact many diagrams which lead to the same final state, all will contribute. The number of vertices is called the *order* of the diagram. As more and more weak or electromagnetic vertices are added, the contribution represented by the diagram decreases; more factors of g_w or g_{em} are contributing and these are small numbers. On the other hand, strong vertices can be added without appreciably changing the transition rate. The possibility of many strong vertices complicates particle interactions. For example, an interaction may occur on a time scale characteristic of a weak interaction but its diagram may in fact have several strong vertices (along with a weak one).

2f. Other Influences. Calculation of the transition rate or decay time may involve other quantities such as mass and momentum, spin, or isotopic spin. However, the product of the fundamental coupling constants gives a reasonable indication of the order of magnitude of the rate.

3. Intermediate States

3a. A Simple Intermediate State. The process $\pi^+ + \mathrm{p} \to \pi^+ \rho$ has contributions from many diagrams, but the simple-looking one in Fig. 7 is a major contributor. It shows a π^+ and a proton coming together and becoming a virtual Δ. The latter exists for awhile, then decays into a π^+ and a proton. An external observer-physicist would say that the process was π^+-p scattering. We will use this diagram for illustrative purposes.

[3]See "Feynman Diagrams; An Introduction" (MISN-0-364) for six diagrams contributing to Σ^0 decay.

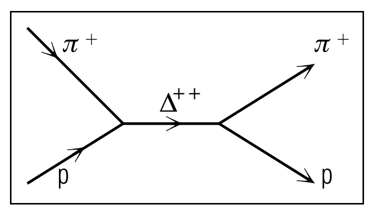


Figure 7.

3b. Conserved Quantities at a Vertex. At each vertex, all appropriate quantities are conserved except energy. By "appropriate" is meant all universally conserved quantities, except energy, plus those additional quantities conserved by the vertex's type of interaction. For example, strangeness must be conserved across a strong-interaction vertex but need not be conserved across a weak one. Parity is conserved across an electromagnetic vertex but not necessarily across a weak one. Momentum, angular momentum, baryon number, etc. are conserved across all vertices.

3c. Vertex ΔE, Virtual Particle Δt. According to the energy-time uncertainty relation,[4] conservation of energy can be violated in the amount ΔE for times less that $\Delta t \simeq \hbar/\Delta E$. Over the entire period of experimental observation of a process Δt is long and energy is conserved. However, at some vertices an intermediate particle is produced and shortly thereafter absorbed at another vertex (for example the Δ^{++} in the π^+-p scattering diagram above). If the intermediate product exists for only a short period of time, the energy can deviate considerably from the value it would have if it were conserved. In the above example, the mass of the Δ^{++} may be greater than the total energy of the original π^+-p system. At low kinetic energies the π^+-p cannot produce the Δ^{++} as a final particle but can produce one as an intermediate particle. If the energy is increased so that Δ^{++} can be produced without violating conservation of energy, the Δ^{++} may appear as a final product. Intermediate particles whose creation violates conservation of energy are called *virtual* particles. Note that the γ is real in the diagrams in Sects. 1e and 2d but virtual in the one in Sect. 1d.

3d. Transmission of ΔE Across a Diagram. In order for energy to be conserved across a whole diagram, the energy surplus or deficit at one vertex, ΔE, must be transmitted across the diagram to later vertices. The latter can then provide equal but opposite compensation, $-\Delta E$. The ΔE information is transmitted via the time phase of the intermediate particles' wave function. Thus in the above diagram the Δ^{++} wave function has an extra time phase proportional to the ΔE across the first vertex. This phase is used at the second vertex to produce a compensating shift and hence overall energy conservation.

[4] See "The Uncertainty Relations" (MISN-0-241).

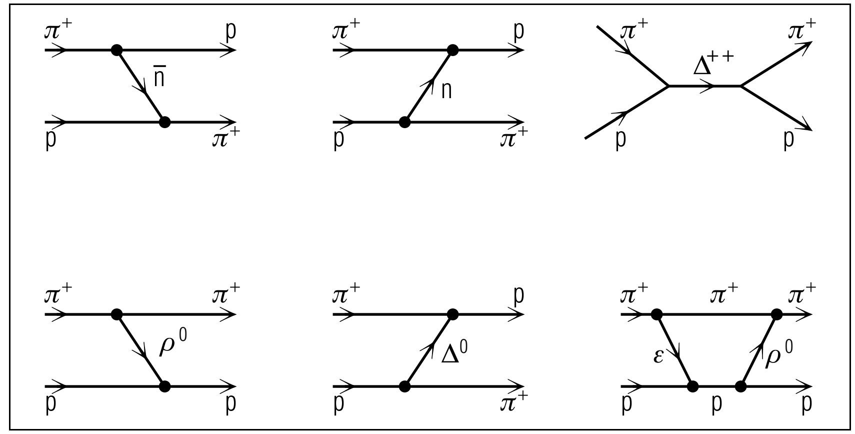


Figure 8.

4. Examples

4a. Pion-Proton Scattering. In the process $\pi^+ + \text{p} \to \pi^+ + \text{p}$, the initial and final particles will all experience the strong interaction since they are hadrons. This means that the process will be dominated by diagrams with strong interaction vertices and intermediate-state hadrons. Here are examples:

4b. Neutral Pion Decay to Two Gammas. The decay $\pi^0 \to \gamma + \gamma$ cannot occur directly since the π^0 is uncharged. Uncharged particles do not experience the electromagnetic interaction, and that is the only interaction experienced by the γ. Thus the π^0 must first undergo a strong decay to charged mesons or to a charged baryon-antibaryon pair (Fig. 9).

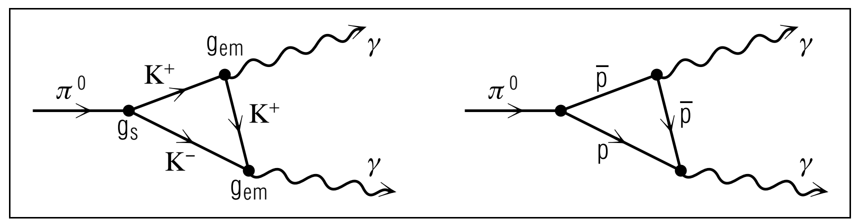


Figure 9.

Acknowledgments

I would like to thank Elliot Lehman for valuable advice on this module. Preparation of this module was supported in part by the National Science Foundation, Division of Science Education Development and Research, through Grant #SED 74-20088 to Michigan State University.

PROBLEM SUPPLEMENT

1. The following reactions are the same as those used in Problems 4-1, 4-2 of M. J. Longo, *Fundamentals of Elementary Particle Physics*, McGraw-Hill Book Co., N.Y. (l973).

   Devise simple diagrams for:

   a. K^--p elastic scattering
   b. e-p elastic scattering
   c. $\pi^- + \mathrm{p} \rightarrow \mathrm{K}^+ + \Sigma^-$
   d. $\pi^- + \mathrm{p} \rightarrow \mathrm{K}^0 + \Lambda^0$
   e. γ-p elastic scattering
   f. $\gamma + \mathrm{p} \rightarrow \pi^+ + \mathrm{n}$
   g. $\overline{\mathrm{p}} + \mathrm{p} \rightarrow \pi^+ + \pi^-$
   h. $\overline{\mathrm{p}} + \mathrm{p} \rightarrow \mathrm{e}^+ + \mathrm{e}^-$

2. Estimate the following rate ratios and compare to experimental fractional decay mode ratios from the latest "Review of Particle Properties," April *Physics Letters*.

   $(\pi^0 \rightarrow \gamma + \gamma)/(\pi^0 \rightarrow e^- + e^+ + \gamma)$

   $(\Sigma^+ \rightarrow \mathrm{p} + \pi^0)/(\Sigma^+ \rightarrow \mathrm{p} + \gamma)$

   $(\eta^0 \rightarrow 3\pi)/(\mathrm{K}^0 \rightarrow 3\pi)$

MODEL EXAM

1. Definitions and relations:

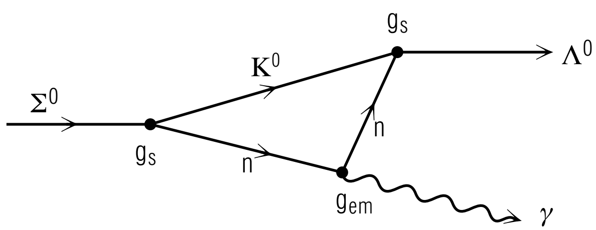
   a. Relate the "intrinsic rate" of an interaction to its coupling constant.

   b. Define "virtual particle."

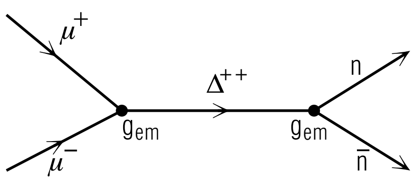
2. Give rough values for the fundamental rates g_s, g_{em}, g_w.

3. Describe, in words, the processes shown in these diagrams:

   a.



   b.



4. For the process shown in part 3a), estimate the intrinsic rate.

5. A hypothetical interaction takes place by exchange of an X particle. Real X particles are produced when the center of mass energy is about 250 MeV. Estimate:

   a. the range of the interaction

   b. the typical time of the interaction

Brief Answers:

1. a. The intrinsic rate equals the coupling constant squared.

   b. A virtual particle is one in an intermediate state, produced by an interaction which violated conservation of energy.

2. $g_s \simeq 1$; $g_{em} \simeq 1/\sqrt{137}$; $g_w \simeq 10^{-7}$.

3. a. A sigma to a neutron and a kaon via the strong interaction. The neutron (which has a magnetic moment!) emits a photon via the electromagnetic interaction, then strongly combines with the kaon to form a lambda via the strong interaction.

   b. A muon pair annihilates to form a photon which then turns into a neutron-antineutron pair.

4. $g^2 \simeq (g_s^2\, g_{em})^2 \simeq \dfrac{1}{137}$

5. a. $R \simeq c\Delta t \simeq c\dfrac{\hbar}{\Delta E} \simeq \dfrac{200\,\text{MeV fm}}{250\,\text{MeV}} \simeq 0.8\,\text{fm} = 8 \times 10^{-16}\,\text{m}.$

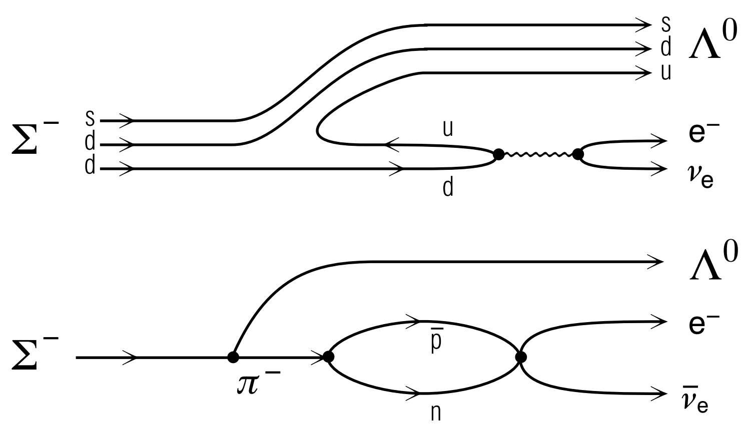
   b. Use $1\,\text{fm} = 10^{-15}\,\text{m}$ to get:

   $$\Delta t \simeq \frac{\hbar}{\Delta E} \simeq \frac{(200\,\text{MeV})(10^{-15}\,\text{m})}{(3 \times 10^8\,\text{m/s})(250\,\text{MeV})} \simeq 0.3 \times 10^{-23}\,\text{s}$$

PHYSNET

MISN-0-278

ISOSPIN: CONSERVED IN STRONG INTERACTIONS



Project PHYSNET Physics Bldg. Michigan State University East Lansing, MI

ISOSPIN: CONSERVED IN STRONG INTERACTIONS

by

J. Christman

Title: **Isospin: Conserved in Strong Interactions**

Author: J.R. Christman, Dept. of Physical Science, U.S. Coast Guard Academy, New London, CT

Version: 11/12/2001 Evaluation: Stage B1

Length: 2 hr; 12 pages

Input Skills:

1. State the algebraic rules for spin (MISN-0-251).
2. Calculate the hypercharge and strangeness of a particle (MISN-0-277).

Output Skills (Knowledge):

K1. Identify the hadron multiplets and list their members.

K2. Give the isotopic spin for each hadron multiplet and explain how it is related to the number of members in the multiplet.

K3. Relate the isospin concept to the hypothesis that members of a multiplet are different states of the same basic object.

K4. Enumerate the allowed values for T_3, for a given T.

K5. Give the assigned values of T and T_3 for each hadron and its antiparticle.

K6. State the relationship between charge, strangeness, baryon number and T_3.

Output Skills (Problem Solving):

S1. For a given system of particles, calculate the allowed values of the total isotopic spin.

S2. Given a decay or interaction, use the conservation laws to determine if it can occur and if so, which force is responsible.

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New authors, reviewers and field testers are welcome.

ISOSPIN: CONSERVED IN STRONG INTERACTIONS

by

J. Christman

1. Abstract

The quantity isospin is conserved only by the strong interaction. Isospin values are assigned only to hadrons: if leptons or photons are involved then the interaction is not strong and isospin is irrelevant. The algebra of isospin is precisely the same as the algebra of angular momentum.

2. Readings

1. Sect. 27.10 in K. W. Ford's *Classical and Modern Physics*, Vol. 3, John Wiley and Sons, NY(1972).

2. Chapter 1 in M. J. Longo's *Fundamentals of Elementary Particle Physics*.

3. Isospin Concept

3a. Multiplet Splitting and Charge. Examples of Hadron multiplets are: a doublet of nucleons and a triplet of pions. The members of a multiplet have the same strangeness, hypercharge, spin, baryon number, electron family number, and muon family number, but differ in charge and differ slightly in mass (about 5 MeV differences in mass). It is believed that if electromagnetic interactions were turned off, the members of a multiplet would be identical.

This hypothesis is born out by the study of nucleon-nucleon scattering where neutron-neutron scattering can be deduced from proton-proton scattering by "turning off the proton's charge" in the scattering equations.

3b. Replace Charge by Isospin. It is convenient to define a new quantity, called isotopic spin or isospin, that accounts for the charges of the various members of a multiplet. Isospin is a little more complicated than it need be just to account for charge: it turns out that isospin is conserved in strong interactions and is more restrictive than simple charge

conservation.

3c. Reason for "Spin" in Isospin. The algebra of isospin is exactly the same as for spin, which accounts for its name. It should be emphasized at the outset, however, that isospin is not spin. Spin exists in real space but isospin exists only in an abstract mathematical isospin space.

4. Isospin Calculations

4a. Magnitude, Components. Each multiplet is assigned a isospin number T (or, more usually, I) that is a positive integer or half an odd positive integer. Isospin may be considered to be a vector with magnitude $\sqrt{T(T+1)}$ (compare with the spin vector of magnitude $\hbar\sqrt{s(s+1)}$). To emphasize that isospin is not a vector in coordinate space, we will not call the axes x, y, z but rather 1, 2, 3. The 3 component, T_3, may take on any one of the values T, $T-1$, $T-2$, …, $-T$ in a fashion similar to the values of the z component of angular momentum. Each of these values corresponds to a different member of the multiplet with $T_3 = T$ assigned to the particle with the most positive charge and $T_3 = -T$ assigned to the particle with the most negative charge. If a particle is in a state with definite T_3 , the uncertainty principle prohibits it from having definite values of T_1 or T_2 so we ignore those components.

4b. Multiplicity. If the isospin of a multiplet is T, there are $2T + 1$ particles in the multiplet. This result follows from counting the possible values of T_3. Thus singlets have $T = 0$, doublets have $T = 1/2$, and triplets have $T = 1$. Later we shall run into a quartet ($T = 3/2$).

4c. Isospin Assignments. A summary of isospin assignments is given in the chart in the Appendix. Note that an antiparticle has the same value of T as the corresponding particle but its T_3 has the opposite sign.

4d. Isospin Addition. Total isospin for a collection of particles is computed in the same manner as for ordinary spin. The magnitude of the total isospin is $\sqrt{T(T+1)}$ where T is a positive integer or half an odd positive integer and, for a given collection of particles, the maximum value T can have is the sum of the individual particle's T's:

$$T_{\max} = \sum_i T(i).$$

The value of T_3 for a collection of particles is the sum of the individual particle's T_3's:

$$T_3 = \sum_i T_3(i).$$

and T must be greater than or equal to $|T_3|$. Thus T can have any one of the values $T_{\max}$, $T_{\max-1}$, $T_{\max-2}$, ..., $|T_3|$.

For example if one considers π^+-p scattering, $T_{\max} = 3/2$ and $T_3 = 3/2$ so T can only have the value 3/2. For $\pi^- - p$ scattering, $T_{\max} = 3/2$ and $T_3 = -1/2$ so T can be either 3/2 or 1/2. In fact, measurement of T for π^--p sometimes produces $T = 3/2$ and sometimes $T = 1/2$ but always $T_3 = -1/2$.

5. Isospin Conservation

5a. Relationship: Q, S, B, T_3; Y, T_3. Charge, baryon number, and the 3 component of isotopic spin are related. The charge of a particle is given by:

$$q = \frac{S+B}{2} + T_3 = \frac{Y}{2} + T_3.$$

The first term, $(S+B)/2$, gives the average charge of the multiplet (or half the hypercharge) and is the same number for all particles in the multiplet. T_3 measures additional charge the specific particle has. Note that the signs of S, B, T_3, and q are opposite for a particle and its antiparticle.

5b. Conservation of T, T_3; Strong, EM, Weak. Both T and T_3 are conserved during strong interactions and decays. Since q, S, and B are conserved there, the conservation of T_3 is not a new conservation law (see equation of Sect. 3a). For weak interactions neither S, T_3, nor T need be conserved although q and B are universally conserved. For electromagnetic interactions, T need not be conserved.

6. Applications

6a. Experimental Consequences. The conservation of isospin prohibits some decays from taking place via the strong interaction.

For example (see the chart of Sect. 2), consider the decay of the Σ^0. The products must include a single baryon (conservation of energy prohibits 2 baryons and an antibaryon). If the decay is to be strong the product particles must have strangeness -1. This requirement leads to

the conclusion that one of the products is either Λ^0 or one of the antikaons. A kaon and any baryon together are too massive so the decay must be to a Λ^0. Any other particles that could be produced must have total mass less than 76.9 MeV: i.e., photons, electrons, or neutrinos.

We have determined that the decay products of Σ^0 must be the Λ^0 plus photons or leptons. Can this decay take place via the strong interaction? The Σ^0 has isotopic spin 1 while the Λ^0 has isotopic spin 0. All the other possible products (leptons and photons) have isotopic spin 0. We conclude that the decay cannot take place via the strong interaction. However all the conservation laws for the electromagnetic interaction do hold—in fact the decay $\Sigma^0 \to \Lambda^0 + \gamma$ does not violate any conservation law we have discussed except isospin. The decay takes place via the electromagnetic interaction.

6b. Deducing Interactions from Decay Products. You are now in a position to figure out which of the three forces is responsible for a decay or interaction for which the products are given. Assume that the interaction proceeds via the fastest route for which the appropriate quantities are conserved.

A few notes:

i. If particles appear for which isospin is not defined (γ, leptons), then the interaction is not strong.

ii. If neutrinos appear, then the interaction must be weak.

iii. If photons appear, the interaction must be electromagnetic.

iv. It is sometimes impossible to distinguish between weak and electromagnetic interactions on the basis of product particles alone. In particular, when C, P, or T violation occurs, the products may not show the violation (neutrinos, of course, do).

v. What you are really doing is finding the dominant part of the interaction for the determination of lifetime. We shall see in the next section that some interactions and decays proceed by two or more steps. Nevertheless it is worthwhile to practice predicting the type of decay from conservation law restrictions.

6c. Examples.

a. $\pi^- + \mathrm{p} \to \mathrm{K}^0 + \Lambda^0$

This interaction conserves charge and baryon number. It can conserve angular momentum, energy, and momentum. It conserves strangeness. For the pion, $T = 1$, $T_3 = -1$; for the proton, $T = 1/2$, $T_3 = 1/2$; for the kaon, $T = 1/2$, $T_3 = -1/2$; and for the lambda $T = 0$, $T_3 = 0$. Initial states could be $T = 3/2$, $T_3 = -1/2$ or $T = 1/2$. The final state must be $T = 1/2$, $T_3 = -1/2$. We conclude that isotopic spin could be conserved. So, barring violation of C, P, or T, the interaction is strong.

b. $\mathrm{p} + \mathrm{p} \rightarrow \mathrm{K}^0 + \mathrm{p} + \mathrm{p}$

This interaction does not conserve strangeness so it must be weak. Charge and baryon number are conserved and angular momentum, energy, and momentum could be conserved.

6d. Predicting Products of Decays or Interactions. In some cases you can use the conservation laws to predict the outcome of a decay or interaction. List all possible outcomes that conserve the universally conserved quantities (to use the conservation of energy law, you will need to know or assume the kinetic energy of the incident particle). Search for a combination that does not violate any conservation laws for the strong interaction. If a combination is found, that will in fact be the outcome. If two or more are found they will generally compete and all of them will be seen experimentally. If none are found proceed to the electromagnetic interaction and repeat the process. If none are found, finally consider the weak interaction.

It is important to proceed in the order: strong, electromagnetic, weak. If an interaction can go via either strong or electromagnetic, it will go via the strong since this is so much faster then the electromagnetic, etc.

Acknowledgments

Preparation of this module was supported by the United States Coast Guard Academy for a Directed Studies Program. Preparation of this module was supported in part by the National Science Foundation, Division of Science Education Development and Research, through Grant #SED 74-20088 to Michigan State University.

A. Table of Isospin Values

Multiplet	T	Members	T_3
Pion	1	π^+	+1
		π^0	0
		π^-	−1
Kaon	1/2	K^+	+1/2
		K^0	−1/2
Eta	0	η	0
Nucleon	1/2	p	+1/2
		n	−1/2
Lambda	0	Λ^0	0
Sigma	1	Σ^+	+1
		Σ^0	0
		Σ^-	−1
Xi	1/2	Ξ^0	+1/2
		Ξ^-	−1/2
Omega	0	Ω^-	0

PROBLEM SUPPLEMENT

1. Give possible values for the total isotopic spin T and for its 3-component T_3 for each of these systems of particles:

a. π^0 - p
b. π^+ - p
c. π^- - n
d. π^0 - n
e. p- n
f. Σ^0 - p
g. Λ^0 - n
h. K^- - K^+
i. K^0 - Ξ^-
j. Λ^0 - Ω^-

2. Figure out which force is responsible for each of the following decays or interactions. Justify your answer. If the decay or interaction does not occur, so state and name the conservation law that is violated.

a. $\mathrm{n} \rightarrow \mathrm{p} + \mathrm{e}^- + \bar{\nu}_\mathrm{e}$
b. $\pi^- + \mathrm{p} \rightarrow \pi^0 + \mathrm{n}$
c. $\Omega^- \rightarrow \mathrm{K}^- + \overline{\mathrm{K}}^0$
d. $\overline{\mathrm{p}} + \mathrm{p} \rightarrow \mathrm{e}^+ + \mathrm{e}^-$
e. $\mathrm{p} + \mathrm{p} \rightarrow \pi^+ + \mathrm{d}$
f. $\mathrm{n} + \mathrm{p} \rightarrow \eta + \mathrm{d}$

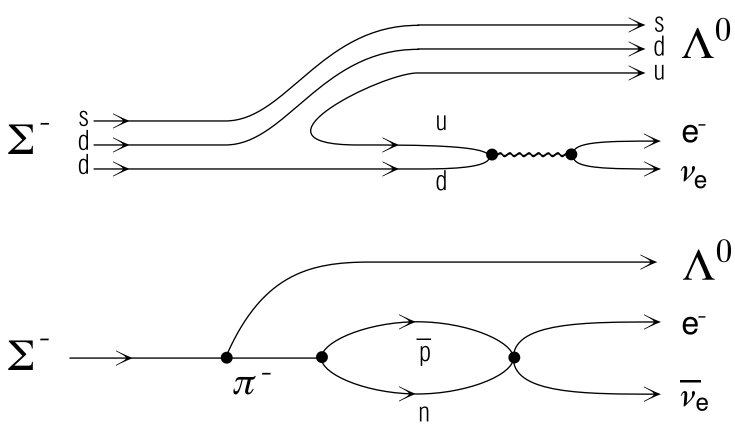
Here d = deuteron = (n + p). Experimentally it is found that $T = 0$ for the deuteron.

3. The Δ^{++} is a particle with mass 1236 MeV, charge $2\,|e|$, spin $\hbar/2$, isospin $T = T_3 = 3/2$, strangeness 0, and baryon number 1. What are the decay products of the Δ? What force is responsible for the decay? Consider only particles stable against strong decay.

4. Suppose the Δ has all the properties listed in problem 3 except that its mass is 1050 MeV. What would then be its decay products? What force would be responsible for the decay? Consider only particles stable against strong decay.

MISN-0-277

PROPERTIES CONSERVED IN STRONG AND EM INTERACTIONS



Project PHYSNET Physics Bldg. Michigan State University East Lansing, MI

PROPERTIES CONSERVED IN
STRONG AND EM INTERACTIONS

by
J. Christman

Title: **Properties Conserved in Strong and Em Interactions**

Author: J. R. Christman, Dept. of Physical Science, U. S. Coast Guard Academy, New London, CT

Version: 11/12/2001 Evaluation: Stage B1

Length: 2 hr; 0 pages

Input Skills:

1. Discuss the concept of operators, and illustrate their use.
2. Give the charge and spin of the strong-force stable particles (MISN-0-274).
3. State which properties are always conserved (MSN-0-275).

Output Skills (Knowledge):

K1. Explain how strangeness conservation or non-conservation can affect particle lifetimes.

K2. Give the definition of the charge conjugation operation and which interactions do and which do not violate charge conjugation invariance.

K3. Give the definition of the time reversal operation and state which interactions do and which do not violate time reversal invariance.

Output Skills (Rule Application):

R1. Calculate the hypercharge and strangeness of a particle, given the charge of the members of its multiplet and its baryon number.

R2. Draw diagrams showing what happens to the spin and momentum of a given single particle or of two colliding particles due to application of $\mathcal{C}$, $\mathcal{P}$, $\mathcal{T}$, $\mathcal{CP}$, $\mathcal{CT}$, $\mathcal{PT}$ and $\mathcal{CPT}$, whether that produces real particles or not.

R3. In general or for a given reaction, relate the possible interaction(s) to the production or non-production of particles with strangeness.

External Resources (Required):

1. See the "Readings" section of this module's *text*.

External Resources (Optional):

1. See the "Readings" section of this module's *text*.

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New authors, reviewers and field testers are welcome.

PROPERTIES CONSERVED IN STRONG AND EM INTERACTIONS

by

J. Christman

1. Abstract

This module deals with the peculiar conservation laws obeyed by both the electromagnetic and strong interactions, which also obey the universal conservation laws. In a given situation, if all possible outcomes violate the laws obeyed by these two interactions, then nature is forced to resort to the weak interaction and this shows up experimentally in much longer interaction times. If the weak interaction is also not possible, then there is no interaction.

2. Readings

1. Ford, Vol. 3, Sect. 27.6 through 27.9, on reserve for you in the Physics-Astronomy Library. Ask for it as "Ford, Volume 3."
2. E. P. Wigner, "Violations of Symmetry in Physics," *Scientific American*, Dec. 1965, on reserve for you in the Physics-Astronomy Library. Ask for it as the CBI readings "Violations of . . . ."
3. Suggested: W. R. Frazer, *Elementary Particles*, if you happen to have it.

3. Multiplets

3a. Type of Property. Strangeness is a number assigned to a particle just like charge or baryon number. It is relevant for mesons and baryons.

3b. Multiplet Groupings: M, S, Q, Y. Hadrons can be arranged in small groups with all members of the group having nearly the same mass. For ease in remembering, the members of a group are called by the same name (pions, kaons, nucleons, antikaons, etc). The groups are called multiplets and the number of particles in a group is called the multiplicity of that group. All members of a multiplet have the same strangeness and it is equal to twice the average charge (in units of e) of the multiplet minus the baryon number. Twice the average charge is called the *hypercharge*

and is denoted by Y. Thus $S = Y - B$. This formula must be altered for particles that have *Charm*, another quantum number which will be introduced in a later lesson. All leptons and photons are assigned $S = 0$.

3c. Table; 9 Multiplets. Here is the baryon number (B), hypercharge (Y) and strangeness (S) of each strong-stable non-charmed meson and baryon:

Multiplet	Members	$Q_{ave.}$	Y	B	S
Pion	π^+, π^0, π^-	0	0	0	0
Kaon	K^+, K^0	1/2	1	0	1
Anti-Kaon	K^-, $\overline{K}^0$	-1/2	-1	0	-1
Eta	η^0	0	0	0	0
Nucleon	p, n	1/2	1	1	0
Lambda	Λ^0	0	0	1	-1
Sigma	Σ^+, Σ^0, Σ^-	0	0	1	-1
Xi	Ξ^0, Ξ^-	-1/2	-1	1	-2
Omega	Ω^-	-1	-2	1	-3

Antiparticles have the same magnitude of strangeness as the respective particle but opposite in sign. Note: the K^- is the anti-particle of the K^+ because the K's are mesons, which are bosons so the antiparticle is just an ordinary particle. The antiparticle of the Σ^+ is negatively charged, but it is not the Σ^-. The Σ's are baryons, which are fermions, so their antiparticles are not just ordinary particles.

4. Strangeness

4a. "Conservation," "Interaction". The total strangeness for a collection of particles is the algebraic sum of the individual strangeness numbers. Every strong and electromagnetic decay or interaction conserves total strangeness. Weak decays and interactions do not.

4b. Strangeness-Violating Decays: Nine Examples. One important consequence of strangeness conservation is that it prevents the decay of certain particles via the strong or electromagnetic interaction. These particles have the long lifetimes (10^{-10} sec) associated with weak decay rather than the shorter lifetimes associated with strong (10^{-23} sec) or electromagnetic (10^{-21} sec) decays.

Refer to the table of Note 3. The following decays are all weak and *all violate conservation of strangeness*:

$$\begin{aligned}
K^0 &\rightarrow \pi^0 + \pi^0 \\
K^0 &\rightarrow \pi^0 + \pi^- \\
K^0 &\rightarrow \pi^0 + e^- + \bar{\nu}_e \\
\Lambda^0 &\rightarrow p + \pi^- \\
\Sigma^0 &\rightarrow n + \pi^0 \\
\Sigma^- &\rightarrow n + \pi^- \\
\Xi^- &\rightarrow n + \pi^- \\
\Xi^- &\rightarrow \Lambda^0 + \pi^- \\
\Omega^- &\rightarrow \Xi^0 + \pi^{\cdot}
\end{aligned}$$

In each case all of the *universal* conservation laws hold and there is no reason (except conservation of strangeness and, in one case, conservation of parity, to be discussed later) why these decays could not go via the stronger and faster interactions. All of the particles (initial and final) in the interactions above, except the electron and neutrino which appear in the K^0 decay, do participate in the strong interaction.

4c. Combining Consv. Laws: S, E, B. It is worthwhile to understand why the K^+, Λ^0, Ξ^0, and Ω^- cannot decay via the strong interaction or the electromagnetic interaction. The K^+ is the lightest strange particle and a combination of the energy and strangeness conservation laws forces its decay to be weak.

The K^+ typically decays according to

$$K^+ \rightarrow \pi^+ + \pi^0$$

with a lifetime on the order of 10^{-8} sec, a weak decay. This decay violates conservation of strangeness.

The Λ^0 is the lightest strange *baryon* and conservation of energy, strangeness, and baryon number precludes its decay via the strong or electromagnetic interactions.

The Ξ^0, if it decayed strongly or electromagnetically, must decay to particles with a net strangeness of -2. Clearly neither two Σ's, nor two Λ's, nor a Σ and a Λ, nor a Σ and a K, nor a Λ and a K, meet energy conservation requirements. Two K's have less total mass and conserve strangeness and charge. However the products must include a baryon and all baryons

have too much mass to join the kaons as decay products. The Ξ^0 must decay weakly. In fact, the most probable decay is

$$\Xi^0 \rightarrow \Lambda^0 + \pi^0$$

with a lifetime of 3.0×10^{10} sec. This decay violates conservation of strangeness.

That the Ω^- also decays weakly can be predicted by invoking conservation of strangeness, energy and baryon number.

4d. A Strange Case: Sigma decay. The decay $\Sigma^0 \rightarrow \Lambda^0 + \gamma$ does not violate conservation of strangeness and is *not* weak. The decays of the charged sigmas are weak but their strong decay would not violate any of the conservation laws discussed so far. These decays will be discussed later.

4e. A Contrast: S vs. $\mathcal{C}$, $\mathcal{P}$, $\mathcal{T}$. The remaining conservation laws of this section will be stated in a form which is different from that of the previous laws. That is, we shall not assign numbers to the particles which are interacting but rather we shall make the statement that two interactions have or do not have certain identical characteristics.

5. Charge Conjugation

5a. The $\mathcal{C}$ Operator. Charge conjugation is the operation of changing a particle into its antiparticle or vice versa. That is,

$$\mathcal{C}(\pi^+) = \pi^-$$

$$\mathcal{C}(p) = \overline{p}$$

$$\mathcal{C}(\overline{p}) = p,$$

etc. Here $\mathcal{C}$ stands for the operation of charge conjugation.

5b. Invariance Under $\mathcal{C}$: Strong, Weak. The strong and electromagnetic interactions are said to be invariant under charge conjugation. This means that the strong interaction between two particles, A and B, is precisely the same as the strong interaction between the two antiparticles $\bar{A}$ and $\bar{B}$ in the sense that these two interactions have the same strength. If, in the first interaction, A is scattered through a certain angle with a certain probability, then, in the second interaction, $\bar{A}$ is scattered through the same angle with the same probability provided, of course, the experimental conditions (initial energy, momentum, and spin) are the same. If

$A \to B + C$ via the strong interaction with a certain decay time, then $\bar{A} \to \bar{B} + \bar{C}$ with the same decay time. Analogous statements about the electromagnetic interaction are also true.

5c. $\mathcal{C}$ Non-Invariance: Weak. Violation of charge conjugation invariance occurs in weak interactions and decays. As an example, consider the two decays

$$\mu^- \to \mathrm{e}^- + \bar{\nu}_\mathrm{e} + \nu_\mu \,,$$

and

$$\mu^+ \to \mathrm{e}^+ + \nu_\mathrm{e} + \bar{\nu}_\mu \,,$$

where both muons are in states with spin in the same direction, for example. Experimentally, it is found that the electrons from the μ^- tend to leave the decay site preferentially in the direction opposite to the spin direction of the μ^- while the positrons from μ^+ tend to leave preferentially in the same direction as the spin of the μ^+. The decay is not invariant under charge conjugation.

6. Parity

6a. The $\mathcal{P}$ Operator, on x, p, L, S. The parity operator, denoted $\mathcal{P}$, reverses the components of all true vectors. For example, the parity operation on radius and momentum vectors produces:

$$\mathcal{P}(\vec{r}) = \mathcal{P}(x\hat{x} + y\hat{y} + z\hat{z}) = -x\hat{x} - y\hat{y} - z\hat{z} = -\vec{r}\,.$$

$$\mathcal{P}(\vec{p}) = \mathcal{P}(p_x\hat{x} + p_y\hat{y} + p_z\hat{z}) = -p_x\hat{x} - p_y\hat{y} - p_z\hat{z} = -\vec{p}\,.$$

Thus a radius vector gets reflected through the coordinate-space origin while any other true vector, like $\vec{p}$, gets reversed in coordinate space. Note that angular momentum is the product of two vectors so it does *not* get reversed by the parity operator:

$$\mathcal{P}(\vec{L}) = \mathcal{P}(\vec{r} \times \vec{p}) = (-\vec{r}) \times (-\vec{p}) = \vec{L}\,.$$

Angular momentum is not a true vector; in mathematics it is called a *pseudovector*. Spin is a form of angular momentum so it too does not change sign under the parity operation.

6b. Inv. Under $\mathcal{P}$: Strong & EM, not Weak. An elementary particle reaction is said to be parity invariant if the parity operator, acting on both sides of the reaction equation, produces another reaction which is

found, experimentally, to occur with exactly the same probability. That is, suppose we have particles A, B, C, and D which appear in this reaction:

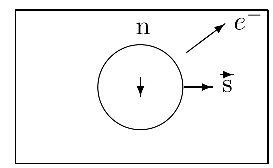
$$A + B \to C + D\,,$$

and

$$\mathcal{P}(A) = A', \quad \mathcal{P}(B) = B', \quad \mathcal{P}(C) = C'\,.$$

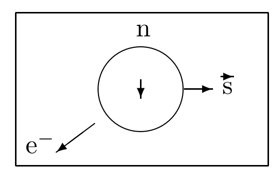
Then the interaction which produces C and D from A and B is said to be parity invariant if the two reactions, $A+B \to C+D$ and $A'+B' \to C'+D'$, are found, experimentally, to occur with equal probability.

6c. $\mathcal{P}$ and Weak Interactions, Neutrinos. Strong and electromagnetic interactions are invariant under the parity operation. The weak interaction is not. The first and most famous experiment to show this is the β decay of the neutron $\mathrm{n} \to \mathrm{p} + \mathrm{e}^- + \bar{\nu}_\mathrm{e}$. It is found experimentally that a neutron at rest emits electrons preferentially into the hemisphere centered on the direction of the spin:



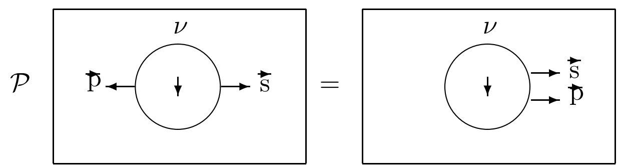
The neutron, represented by the circle, exists only before the decay while the electron exists only after the decay. The downward arrow on the "surface" of the neutron represents the direction the surface of the neutron is traveling to produce the spin vector $\vec{s}$.

Now the picture after the parity operation looks like this:



The spin of the neutron does not change but the position and momentum of the electron do. Now there is nothing physically different about the *initial conditions* in the two cases—both start with the same neutron. If parity invariance were not violated, the electrons should come out with equal probability in the hemisphere centered on the neutron's spin and in the opposite hemisphere. The fact that the electrons come out preferentially in the hemisphere centered on the spin constitutes proof that the weak interaction violates parity invariance.

6d. E & S & B & ℓ & $\mathcal{P}$ for Weak Decays. The neutrino by itself is a violator of parity invariance. The only neutrinos which exist are those with spins that are in the exact opposite direction to their momenta. If we operate on the neutrino with the parity operator, we get this:



which is a neutrino with its spin in the same direction as it momentum, and this does *not* exist in nature. That is, there is zero probability of finding it in nature.

Violation of parity invariance by the weak interaction is more general than the violation which must occur when a neutrino is produced. The decay

$$\Lambda^0 \to \mathrm{p} + \pi^- \,,$$

is weak, does not include a neutrino, and violates parity invariance.

6e. $\mathcal{CP}$ Invariance and Violation. The K^0, Σ^+, Σ^-, and Ξ^- decay weakly. To show that no strong or electromagnetic decays are possible, one must invoke conservation of energy, charge, strangeness, baryon number, electron family number, and parity.

As an example, consider the strong or electromagnetic decay of the Σ^+. To conserve strangeness, which strong and electromagnetic decays require, the Σ^+ must decay either to a Λ^0 or a kaon, the only strange particles with less mass.

Consider first the decay to a Λ^0. The other decay products must have net strangeness 0, total mass less than 73.8 MeV, and charge +1. These conditions can be met only if a positron appears among the products. Other products must have net charge 0, and electron family number +1.

Only the electron's neutrino will do. Since a neutrino is required, parity invariance is violated and the decay cannot be strong or electromagnetic. The appearance of the neutrino signals the violation of parity invariance.

Now consider the possibility of decay to a $\overline{\mathrm{K}}^0$. Strangeness is conserved. Other products must have net charge +1, mass less than 695.6 MeV, and strangeness 0. In addition, at least one baryon must be involved. There is no baryon with mass less than 695.6 eV. Thus this decay cannot occur via the strong or electromagnetic interactions.

In fact, the decay most often observed is

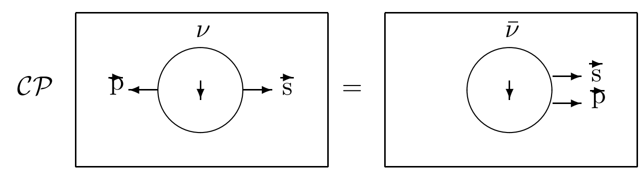
$$\Sigma^+ \to \mathrm{n} + \pi^+ \,,$$

and its lifetime of 0.8×10^{-10} sec shows that it is a weak decay.

Arguments for the weak decay of the K^0, Σ^-, and Ξ^- follow the same lines.

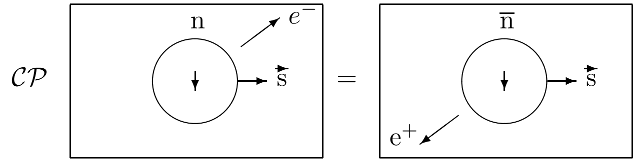
7. Time Reversal

7a. The $\mathcal{T}$ Operator: Inv. for Strong, EM. Some weak interactions are $\mathcal{CP}$ invariant. The product of $\mathcal{C}$ and $\mathcal{P}$, $\mathcal{CP}$ is an operator which simultaneously reflects the system in the origin and changes particles to antiparticles (and antiparticles to particles). For example, $\mathcal{CP}$ operating on a neutrino produces



and this particle *does* exist in nature. In fact, the antineutrino's spin is *always* in the same direction as its momentum.

$\mathcal{CP}$ operating on neutron decay produces



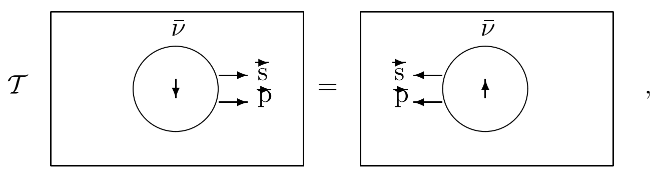
This also occurs in nature, with probability equal to that of neutron decay. That is, antineutrons decay with positrons coming out preferentially in a directions opposite to the antineutrons' spins.

There is some evidence for $\mathcal{CP}$ violation in the decay of the kaon and it is not clear whether or not $\mathcal{CP}$ is violated in a small way in other weak interactions. For example the distribution of positrons from $\bar{n}$ decay might be slightly different from the reflection in the origin of the distribution of electrons from n decay. The difference, if any, has not been detected.

7b. $\mathcal{T}$ Inv. and Weak Decays: $\mathcal{CPT}$ Inv. The time reversal operator $\mathcal{T}$ makes time run backwards. This reverses both momentum and spin and changes ingoing particles to outgoing and outgoing to incoming. If the reaction and the time-reversed reaction take place with equal probability, then the interaction is said to be time reversal invariant.

Strong and electromagnetic interactions have been found to be time reversal invariant.

As an example, here is the effect of time-reversal on the anti-neutrino:



which is also just an anti-neutrino (but going the other way and with its spin reversed!).

There is some question about whether or not weak interactions are time reversal invariant. The argument is indirect. Many physicists believe that all interactions must be $\mathcal{CPT}$ invariant. That is, if an interaction can occur, then the interaction obtained from it by successive operation

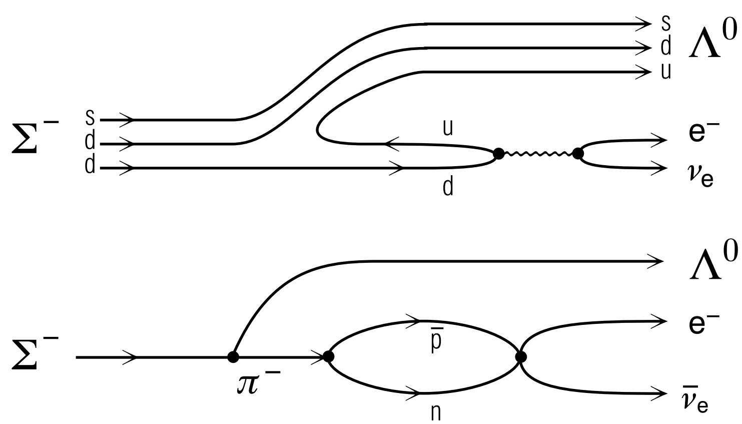
of the three operators also occurs with the same probability. If this is true and it is also true that $\mathcal{CP}$ is violated in weak interactions, then $\mathcal{T}$ must also be violated in just the right way to make weak interactions $\mathcal{CPT}$ invariant.

Acknowledgments

Preparation of this module was supported in part by The United States Coast Guard Academy for a Directed Studies Program. Preparation of this module was supported in part by the National Science Foundation, Division of Science Education Development and Research, through Grant #SED 74-20088 to Michigan State University.

MISN-0-276

UNIVERSALLY CONSERVED QUANTITIES



Project PHYSNET Physics Bldg. Michigan State University East Lansing, MI

UNIVERSALLY CONSERVED QUANTITIES

by

J. R. Christman

Title: **Universally Conserved Quantities**

Author: J. R. Christman, Dept. of Physical Science, U. S. Coast Guard Academy, New London, CT

Version: 11/13/2001 Evaluation: Stage B1

Length: 2 hr; 8 pages

Input Skills:

1. State the family and family group to which a particle belongs (MISN-0-274).
2. Explain what is meant by saying a property is conserved (MISN-0-275).

Output Skills (Knowledge):

K1. For each strong-stable particle, give its charge, baryon number, electron number, muon number, and lepton number.

K2. State the conservation laws which ensure the stability of the: (a) proton; (b) electron; (c) electron neutrinos and antineutrinos; and (d) muon neutrinos and antineutrinos.

Output Skills (Problem Solving):

S1. Given a decay or interaction in the form $A \to B + C + D$, show that it satisfies the universal conservation laws.

S2. Given a list of possible interactions, identify those that cannot occur because a universal conservation law would be violated.

External Resources (Required):

1. K. W. Ford, *Classical and Modern Physics*, Vol. 1. John Wiley and Sons (l972).

Post-Options:

1. "Additional Properties Conserved in Electromagnetic and Strong Interactions: Strangeness, Charge Conjugation, Parity, Time Reversal" (MISN-0-277).

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Our publications are designed: (i) to be updated quickly in response to field tests and new scientific developments; (ii) to be used in both classroom and professional settings; (iii) to show the prerequisite dependencies existing among the various chunks of physics knowledge and skill, as a guide both to mental organization and to use of the materials; and (iv) to be adapted quickly to specific user needs ranging from single-skill instruction to complete custom textbooks.

New authors, reviewers and field testers are welcome.

UNIVERSALLY CONSERVED QUANTITIES

by

J. R. Christman

1. Introduction

This module deals with additive quantum numbers associated with conserved quantities, and the specific values assigned to various particles.

2. Charge

2a. Q is an Interaction Strength. Charge determines the strength of the electromagnetic field produced by the particle. It also determines the force on the particle if it is put into the electromagnetic field produced by other particles.

2b. Particle and Antiparticle Q. The charge associated with each particle is listed in the table in "The Strong-Stable Particles," MISN-0-274. The charge on an antiparticle is equal in magnitude to the charge on the corresponding particle, but opposite in sign.

2c. Q is Integer Multiple of e. The magnitude of the charge on any particle so far observed is a small integer times the charge e on the proton. All particles listed have charge $+e$ or $-e$ but other fundamental particles have greater amounts of charge. The Δ^{--}, for example has charge $-2e$.

2d. Q Conservation: Consequences. Charge is conserved in the sense that the sum of all charge entering an interaction is always the same as the sum of all charge leaving. The sum must take into account the sign of the charge. For example, in the interaction

$$\mathrm{e}^+ + \mathrm{e}^- \rightarrow \gamma + \gamma,$$

there is zero net charge both before and after the interaction. Note that the identity of the particles may change but the total charge does not.

2e. Conservation of Q and E; Consequences. We shall see later that if a *single* particle undergoes decay, the sum of the masses of the particles in the final state of the system must be less than the mass of the original particle. This is a consequence of Conservation of Energy. The electron is stable because energy and charge are conserved and there are

no charged particles with mass less than the mass of the electron. The positron (antielectron) is stable for the same reason.

3. Baryon Number

3a. B Assignments. Each baryon is assigned the baryon number $B = +1$, each antibaryon is assigned $B = -1$, and all other particles are assigned $B = 0$.

3b. B Conservation. The net baryon number is conserved in an interaction in exactly the same manner that net charge is conserved. For example. in the interaction

$$n + p \rightarrow \Lambda^0 + n + K^+ + \pi^0,$$

a net baryon number of +2 on the left ($B = 1$ for the neutron, $B = 1$ for the proton) and a net baryon number of +2 on the right (carried by the Lambda and neutron).

3c. B In Baryon Production. The net number of baryons in the world can increase only if there is a like increase in the number of antibaryons.

3d. Conservation and Proton Stability. Baryon number conservation (in conjunction with energy conservation) is responsible for the stability of the proton and the antiproton: the proton is the lightest baryon.

4. Lepton Numbers

4a. Electron No. and Stability of the Electron's Neutrino. Electrons and electron neutrinos are assigned electron number +1, positrons and electron antineutrinos are assigned −1, and all other particles are assigned 0. Electron number is conserved in all interactions in precisely the same way as baryon number and charge. Simultaneous conservation of electron number and energy explains the stability of the electron neutrino and electron anti-neutrino.

4b. Muon No. and Stability of the Muon's Neutrino. In a like manner, the muon (μ^-) and its neutrino are each assigned muon number +1, the antimuon (μ^+) and the muon antineutrino are assigned −1, and all other particles are assigned 0. Muon number is conserved in the same manner as electron number. This conservation law, along with energy conservation, accounts for the stability of the muon neutrino and antineutrino.

4c. Tauon No. and Stability of the Tau's Neutrino. In a like manner, the tau (τ^-) and its neutrino are each assigned tau number +1, the antitau (τ^+) and the tau antineutrino are assigned −1, and all other particles are assigned 0. Tau number is conserved in the same manner as muon and electron number. This conservation law, along with energy conservation, accounts for the stability of the tau neutrino and antineutrino.

4d. Lepton Number. Another quantum number which is widely used is *lepton* number. Leptons (e^-, μ^-, τ^-, ν_e, ν_μ, ν_τ) are assigned +1, antileptons (e^+, μ^+, τ^+, $\bar{\nu}_e$, $\bar{\nu}_\mu$, $\bar{\nu}_\tau$) are assigned −1, and all other particles are assigned 0. It is obvious that the validity of individual conservation laws for electrons, muons, and taus implies a conservation law for leptons.

5. Family Number Conservation

5a. Mesons, Photons: No. There are no analogous conservation laws for mesons or for photons. These particles can be created or destroyed in any number, provided other conservation laws, such as the energy conservation law, are not violated.

5b. Fermions, Yes; Bosons, No. Family number conservation laws exist for fermions but not for bosons. These conservation laws are linked to spin by the laws of relativistic quantum mechanics (fermions have half-integral spin while bosons have integral spin).

For fermions, there exist negative energy states, separated from positive energy states by a gap of magnitude $2mc^2$. The vacuum state is described as one in which all negative energy states are filled and no positive energy states are filled. In this widely accepted model, the particles in the negative energy states are said to be unobservable. When enough energy is supplied to an (unobservable) electron in a negative energy state, it jumps up to one of the empty positive energy states and thereby becomes observable. The empty negative energy state it left behind (a "hole" in the negative energy states) acts like a positively charged particle and is an observable positron.

For example, in this model any *positron* is a hole in the all-pervasive "sea" of unobservable negative energy *electrons*. In the diagram below, energy

is plotted vertically. There are no states between $-mc^2$ and $+mc^2$. All energies below $-mc^2$ and above $+mc^2$ are possible states that electrons can occupy.

Vacuum = all states empty. *Electron* = 1 state filled

$E = +mc^2$ ————————————————

$E = 0$ ————————————————

$E = -mc^2$ ————————————————

Vacuum = all states filled. *Positron* = 1 state unfilled

No negative energy states exist for the bosons. However, boson antiparticles can be defined through the operation of *charge conjugation*, which is discussed in a later lesson. The force property of *invariance under charge conjugation* allows one to deduce properties of boson antiparticles from the corresponding particle properties.

6. Work These Problems

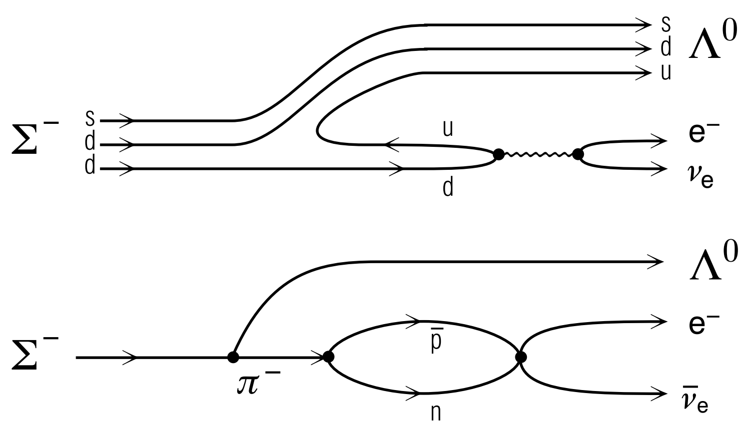
▷ Ford, Chap. 4 - E4.2, E4.3, E4.5, E4.12, E4.13, E4.14, P4.1, P4.2.

Acknowledgments

Preparation of this module was supported in part by The United States Coast Guard Academy for a Directed Studies Program. Preparation of this module was supported in part by the National Science Foundation, Division of Science Education Development and Research, through Grant #SED 74-20088 to Michigan State University.

MISN-0-275

CONSERVED QUANTITIES: AN OVERVIEW



Project PHYSNET Physics Bldg. Michigan State University East Lansing, MI

CONSERVED QUANTITIES: AN OVERVIEW

by

J. Christman, U. Coast Guard Academy

Title: **Conserved Quantities: An Overview**

Author: J.R. Christman, Dept. of Physical Science, U. S. Coast Guard Acad., New London, CT

Version: 2/1/2000 Evaluation: Stage B1

Length: 2 hr; 8 pages

Input Skills:

1. Vocabulary: conservation law (MISN-0-15).
2. State the family and family group to which a given strong-stable particle belongs (MISN-0-274).

Output Skills (Knowledge):

K1. Explain what is meant when an elementary-particle quantity is said to be "conserved" and when it is said to be "universally conserved."

K2. List the names of all the quantities which may be conserved in elementary particle interactions.

K3. For each type of elementary particle interaction, list the conserved quantities.

External Resources (Required):

1. K. W. Ford, *Classical and Modern Physics*, Vol. 1, John Wiley and Sons (1972).

Post-Options:

1. "Universally Conserved Quantities in Elementary Particle Interactions" (MISN-0-276).
2. "Additional Properties Conserved in Electromagnetic and Strong Interactons" (MISN-0-277).

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New authors, reviewers and field testers are welcome.

CONSERVED QUANTITIES: AN OVERVIEW

by

J. Christman, U. Coast Guard Academy

1. Introduction

1a. Overview. Certain particle properties are conserved in interactions and decays. The purpose of this unit is to enumerate such properties and to discuss conservation laws in a general way. Details of the properties are discussed elsewhere.[1]

1b. Definition of "Conserved Quantity." . Here is the meaning of "conserved property" in a particle interaction or decay:

> A number representing the property can be assigned to each particle entering the interaction and to each particle produced by the interaction. There exists a scheme for calculating the total amount of the property both for particles entering and for particles leaving the interaction. If the entering total and leaving total are the same, the property is said to be conserved.

1c. Quantities Conserved Depend on Interaction. The list of properties which are conserved is characteristic of the type of interaction. Those properties which are conserved in all 4 types of interaction are called "universally conserved quantities." In addition, there is a property which is conserved in strong interactions but not in the others, and there are properties which are conserved in electromagnetic interactions but not in weak ones.

1d. Correlation With Interaction Strength. It is interesting, but perhaps coincidental, that if a property is conserved by one type of interaction it is conserved by all stronger interactions. That is, the list of conserved properties for the electromagnetic interaction contains all those on the list for the weak interaction plus four others and the list for the strong interaction includes all those for the electromagnetic interaction plus one other. Little experimental information exists for the gravitational interaction.

[1] "Universally Conserved Quantities in Elementary Particle Interactions" (MISN-0-276). Also see Reference 3.

2. Assigned Reading

Sections 4.1 through 4.7 in K.W. Ford's *Classical and Modern Physics*, Vol. 1, John Wiley and Sons, NYC (1972), on reserve for you in the PA Library: Ask for "the readings for Unit 275."

3. Universally Conserved Quantities

3a. List of Properties. These quantities are universally conserved in elementary particle interactions:[2]

a. energy

b. linear momentum

c. angular momentum

d. electric charge

e. baryon number

f. electron-muon-tauon number.

g. lepton number

In special relativity, energy becomes the fourth component of the (now four-dimensional) momentum vector. Conservation of this "four-momentum" implies conservation of both energy and momentum.

3b. Intrinsic vs. Dynamic Properties. The last four properties in Sect. 3a, items d-g, are intrinsic properties of the particle. The first three, items a-c, depend on the dynamical situation. For example, electrons may have different momenta, depending on the extent to which they have been accelerated, but all electrons have the same charge, baryon number, electron number, muon number, tauon number, and lepton number.

[2] Older references may have lists of conserved quantities that are obsolete and hence differ from the up-to-date list given here.

3c. CPT. There is another quantity, called CPT, which is believed to be universally conserved. This quantity is the product of the quantities for charge conjugation invariance (C), parity invariance (P), and time reversal invariance (T). The meaning of C, P, and T are discussed elsewhere.[3]

4. Properties Conserved by Each Interaction

4a. Weak Force: Only Universal Quantities. All of the universal quantities listed in Sect. 3a are conserved by the weak interaction. With the exceptions noted in Sections 3c and 4c, no other known quantities are conserved by the weak interaction.

4b. EM and Strong Forces: Additional Constraints. In addition to the seven universal properties, these properties are also conserved by the electromagnetic interaction:

h. parity

i. strangeness

j. charge conjugation

k. time reversal

In addition to the above listed properties, the strong interaction also conserves a quantity called "isospin."

4c. Time Reversal Indicates Two Weak Forces. Effects which stem from the violation of time reversal invariance by the weak interaction are roughly one-thousandth as strong as the more usual effects of the weak interaction. This has led some physicists to believe that the force which has been traditionally called the weak interaction is, in reality, two forces, the stronger of which is time reversal invariant and the weaker of which is not. This super weak force, if it exists, is a fifth force of nature. Here we do not make a distinction between these two weak forces.

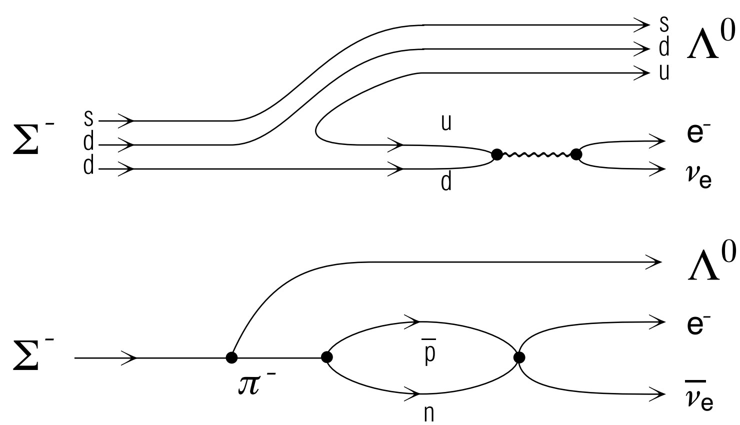
Acknowledgments

Preparation of this module was supported in part by the United States Coast Guard Academy for a Directed Studies Program. Preparation of this module was supported in part by the National Science Foundation, Division of Science Education Development and Research, through Grant #SED 74-20088 to Michigan State University.

[3] "Additional Properties Conserved in Electromagnetic and Strong Interactions" (MISN-0-277).

MISN-0-274

PARTICLES STABLE AGAINST THE STRONG FORCE



Project PHYSNET Physics Bldg. Michigan State University East Lansing, MI

PARTICLES STABLE AGAINST THE STRONG FORCE

by

J. R. Christman

Title: **Particles Stable against the Strong Force**

Author: J.R. Christman, Dept. of Physical Science, U.S

Version: 11/15/2000 Evaluation: Stage B1

Length: 2 hr; 11 pages

Input Skills:

1. Express the mass, spin (angular momentum) and charge of a particle in units of MeV, $\hbar$, and e respectively (MISN-0-273).
2. Give the typical decay times of particles which decay via the strong, weak, and electromagnetic interactions (MISN-0-273).

Output Skills (Knowledge):

K1. Define: hadron, lepton, fermion, boson.

K2. List the names of the elementary particles, and write their symbols and the symbols for their antiparticles.

K3. For each strong-stable particle, state the name of the family and the family group to which it belongs.

K4. List the names of the absolutely stable particles.

K5. List the names of the zero mass particles and give the orders of magnitude of the particle masses in each family.

K6. Give the spin of each particle.

Output Skills (Problem Solving):

S1. Given the typical decay mode for a particle, write the corresponding decay mode for its antiparticle.

External Resources (Required):

1. K.W. Ford, *Classical and Modern Physics*, Vols. 1 and 3, John Wiley and Sons (l972).

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New authors, reviewers and field testers are welcome.

PARTICLES STABLE AGAINST THE STRONG FORCE

by

J. R. Christman

1. Introduction

This module deals with some of the basic "facts" about those particles that are stable under strong interactions (strong interactions cannot cause them to spontaneously decay). Elsewhere we will discuss theories built on those facts.

2. Readings

1. Ford, Vol. 1, Sect. 3.1 - 3.5, and Vol. 3, Sect. 27.2 - 27.4, on reserve for you in the Physics-Astronomy Library. Ask for them as "Ford, Volume 1," and "Ford, Volume 3."

3. Classification

3a. Explanation of the Table. Refer to the table at the end of this module. It lists important properties of all particles which do not decay via the strong interaction. There are, of course, many more particles than these but their lifetimes are extremely short ($\approx 10^{-22}$ sec), and the strong interaction brings about their decay.

In the table there is a quantity called *mass.* In high energy physics the quantity called "mass" is really mass energy: to obtain mass itself, divide by the square of the speed of light ($1\,\text{MeV} = 10^6\,\text{eV}$ and $1\,\text{eV} = 1.60 \times 10^{-19}\,\text{J}$). *Time* indicates the particle's lifetime. The second column in the table shows the symbol used to represent the particle.

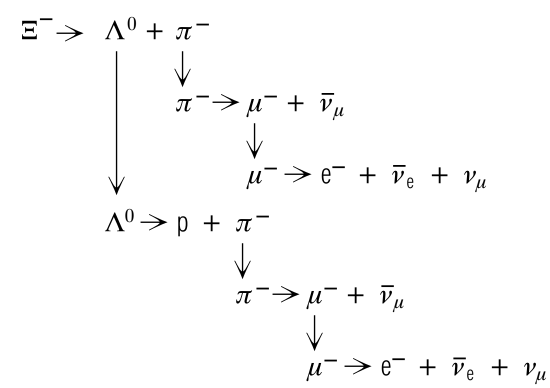
3b. Families of Particles. The particles can be classified according to family: electron, muon, meson, and baryon. The photon does not have any brethren (although some have been postulated). There are also names for groups of families. Members of the electron, muon, and tau families are collectively called *leptons.* Mesons and baryons are collectively called *hadrons.*

3c. Antiparticles. An antiparticle is associated with each particle. The antiparticle is usually denoted by the same symbol as the particle but with a bar over it. For some particles (the photon and some mesons), the antiparticle is indistinguishable from the same or another particle: $\bar{\gamma} = \gamma$, $\bar{\pi}^- = \pi^+$, $\bar{\pi}^+ = \pi^-$, $\bar{\pi}^0 = \pi^0$, and $\bar{\eta} = \eta$. The distinction between particle and antiparticle is to some extent an arbitrary one, but we shall see that in the case of baryons, electrons, and muons, the classification leads to a conservation law. For the mesons, no such conservation law exists and it is immaterial whether π^- is listed as a separate particle or as the antiparticle associated with π^+.

3d. Particle/Antiparticle Properties. A particle and its associated antiparticle have exactly the same mass, spin angular momentum, and lifetime. They have the same magnitude, but opposite sign, for charge, and the typical decay mode of the antiparticle can be found from that of the particle by putting a bar over all un-barred symbols and removing the bar over barred symbols.

3e. Baryons. Baryons are particles whose final decay products include one, or some other odd number, of protons and antiprotons.

For example:



The final products of a Ξ decay are a proton, 2 electrons, and 6 neutrinos. The key to the classification of a Ξ as a baryon is the appearance of a proton among the final products.

3f. Hadrons. Mesons and baryons, collectively called hadrons, participate in the strong interaction. That is, these particles interact with each other via the strong interaction, although the hadrons listed do not *decay* via the strong interaction. No other particles, except the hadrons, participate in the strong interaction and this interaction may be taken as the definition of a hadron. (Mesons are then hadrons which are not baryons; they are hadrons which do not decay to protons or antiprotons.)

3g. Absolutely Stable Particles. The photon, electron, electron's neutrino, muon's neutrino, the tau's neutrino, and the proton are absolutely stable. They do not decay under any interaction, as far as is known. Lower limits on the lifetimes of these particles are suspected to exceed the age of the universe.

4. Particle Decay

4a. Lifetimes and Interactions. Most of the unstable particles on the list decay on the order of 10^{-10} to 10^{-8} sec after they are produced. These are characteristic times for decay via the weak interaction. Muon decay is also via the weak interaction but it is slowed by a factor which has to do with the muon's mass. The π^0 and ηparticles, which decay via the electromagnetic interaction, have much shorter lifetimes. (At fist glance, the lack of charge would seem to preclude an electromagnetic decay for these particles. We shall see later that the electromagnetic decay is preceded by a strong decay to positive and negative charged particles. These charged particles then decay to the final products we see.)

4b. Lifetime to Interaction Strength. The strength of an interaction can be inferred from typical decay times. For strong decays, the particle lifetime is on the order of 10^{-23} sec, for electromagnetic decays the particle lifetime is on the order of 10^{-21} sec, and for weak decays the particle lifetime is on the order of 10^{-10} sec. The ratio of strengths is roughly

$$\text{strong: em: weak } = 1 : \ 10^{-2} : \ 10^{-13} \,.$$

For particular decays, other factors (such as mass) may alter the lifetime from the typical value. Stronger interactions can mask weaker interactions. Foe example, if a particle can decay via more than one route it will be observed to go via the faster route more often than via the slower route. In fact, the slower route may be too slow to be seen.

4c. Spin and Fermions/Bosons. Each particle has associated with it a specific amount of intrinsic angular momentum, called *spin angular momentum* or, usually, just *spin*.[1] *Spin* is always either an integer or a half integer times $\hbar$. Particles with integer spin (including zero) are said to obey Bose-Einstein statistics and are called *bosons*. Particles with half odd integer spin are said to obey Fermi-Dirac statistics and are called *fermions*. The significance of this classification will be discussed later.

Acknowledgments

Preparation of this module was supported in part by The United States Coast Guard Academy for a Directed Studies Program. Preparation of this module was supported in part by the National Science Foundation, Division of Science Education Development and Research, through Grant #SED 74-20088 to Michigan State University.

[1] For a discussion of spin, see "Spin" (MISN-0-244). For quantized angular momenta in general and a discussion of $\hbar$, see "Quantized Angular Momentum" (MISN-0-251).

A. Table of Particle Properties

These are particles stable against decay via the strong interaction.[2]

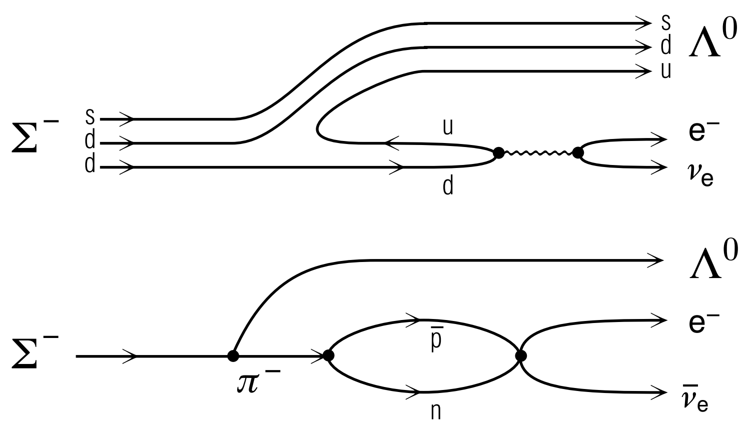
Name		Mass (MeV)	Spin ($\hbar$)	Charge ($\lvert Q_e \rvert$)	Time (sec)	Dominant Decay
Photons						
Photon	γ	0	1	0	∞	
e Family						
Electron	e	.511	1/2	-1	∞	
e's neutrino	ν_e	0	1/2	0	∞	
μ Family						
Muon	μ	105.66	1/2	-1	10^{-6}	$\mu^- \to e^- + \bar{\nu}_e + \nu_\mu$
μ's neutrino	ν_μ	0	1/2	0	∞	
τ Family						
Tau	τ	1784	1/2	-1	10^{-13}	$\tau^- \to e^- + \bar{\nu}_e + \nu_\tau$ (not dominant)
τ's neutrino	ν_τ	0	1/2	0	∞	
Mesons						
Pion	π^+	139.57	0	1	10^{-8}	$\pi^+ \to \mu^+ + \nu_\mu$
	π^-	139.57	0	-1	10^{-8}	$\pi^- \to \mu^- + \bar{\nu}_\mu$
	π^0	134.97	0	0	10^{-16}	$\pi^0 \to \gamma + \gamma$
Kaon	K^+	493.6	0	1	10^{-8}	$K^+ \to \pi^+ + \pi^0$
	K^-	493.6	0	-1	10^{-8}	$K^- \to \pi^- + \pi^0$
	K^0	497.7	0	0	10^{-10}	$K^0 \to \pi^+ + \pi^-$
Eta	η	549	0	0	10^{-19}	$\eta \to \gamma + \gamma$
Baryons						
Nucleon	p	938.27	1/2	1	∞	
	n	939.57	1/2	0	10^3	$n \to p + e^- + \bar{\nu}_e$
Lambda	Λ^0	1115.6	1/2	0	10^{-10}	$\Lambda^0 \to p + \pi^-$
Sigma	Σ^+	1189.4	1/2	1	10^{-10}	$\Sigma^+ \to n + \pi^+$
	Σ^-	1197.4	1/2	-1	10^{-10}	$\Sigma^- \to n + \pi^-$
	Σ^0	1192.6	1/2	0	10^{-20}	$\Sigma^0 \to \Lambda^0 + \gamma$
Xi	Ξ^-	1321.3	1/2	-1	10^{-10}	$\Xi^- \to \Lambda^0 + \pi^-$
	Ξ^0	1314.9	1/2	0	10^{-10}	$\Xi^0 \to \Lambda^0 + \pi^0$
Omega	Ω^-	1672.4	3/2	-1	10^{-10}	$\Omega^- \to \Xi^0 + \pi^-$

[2]Lifetimes have been rounded to the nearest power of ten. All values are from "Review of Particle Properties," R. Gatto et al., *Physics Letters B*, **204**, 1988.

PHYSNET

MISN-0-273

PARTICLES AND INTERACTIONS



Project PHYSNET Physics Bldg. Michigan State University East Lansing, MI

PARTICLES AND INTERACTIONS

by

J. Christman

Title: **Particles and Interactions**

Author: J.R. Christman, Dept. of Physical Science, U.S. Coast Guard Academy, New London, CT

Version: 2/1/2000 Evaluation: Stage B1

Length: 2 hr; 11 pages

Input Skills:

1. State the definition of an electron volt (eV) (MISN-0-117).
2. Use the relativistic expression relating energy, momentum, and mass, $E = (p^2c^2 + m^2c^4)^{1/2}$ (MISN-0-24).

Output Skills (Knowledge):

K1. Express energy, mass, or momentum in terms of MeV.

K2. Express angular momentum in units of $\hbar$.

K3. Express charge in units of e, the charge on a proton.

K4. List the four types of forces at work in nature, along with their ranges, and give an example of each force.

K5. Give the relative strengths of the four forces and the typical decay times corresponding to each.

K6. List the class or classes of particles acted on by each of the four forces in nature.

External Resources (Required):

1. K.W. Ford, *Classical and Modern Physics*, Vols. 1 and 3, John Wiley and Sons (1972).

Post-Options:

1. "Particle Properties" (MISN-0-275).

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PARTICLES AND INTERACTIONS

by

J. Christman

1. Reading

1. Sect. 3.5, 3.7, 27.1 in K. W. Ford's *Classical and Modern Physics*, Volumes 1 and 3, John Wiley and Sons, NYC (1972).

2. Forming a Theory

2a. The Goals of Particle Physics. In this unit we present the interactions studied by Particle ("High Energy") physicists. The goal of high energy ("Particle") physics is to understand why particles have the properties they have and why they interact as they do, in terms of a comprehensive theory. Such a comprehensive theory does not exist today but various leads are being followed and some of the characteristics of particles and interactions can be understood in terms of fundamental ideas.

2b. Large Numbers of Particles: Probably None Fundamental. There are now several hundred different types of particles that have been observed. Most physicists believe that few, if any, of these are truly fundamental and that there must be even more fundamental building blocks out of which the known particles are constructed. Quarks are candidates for fundamental building blocks but they have probably not been observed directly. The quark model and the classification of particles that led to it are discussed elsewhere.[1]

3. Interaction of Particles

3a. Identity, Transmutation, and Decay. Particles are identified by their properties, such as mass, charge, intrinsic angular momentum, and others. The overwhelming characteristic of the particle world is that the identities of particles change. To be sure, two particles can interact and just change their directions of motion - they scatter each other. However, if conditions are right, the particles that leave the interaction are not the same particles that entered.

[1] For example, in "SU(3) and the Quark Model" (MISN-0-282).

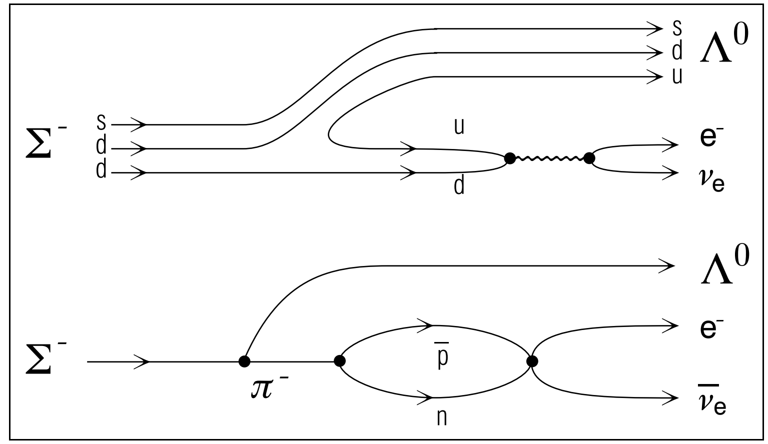


Figure 1. Two particles interact and become three particles.

For the interaction shown in Fig. 1, particles A and B disappear and particles C, D, and F appear. In addition, a single particle may decay into other particles. In fact only a small number of particles are stable against decay.

3b. Relevance of Conserved Quantities. Even though the identities of the particles may change, some quantities remain constant. These are the conserved quantities. Most of our knowledge of particles and their interactions centers around these.

3c. The Four Interactions: Properties, Manifestations. The dynamics of the particle interactions are not well understood at this time. It appears that there are four types of forces at work in nature, although some physicists are trying to show that at least some of them are really different aspects of a single force. The four are named gravitational, weak, electromagnetic and strong.

Although important in the macroscopic world, the gravitational force is so weak that it can be neglected in almost all aspects of particle physics. The other three forces are important.

The various forces are characterized and identified by their ranges, strengths, and the quantities that are conserved when they operate. A summary of some of the properties is given in the table in Appendix A.

The range of a force is an indication of how close the participating particles must come to each other before they influence each other via the force. Relative strengths of forces give some indication of the relative probability that the particles interact when they are within range. Other factors, such as energy and spin, enter the calculation of this probability: our calculation of their influences will be rough and in particular instances could be wrong by an order of magnitude.

The forces not only cause interactions between two particles, but they are also responsible for the decay of a single particle. A particle has a lifetime, the time between creation and decay, that is typical of the species of particle. A species' lifetime is indicative of the type of force that causes the decay. In fact, a measurement of the lifetime is the principal experimental method of determining the force causing a particular decay. Note in the Appendix A table that typical lifetimes are inversely proportional to relative strengths.

In the Appendix A table, under the heading "Particles Acted Upon," are listed those classes of particles that decay or interact with each other via the named force. We shall see later that perhaps not all particles interact via the weak force and that a strong interaction, too fast to be observed, sometimes precedes the weak interaction.

Some nuclei transmute into other nuclei by emitting an electron or positron. This happens when a proton or neutron within a nucleus decays:

$$\mathrm{n} \rightarrow \mathrm{p} + \mathrm{e}^- + \bar{\nu}_\mathrm{e} \quad \text{or} \quad \mathrm{p} \rightarrow \mathrm{n} + \mathrm{e}^+ + \nu_\mathrm{e}$$

where ν_e and $\bar{\nu}_\mathrm{e}$ are neutrinos. Isolated protons can never decay but those in nuclei sometimes can.

3d. Signatures of Specific Interactions. Neutrinos participate in the weak interaction only, so if a neutrino appears as a product particle at least part of the interaction must be weak. There are other weak interactions in which neutrinos do not appear.

A photon in a reaction signals you that an electromagnetic interaction was present. That is, if a photon appears, at least part of the interaction must have been electromagnetic.

Hadrons (mesons and baryons) interact via the strong interaction unless they are prevented from doing so by conservation laws.

4. Problems

Do these problems: Ford, Chap. 3 – E3.5, E3.6, E3.7.

Acknowledgments

Preparation of this module was supported by the United States Coast Guard Academy for a Directed Studies Program. Preparation of this module was supported in part by the National Science Foundation, Division of Science Education Development and Research, through Grant #SED 74-20088 to Michigan State University.

A. Table: Forces of Nature

Range:	Relative Strength:	Typical Decay Time:	Particles Acted Upon:	Examples:
STRONG:				
$\approx 10^{-15}$ m	1	10^{-23} sec.	Hadrons (mesons and baryons)	Binds nucleons in nuclei (nuclear forces)
ELECTROMAGNETIC:				
∞	10^{-2}	10^{-21} sec.	Charged particles	Binds electrons in atoms (atomic forces)
WEAK:				
$<< 10^{-16}$ m	10^{-13}	10^{-10} sec.	Hadrons and leptons	Radio-activity
GRAVITATIONAL:				
∞	10^{-40}	10^{17} sec. ?	All	Attraction of macro-scopic bodies for each other

B. Notes About Units

i. MeV is a common energy unit in particle physics. One electron volt, abbreviated 1 eV, is the kinetic energy obtained by an electron when it moves through a 1 volt potential difference:

$$1\,\text{eV} = 1.602 \times 10^{-19}\,\text{joules.}$$

A common energy unit is the MeV (one million electron volts): $1\,\text{MeV} = 10^6\,\text{eV}$. Another common energy unit is the GeV: $1\,\text{GeV} = 10^3\,\text{MeV} = 10^9\,\text{eV}$.

ii. Since the product mc^2 (where $m =$ mass and $c =$ speed of light) has units of energy, mass is sometimes given in MeV/c^2 or sometimes just MeV (it is common to pick units in which $c = 1$). Given mass in MeV, to get mass in kilograms, multiply by 1.602×10^{-13} and divide by c^2.

iii. The product pc (where $p =$ momentum) has units of energy so momentum is sometimes given in GeV/c or MeV/c or just MeV (using the convention $c = 1$).

iv. Angular momentum is often given in units of $\hbar$ (1.0545×10^{-34} joule seconds).

v. Charge is given in units of the charge on the proton, e (1.602×10^{-19} coulomb).

C. Problem Answers

E3.5 (1) $t < d/v$, and $d/v = 10^{-14}$ s, so answer is η, Σ^0.

(2) $t > d/v$, and $d/v = 12.74 \times 10^{-2}$ s, so answer is n.

E3.6 (1) $\text{p} + \text{e}^- + \bar{\nu}_\text{e} + \nu_\mu + \bar{\nu}_\mu + \gamma + \gamma$.

(2) progressively more negative.

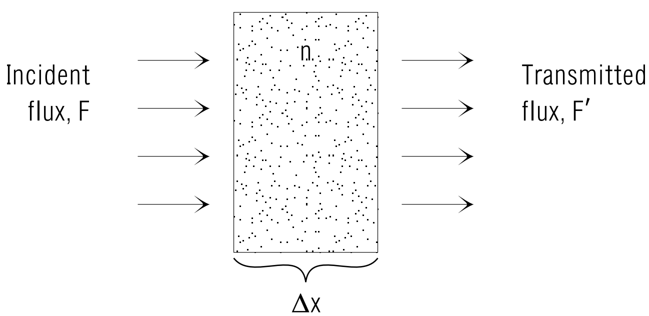
E3.7 (1) [a] positive bends counterclockwise, negative clockwise;

[b] slower is more intense.

(2) negative; more intense is slower, therefore is later in time.

MISN-0-266

FUNDAMENTAL FORCES: RANGES, INTERACTION TIMES, CROSS SECTIONS



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FUNDAMENTAL FORCES: RANGES, INTERACTION TIMES, CROSS SECTIONS

by

J. S. Kovacs and William C. Lane

Title: **Fundamental Forces: Ranges, Interaction Times, Cross Sections**

Author: J. S. Kovacs and William C. Lane, Michigan State University

Version: 1/22/2001 Evaluation: Stage 0

Length: 1 hr; 16 pages

Input Skills:

1. Vocabulary: electromagnetic interaction, elementary particle, fundamental force, hadron, meson, muon, neutrino, nucleons, strong interaction, weak interaction (MISN-0-255).
2. Integrate functions such as x and $1/x$ (MISN-0-1).

Output Skills (Knowledge):

K1. Vocabulary: cross section, flux, total cross section, intermediate vector boson.

K2. State the characteristic times associated with the strong, weak and electromagnetic interactions among elementary particles.

K3. State the characteristic ranges associated with the strong and weak interactions.

Output Skills (Problem Solving):

S1. Determine the fractional number of strong (or weak) interactions that can occur in an interaction in which the target particle density is given.

S2. Given the target particle density and the fractional loss of projectile beam flux through the targets, calculate the total cross section for all reactions.

Post-Options:

1. "Conservation Laws for Elementary Particle Reactions" (MISN-0-256).

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FUNDAMENTAL FORCES: RANGES, INTERACTION TIMES, CROSS SECTIONS

by

J. S. Kovacs and William C. Lane

1. Introduction

There are only three fundamental forces responsible for all interactions among elementary particles.[1] Each of these interactions (electromagnetic, weak, and strong) has its own characteristic range and characteristic interaction time which determine the likelihood that a given elementary particle reaction will occur. These characteristic properties vary over such a wide range among the three interactions that certain elementary particle reactions are overwhelmingly more likely to proceed via one interaction than the others. The probability that a certain reaction will occur via a given fundamental interaction may be expressed as a quantity called the "cross section."

2. Characteristic Force Ranges

2a. The Range of the Strong Interaction. The strong interaction is a "short-range" force, responsible for the mutual attraction of nucleons within the nucleus. Because the nucleus is on the order of 10^{-15} m in diameter, we infer that this is the approximate range of the strong interaction. The reason the strong interaction has such a short range is related to the fact that the quanta for this force field, the mesons, are particles with mass, unlike the electromagnetic field quanta (photons). It may be shown that the range of an interaction "mediated" by a massive particle is given approximately by:

$$\rho \simeq \frac{\hbar}{mc}, \qquad (1)$$

where ρ is the characteristic range and m is the mass of the force field's quanta. If we use the mass of the least massive meson, the pion, then $m = 140\,\mathrm{MeV}/c^2$, and ρ may be calculated to be about 1.4×10^{-15} m.

[1] The gravitational interaction is so weak that its presence may be virtually ignored at the elementary particle level of interactions.

2b. The Range of the Weak Interaction. Newly acquired experimental data have established that the range of the weak interaction is roughly 10^{-18} m. This number is the result of the observation of the intermediary of the weak interaction, the "intermediate vector boson."[2] This weak interaction quantum has a mass of $84\,\mathrm{GeV}/c^2$, which, using Eq. (1), produces a value for ρ of 2.3×10^{-18} m. This range is exceedingly small compared to the size of a nucleus, and is taken to be zero in most weak interaction calculations.

2c. The Range of the Electromagnetic Interaction. The range of the electromagnetic interaction is infinite; that is, it has no characteristic range. All charged particles may interact electromagnetically at any separation, although the interaction depends inversely on the square of the separation.

3. Characteristic Interaction Times

3a. Characteristic Time for Strong Interactions. The characteristic time for the strong interaction is approximately 10^{-23} seconds. This can be seen by imagining the following "thought experiment." Suppose two hadrons approach one another, moving at speeds comparable to the speed of light, i.e. $\approx 10^8$ m/s. If the particles approach within 10^{-15} m of each other, they will interact via the strong interaction. The amount of time that either particle spends near the other is thus approximately the time it takes for one particle to traverse a distance of 10^{-15} m, i.e. 10^{-23} s. From that little thought experiment you are led to conclude that whenever hadrons spend at least 10^{-23} seconds within the range of their mutual strong interactions they will interact. If some unstable hadron decays into two or more particles via the strong interaction, e.g.

$$X \Longrightarrow Y + Z$$

the mean lifetime of X is $\approx 10^{-23}$ seconds. Thus when such a particle X is produced through some other interaction it doesn't last very long.

3b. Characteristic Time for Weak Interactions. The characteristic times for weak interactions are from 10^{-6} to 10^{-10} seconds. These times are observed when unstable particles in nature decay via the weak interaction. For example, the π^+ meson (a pion) is unstable and it usually

[2] For a more detailed account of the discovery of the vector boson see the "Science and Citizen" column of *Scientific American*, April 1983.

decays to an antimuon and a neutrino:

$$\pi^+ \Longrightarrow \mu^+ + \nu_\mu .$$

Occasionally it decays to a positron and a neutrino:

$$\pi^+ \Longrightarrow e^+ + \nu_e .$$

If the interaction that took the initial pion to the final group of particles was the strong interaction, the pion lifetime would be very short, about 10^{-23} seconds. In that case a pion, even if it moved at the speed of light, would not travel more that 10^{-15} meters before it decayed. In actuality, charged pions travel path lengths of several tens of centimeters before decaying, even when moving at low speeds.[3] The mean lifetime of the π^+, as determined by observing a great many particle decays, is 2.6×10^{-8} seconds hence it decays via the weak interaction. Although weak interactions can go no faster than 10^{-10} seconds, they can go slower than the 10^{-6} seconds we quoted above. Other factors may cause a weak interaction process to go more slowly. For example, the nucleus ^{210}Pb is radioactive, and decays via the weak interaction with a half-life of about 22 years.

3c. Characteristic Time of Electromagnetic Interactions. The electromagnetic interaction has a characteristic time that falls somewhere between the strong and weak interaction times. Because the range of the electromagnetic interaction is not a sharply defined quantity, we cannot use the same simple model for calculating the interaction time that we used for the strong interaction. About all that we can say is that the electromagnetic interaction time is greater than 10^{-21} seconds. We obtain this value by noting that the electromagnetic interaction is intrinsically about a hundred times weaker than the strong interaction. As an illustration of an electromagnetic interaction, the neutral pion, π^0, with a rest energy of 135 MeV, decays into two photons of total energy 135 MeV:

$$\pi^0 \Longrightarrow \gamma + \gamma .$$

The mean lifetime for this decay is 0.84×10^{-16} seconds. Comparing this lifetime to the lifetime of the weakly decaying π^+ (2.6×10^{-8} seconds) illustrates the relative strengths of the electromagnetic and weak interactions. The characteristic time for electromagnetic interactions also depends on

[3] Charged particles can be observed by the tracks of bubbles they leave in a bubble chamber or by the trail of moisture droplets they leave in a cloud chamber.

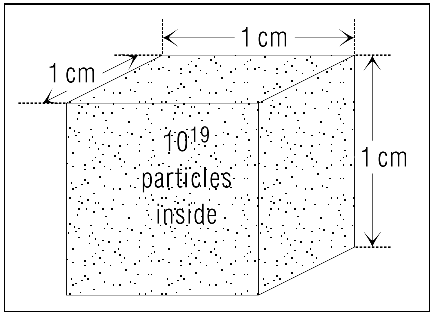


Figure 1. A simple model for an elementary particle collision target.

the size of the system involved, (e.g. an elementary particle, a nucleus, an atom, etc.). The lifetimes of elementary particles that decay via the electromagnetic interaction are typically between 10^{-16} and 10^{-21} seconds. Interactions such as $A^* \Longrightarrow A + \gamma$, where A^* is an excited state of a nucleus or an atom and A is the nuclear or atomic ground state, also proceed via the electromagnetic interaction. The characteristic time for these processes is 10^{-16} seconds or longer for nuclear decays and 10^{-8} seconds or longer for atomic transitions.

4. Reaction Cross Sections

4a. Introduction. Aside from giving you an order of magnitude estimate of the relative strengths of the basic interactions, the characteristic time associated with a reaction is scientifically useful when you are dealing with decay reactions. However, for a reaction that is brought about by firing projectile particles at target particles a characteristic time is not readily measurable, and when you have it, it's not particularly informative. To determine whether a projectile particle and a target particle will interact, we must first calculate the collision probability as a function of target particle size and density. As a convenient model, consider the cubical system shown in Fig. 1, containing 10^{19} target particles (for the time being, consider them as small spheres) randomly distributed throughout the cubical volume. Assume, for ease of computation, that when you "look into" this cubical region, the distribution of the particles is such that no particle in the box is obscured by another particle (no one particle is even partially behind another).[4] Suppose now that a projectile

[4] Of course, that assumption can't be made if the particles are "large enough" in extent, or if their number is large enough so that they are densely packed. For a dilute gas of small particles this assumption is reasonable.

particle is fired into this box in a direction normal to one of the cube faces. The probability that the projectile particle will hit a target particle is simply the ratio of the total effective cross-sectional area presented by the targets to the total area of the cube face.

4b. Definition of Cross Sections. The total effective target area associated with a given reaction is called the "cross section" for that reaction. The cross section for all possible reactions within the target can, in effect, be determined by direct measurement. The cross section so determined is the "total cross section," related to the fraction of incident projectile particles that interact in any way with the target particles. This microscopic total cross section is completely analogous to your usual conceptions about the cross-sectional area presented by a target to a projectile. Less familiar is the cross section for an interaction to result in a specific outcome (such as, for example, the outcome where the incident and target particle disappear and two other specified particles result from the interaction). The total cross section is a measure of the probability of all possible reaction outcomes.

4c. Flux of Particles. The model developed in Sect. 4a is modified for real experiments, where the beams are generally smaller than the target face and "flux" is measured rather than numbers of particles. In our simple model, where the incident particles were fired randomly at the entire face of the container of target particles, the probability for interacting with the target particles was the ratio of the area presented by the target particles to the area A of the face of the container. In actual practice the incident particles are generally localized in a beam smaller in cross section than the target area, so the probability of striking a point on the target is not the same for every part of the target. Instead of the total number of particles incident on the target, you need to deal with the number of particles per unit area, thus taking account of the concentration of the beam. Also, the incident beam is generally a steady current of particles so you really measure currents of particles (particles per unit time) which emerge from the target. These two modifications of the counting of the projectile particles define a quantity called the "flux." The flux of projectile particles is the number of projectile particles per second per unit cross-sectional area of the beam. The flux of particles that emerge unscathed from the other side of the target should then tell you about the total cross section.

4d. Relation Between Flux and Cross Section. The fractional change in flux due to projectile particles that interact in any way with

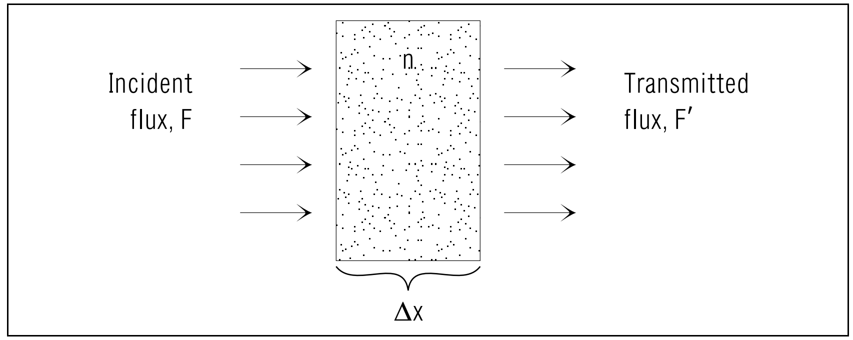


Figure 2. Attenuation of particle flux by a given thickness, Δx, of absorbing material in a target that has particle density n.

the target particles is directly related to the total cross section. Figure 2 shows a slab of target material, of cross-sectional area A and thickness Δx, containing N target particles.

A flux of projectile particles, F, is incident on the left side of the slab. Emerging unscathed from the right side of the slab is a smaller flux, F', of projectile particles. The fractional number of particles that do interact is equal (for large numbers) to the probability that a projectile particle collides with a target particle, and is given by

$$\frac{F - F'}{F} = \frac{N\sigma}{A} \tag{2}$$

where σ is the total cross section for all possible reactions. We may express N/A, the number of target particles per unit area as

$$\frac{N}{A} = \frac{N\Delta x}{A\Delta x} = \frac{N\Delta x}{V} = n\Delta x \tag{3}$$

where we have used the fact that $A\Delta x$ is the volume V of the target, and have defined n as the target particle density, N/V. The change in flux, ΔF, is equal to $F' - F$, so we may express Eq. (2) as

$$\frac{\Delta F}{F} = -n\sigma\Delta x. \tag{4}$$

In the limit that Δx is infinitesimally small, ΔF goes to dF, the infinitesimal change in the flux of the beam when passing through an infinitesimal thickness dx of target material. Integrating this expression as an indefinite integral, we obtain

$$\int \frac{dF}{F} = -\int n\sigma\, dx \Longrightarrow \ell nF + C = -n\sigma x, \qquad (5)$$

where C is a constant of integration obtained by applying the boundary conditions. When x is zero, F is equal to the initial flux, which we will call F_0. This condition, applied to Eq. (5), establishes C as $-\ell nF_0$, so we may rearrange Eq. (5) and get

$$F = F_0 e^{-n\sigma x}, \qquad (6)$$

the expression for the flux that gets through a thickness x of target material unscathed. Thus if we are given the incident and transmitted particle fluxes, and the target thickness and density, we can calculate the cross section for the scattering process.

Acknowledgments

We would like to thank Professors Wayne Repko and Dan Stump for helpful discussions on some of the topics in this module. Preparation of this module was supported in part by the National Science Foundation, Division of Science Education Development and Research, through Grant #SED 74-20088 to Michigan State University.

Glossary

- **cross section**: a measure of the probability of a given reaction between target particles and projectile particles, in terms of the total effective cross-sectional area of the target particles.
- **flux**: the number of projectile particles per unit time per unit cross-sectional area of a particle beam.
- **intermediate vector boson**: the particle that mediates the weak interaction.
- **total cross section**: a measure of the probability of all possible reactions between target particles and projectile particles, in terms of the total effective cross-sectional area of the target particles.

PROBLEM SUPPLEMENT

1. Consider the target chamber described in Sect. 4a of the module text.
   a. If each of the particles in the box has a "radius" of 10^{-15} m, what fraction of the cross-sectional area of the box do these particles cover?
   b. Suppose you fire point particles into the box. What is the probability that a projectile particle will come to within 10^{-15} m of the center of any one of the target particles?
   c. What is the probability that a projectile will come to within 10^{-2} m of one of the target particles?
   d. If all of the projectiles and targets are hadrons, what is the probability that a projectile comes within range of the strong interaction with one of the targets?
   e. Suppose a burst of 10^8 projectile particles is fired into the box. How many of these can be expected to interact with the targets?
   f. Suppose that in a sequence of five such bursts, the number of interactions observed in each "experiment" was: 35, 29, 32, 32, and 40. What conclusion can you draw from this "experimental" result?
2. Suppose that the projectile particles and the target particles described in Problem 1 interact only via the weak interaction. Since the weak interaction is less likely by a factor of about 10^{-13}, how many of these $1\,\text{cm}^3$ boxes should you place end-to-end, sending the beam through their length, for you to observe one weak interaction out of the 10^8 projectile particles?
3. Suppose we have a target chamber filled with iron nuclei of particle density $8.4\times10^{28}\,\text{m}^{-3}$. If the thickness of iron nuclei needed to decrease an incident flux of neutrons by a factor of one half is 5 cm, what is the total cross section for all iron nuclei-neutron reactions?

Brief Answers:

1. a. $\pi \times 10^{-7}$

   b. $\pi \times 10^{-7}$

   c. 1 (certainty)

   d. $\pi \times 10^{-7}$

   e. $10\,\pi$

   f. We can conclude that only the average number of interactions may be measured, and that this average will have some statistical uncertainty.

2. $\approx 10^{12}$

3. $1.65 \times 10^{-28}\,\text{m}^2$

MODEL EXAM

1. See Output Skills K1-K3 on this module's *ID Sheet*.

2. A target chamber contains 10^{23} hadronic particles. The dimensions of the face of the chamber are 5 cm by 8 cm. A beam of projectile particles is directed at the face of the chamber into the region containing the target particles.

   a. If the particles of the beam are hadrons, approximately how many of 10^{11} beam particles can you expect to undergo strong interactions with particles of the target? An order of magnitude answer is all that is asked here.

   b. If the projectile particles interact with the target particles only via the weak interaction, how many beam particles need to be sent into the chamber for there to occur approximately one interaction between a beam particle and target particle?

   c. If the density of target particles in a chamber is $2.5 \times 10^{26}\,\text{m}^{-3}$, and if the thickness of target chamber needed to reduce the incident intensity to 1.0 % of the original intensity is 8 cm, what is the total cross section for interaction of the beam particles with those of the target?

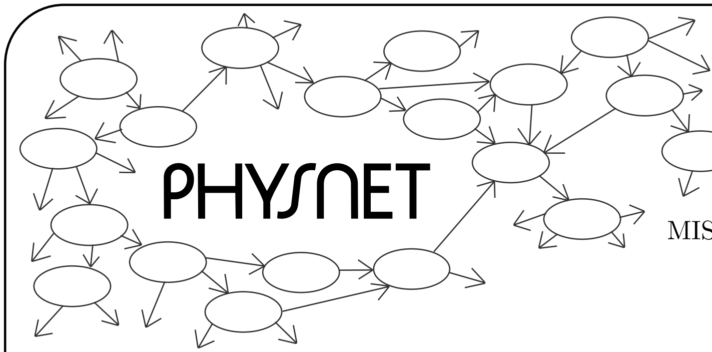
Brief Answers:

1. See this module's *text*.

2. a. 10^7

   b. 10^{17}

   c. $2.3 \times 10^{-25}\,\text{m}^2$



MISN-0-256

CONSERVATION LAWS FOR ELEMENTARY PARTICLE REACTIONS

$\pi^0 + p \rightarrow \Lambda^0 + K^+$

$\Lambda^0 \rightarrow p + \pi^-$

$\pi^- + p \rightarrow \Lambda^0 + K^0$

$n + p \rightarrow n + p + \gamma$

$K^+ \rightarrow \pi^+ + \pi^0$

$K^- + \Xi^0 \rightarrow \Omega^- + \pi^0$

$K^- + p \rightarrow \Lambda^0 + \pi^- + \pi^+$

$\Sigma^+ + p \rightarrow \Lambda^0 + p + \pi^0$

$n \rightarrow p + e^- + \nu_e$

$\pi^- + p \rightarrow n + \pi^0$

Project PHYSNET Physics Bldg. Michigan State University East Lansing, MI

CONSERVATION LAWS FOR
ELEMENTARY PARTICLE REACTIONS

by

J. S. Kovacs and William C. Lane
Michigan State University

Title: **Conservation Laws for Elementary Particle Reactions**

Author: J. S. Kovacs and William C. Lane, Michigan State University

Version: 11/14/2001 Evaluation: Stage 0

Length: 1 hr; 24 pages

Input Skills:

1. Determine whether a given reaction goes via the strong, electromagnetic, or weak interaction from the nature of the particles involved in the reaction (MISN-0-255).
2. Given a two component system with angular momentum quantum numbers j_1 and j_2, determine the possible values of the resultant total angular momentum and of the projection of the angular momentum along any axis (MISN-0-251).

Output Skills (Problem Solving):

S1. Determine whether a proposed elementary particle reaction is allowed or forbidden by each of the absolute conservation laws (conservation of energy, momentum, charge, baryon number, electron number,and muon number).

S2. Determine whether a proposed elementary particle reaction is allowed or forbidden by conservation of isotopic spin or strangeness.

S3. Given a proposed elementary particle reaction, determine which interactions allow it, which forbid it.

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CONSERVATION LAWS FOR ELEMENTARY PARTICLE REACTIONS

by

J. S. Kovacs and William C. Lane
Michigan State University

1. Introduction

Any particle reaction equation that you might write down won't necessarily represent a reaction that can occur in nature. Energy, momentum, and electrical charge, for example, must be conserved. These are conservation laws whose importance was discovered on the macroscopic scale, and whose absolute validity extends to the microscopic scale, the elementary particle interaction level. For example, consider the hypothetical reaction involving only baryons on both sides of the reaction equations

$$\mathrm{p} \Rightarrow \mathrm{n}\,,$$

or

$$\mathrm{n} \Rightarrow \mathrm{p}\,.$$

These reactions violate conservation of charge, and conservation of energy and momentum. These reactions are thus forbidden and won't go via the strong interaction, the weak interaction, the electromagnetic interaction, or any combination of these. There are other hypothetical reactions, which don't violate these familiar conservation laws which don't seem to occur, which appear to be absolutely forbidden. For example:

$$\mathrm{n} \Rightarrow \mathrm{e}^+ + \mathrm{e}^-$$

or

$$\mathrm{p} + \mathrm{e}^- \Rightarrow \gamma + \gamma$$

or

$$\mathrm{p} \Rightarrow \pi^+ + \pi^0\,.$$

These reactions conserve charge. Considering the last of these, if the π^+ and π^0 that resulted from the decay of the proton went off in opposite directions with equal speeds, then momentum would be conserved. Conserving energy as well would require each pion to have total energy equal to one half the proton's rest energy. Furthermore, all particles involved are hadrons so the reaction should go very quickly with a lifetime of

$\approx 10^{-23}$ sec. However, our existence testifies to the fact that this reaction does not occur at the expected rate; protons do not decay spontaneously! In fact, all attempts to seek any evidence of the instability of the proton result in the currently accepted conclusion that the proton is absolutely stable (its mean lifetime is infinite).[1] This suggests that there are other conservation laws operative on this fundamental interaction level which are not readily discernible on the macroscopic level. Such laws could account for the observed forbiddenness of certain reactions such as the decay of the proton. Such conservation laws are the topic of this module.

2. Universally Conserved Quantities

2a. Conservation of Momentum. If we consider the particles in a reaction and the forces between them as a closed, isolated system, then the system's total momentum must be conserved. Rigorous calculations to check conservation of momentum for most particle reactions require the use of relativistic dynamics and so are beyond the scope of this module. However we may make certain qualitative observations about the momenta of the products of a decay reaction. If the reacting particle decays into two products, then the two resulting ("product") particles must move in opposite directions with momenta of equal magnitude, when observed in the rest frame of the decaying particle. If the initial particle decays into three products, then the momenta of the three product particles must add to zero and hence must be coplanar, when observed in the rest frame of the decaying particle.

2b. Conservation of Energy. With the common assumption that the interacting system is closed and isolated, total energy is conserved in a particle reaction. Relativistic calculations are required to verify that the energy actually is conserved in any given reaction. However we may make these observations:

1. For two or more colliding particles, the reaction is energetically possible if sufficient kinetic energy is supplied to the reaction.

[1] However, current theories which propose to combine strong, weak and electromagnetic interactions into a unified fundamental theory, have as a consequence the possible very slow decay of the proton - with a characteristic lifetime of $\approx 10^{31}$ years (compare with the $\approx 4 \times 10^9$ years for the age of the universe!). Extensive research is now being conducted whose goal is detection of the decay of a proton.

2. For particle decays, conservation of energy requires that the mass of the decay products be less than or at most no greater than the mass of the decaying particle. This can be seen by considering the decaying particle in its rest frame. Before the decay its total energy is merely its rest energy, mc^2. After the decay, the product particles typically have some kinetic energy. To balance energy on both sides of the reaction equation, the total rest energy of the products must be less than the rest energy of the decaying particle. This may be stated mathematically as:

$$Mc^2 = \sum_{\text{products}} \left(mc^2 + E_k \right) . \tag{1}$$

Since $E_k \geq 0$, the total mass of the products must be less than the mass M of the initial particle.

2c. Conservation of Angular Momentum. In the absence of any external torques, the total angular momentum of an isolated system of interacting particles must be conserved. To check for conservation of angular momentum, you need to know the spins of the particles involved and the rules for adding quantized angular momenta.[2] As a quick check, use the following facts obtained from the rules for adding quantized angular momenta:

1. The total angular momentum of two particles of integer spin ($S = 0, 1, 2, \ldots$) is an integral multiple of $\hbar$.
2. The total angular momentum of two particles of half-odd-integer spin ($S = 1/2, 3/2, 5/2, \ldots$) is also an integral multiple of $\hbar$.
3. The total angular momentum of two particles, one of integer spin and the other of half-integer spin, is a half-odd-integer multiple of $\hbar$.

As an example, consider the reaction

$$\mathrm{p} + \pi^- \Rightarrow \mathrm{e}^+ + \mathrm{e}^- .$$

The spin of the proton is 1/2 and the spin of the π^- is zero, so the total angular momentum of the reacting particles is a half-odd-integer multiple of $\hbar$. However the electron and positron each have spin 1/2, so the resulting particles have a total angular momentum that is an integer multiple of $\hbar$. Angular momentum cannot be conserved, so this reaction will never take place.

[2] See "Quantized Angular Momenta" (MISN-0-251).

2d. Conservation of Charge. All elementary particle reactions must conserve charge. Unless you are considering the sub-hadronic world, all charged elementary particles have a charge that is a positive or negative integer multiple of the electron's charge.[3] Given the charge of all particles involved in a reaction, the net charge of the initial particles must equal the net charge of the final particles.

3. Family Particle Number

3a. Conservation of Baryon Number. A conservation law which accounts for the stability of the proton is the conservation of "baryon number," B. This also accounts for the fact that the neutron and all of the other heavier "elementary particles," the baryons, decay in such a way that the final product is the proton. This conservation law is similar to electrical charge conservation.

Just as all particles can be assigned electrical charge values of $0, \pm 1$, or ± 2, etc., (in units of the quantum of electric charge, the charge on the protons), every particle has a "baryon charge" of $B = 0$, $B = +1$, or $B = -1$. Furthermore, in any reaction the total baryon number of the products of the reaction must equal the sum of the baryon numbers of the initial particles. The proton is the lightest particle with baryon charge $B = +1$ so this accounts for the stability of the proton. All baryons have $B = 1$, their anti-particles have $B = -1$, and all mesons, leptons and the photon have $B = 0$. Thus

$$\begin{array}{rccccc} & \mathrm{p} & \Rightarrow & \pi^+ & + & \pi^0 \\ B: & +1 & & 0 & & 0 \end{array}$$

is forbidden by baryon conservation, while

$$\begin{array}{rccccc} & \Delta^{++} & \Rightarrow & \mathrm{p} & + & \pi^+ \\ B: & +1 & & +1 & & 0 \end{array}$$

is allowed by baryon number conservation as well as by all other conservation laws. This reaction involves only hadrons and it occurs with the characteristic time of 10^{-23} second. Similarly

[3] Current theories of elementary particles state that hadrons (baryons and mesons) are composed of particles called "quarks" which have charge 1/3 or 2/3 times the charge of an electron. See "SU(3) and the Quark Model" (MISN-0-282).

	n	⇒	p	+	e^-	+	$\bar{\nu}_e$
B:	1		1		0		0

is also allowed by baryon number conservation. Since it involves leptons as well as hadrons, the reaction goes at a much slower rate (the neutron mean life being 1000 seconds).

3b. Conservation of Lepton Quantum Numbers. There appears to be a set of two quantum numbers associated with leptons that has similarities to baryon number. These two quantum numbers are similar to baryon number in the sense that the conservation laws associated with them are absolute; they must be satisfied in all processes. These quantum numbers are the electron lepton number and the muon lepton number. Their assigned values are:

Particle	Electron No.	Muon No.
e^-	+1	0
μ^-	0	+1
ν_e	+1	0
ν_μ	0	+1

The antiparticle to any of these particles has the opposite lepton number; for example, the e^+, the anti-electron, has an electron lepton number of −1. All other particles have zero value for both these lepton numbers. Consider the process: $\pi^- \Rightarrow \mu^- +$ "*neutrino.*" Which one of the 4 kinds of neutrino-type particles (neutrinos and anti-neutrinos) must this "neutrino" be if both electron lepton number and muon lepton number are to be conserved in this process? *Help: [S-7]*

4. Strangeness

4a. "Strange" Decay Modes. The failure of certain hadrons to decay via the strong interaction has led physicists to infer the existence of a hadron quantum number called "strangeness." Consider the hadron Λ^0 which is a baryon (the Λ^0 has a rest energy of 1116 MeV compared to 938 MeV for the proton). The Λ^0 is produced via Strong Interactions such as

$$\pi^- + p \Rightarrow \Lambda^0 + K^0$$

and

$$\pi^0 + p \Rightarrow \Lambda^0 + K^+ ,$$

where the K^0 and K^+ are mesons. The K's are $B = 0$ particles as are the pions, so baryon number is conserved in the reaction since the Λ^0 has $B = 1$. So Λ^0 must be a hadron (it's produced in a reaction which goes at the characteristic rate of strong interactions) and it is a baryon. The Λ^0 is unstable and decays via

$$\Lambda^0 \Rightarrow p + \pi^- .$$

This reaction involves only hadrons and baryon number is conserved, so it should go at the SI (strong interaction) rate of $\propto 10^{-23}$ seconds. In fact the observed Λ^0 lifetime is much longer, about 10^{-10} seconds, which indicates that the reaction occurs via the WI (weak interaction) and not the SI. This is not an isolated case. For example, the K^0 and K^+ decay via:

$$K^+ \Rightarrow \pi^+ + \pi^0 ,$$

Again these are hadrons, $B = 0$ on both sides, and you'd expect the reaction to be carried out by the strong interaction. Again the lifetime is about 10^{-8} seconds, which reveals that it is the WI that brings about the process. What forbids the occurrence, via the SI, of such reactions involving hadrons? Strangeness!

The answer is that these observations reveal the existence of another conservation law. This one is unlike the law of baryon number conservation or the law of charge conservation, which are absolute conservation laws (absolute in the sense that the laws are satisfied in all processes,[4] no matter what interaction meditates the process). This conservation law is absolute only for Strong Interaction and Electromagnetic processes; it may be violated in processes which go via the weak interaction.

4b. Conservation of Strangeness. Analogous to the assignment of baryon number, another quantum number is assigned to all hadrons, the Strangeness quantum number S. The strangeness assignments are to be made in a way which is consistent with what is observed. That is, those processes involving hadrons which are observed not to go via the SI must violate Strangeness conservation. As an example, consider the three processes

[4] That is to say, no processes violating these conservation laws have ever been observed.

$$\pi^0 + \mathrm{p} \Rightarrow \Lambda^0 + \mathrm{K}^+ \qquad \text{(goes via SI)}$$
$$\Lambda^0 \Rightarrow \mathrm{p} + \pi^- \qquad \text{(goes via WI)}$$
$$\mathrm{K}^+ \Rightarrow \pi^+ + \pi^0 \qquad \text{(goes via WI)}$$

Try an assignment scheme for yourself. Assign Strangeness quantum numbers to the particles involved in these three processes such that Strangeness is conserved in the first reaction but violated in the last two. As a simplifying ground rule, use only $S = 0$, 1, and -1 and assign the same S to each of the three members of the pion family *Help: [S-6]*. Of course, based on only these three experimental facts, there is no unique set of S values which will satisfy the observations. You could make assignments that would work just as well as the values you'll find in the physics literature. However, the assigned numbers must be restricted to those sequences that match the experimental results. Physicists always use the "commonly accepted" values found in the literature and we ask that you do likewise. The values you used in the above three reactions are indeed those values (see the complete table of them at the beginning of the Problem Supplement). There are many other hadrons and many other reactions, and the S quantum number assignments made to all the hadrons must be consistent with what is observed for all processes involving only hadrons: if a process is observed to go via the SI or the E-M (electromagnetic) interaction, Strangeness must be conserved; if it is to be allowed to go via the WI, then Strangeness must not be conserved.

5. Isospin

5a. Introduction to Isospin. There is yet another internal quantum number that is associated with elementary particles. This one, like strangeness, is associated with hadrons only. Note that the number of conservation laws that must be satisfied in a given interaction increases with increasing strength of the interaction, the SI having the most conservation laws, the WI the fewest (so hadrons have more quantum numbers than leptons). This internal quantum number is called the "isotopic spin," "I-spin," or "isospin," but it has nothing to do with angular momentum or spin.

5b. Assigning Isospin Quantum Numbers. The assignment of isospin quantum numbers may be made if we consider the situation which prevails with the spectrum of hadrons (baryons and mesons). If you

plot the masses of the baryons in a way similar to the plot of atomic energy levels and separately do the same for the meson masses you observe degeneracies, places where several particles have about the same mass. The spectrum is said to consist of degenerate multiplets. There are singlet levels (such as the Λ^0), doublets (such as the proton and neutron), triplets (such as the Σ^+, Σ^0, and Σ^-.), quartets (such as the Δ^{++}, Δ^+, Δ^0, Δ^-), etc.. Each grouping consists of particles of nearly equal mass and differing only in charge. This suggests that we can assign, to those particles, pairs of quantum numbers analogous to the (j, M_j) pair that are assigned to atomic levels. These quantum numbers have nothing to do with angular momentum, however. They are internal quantum numbers just as are B and S. These quantum numbers are usually labeled (I, I_3).[5] Consider the Σ^+, Σ^0, Σ^- triplet. The isospin I obviously is 1 and so I_3 has possible values of 1, 0, -1 *Help: [S-10]*. Note that I_3 is directly related to the electrical charge of the particle: values of I_3 are assigned in order of decreasing value as Q decreases. Thus I and I_3 for the Δ^{++} are 3/2 and +3/2, respectively; for the Δ^- they are 3/2 and $-3/2$, respectively *Help: [S-11]*.

5c. The Gell-Mann-Nishijima Formula. There is a relationship between electric charge Q and the quantum numbers I_3, S and B given by the Gell-Mann-Nishijima formula:

$$\frac{Q}{e} = I_3 + \frac{S+B}{2}\,.$$

Note that (Q/e) has integer absolute value. That is, (Q/e) may be 0, ± 1, ± 2, ... The sum $S + B \equiv Y$ is called "hypercharge."

5d. When and How Isospin is Conserved. The assignment of isospin quantum numbers would be an empty exercise except for the observation that I is conserved in all strong interactions and I_3 is conserved in all strong and electromagnetic interactions.

Conservation of I_3 is similar to the conservation of charge and of B and S (and not independent of these conservation laws because of the relationship $Q/e = I_3 + B/2 + S/2$): just add the I_3 values on the left side of a reaction equation and it must equal the sum of the I_3 values on the right side for the reaction to go via SI or the electromagnetic interaction.

Conservation of I is a little more complicated. To see how this works, consider first this reaction:

[5] In some references you'll see (T, T_z).

$$\pi^- + \mathrm{p} \Rightarrow \Lambda^0 + K^0 \qquad \text{(goes via SI)}$$

Given that: π^- belongs to a triplet, K^0 belongs to a doublet, both K and π are mesons, and Λ^0 has strangeness $S = -1$ and is a baryon, fill in this table *Help: [S-9]*:

	π^-	+	p	$\Rightarrow$	Λ^0	+	K^0
Q		&		$\Rightarrow$		&	
I_3		&		$\Rightarrow$		&	
S		&		$\Rightarrow$		&	
B		&		$\Rightarrow$		&	
I		&		$\Rightarrow$		&	

On the right side the total I is 1/2. Therefore on the left side the total I must be 1/2 for Isospin to be conserved. But the I-values on the left side are $I = 1$ and $I = 1/2$. How do you add these to get 1/2? Getting the resultant of these two is completely analogous to getting the resultant of two angular momenta.

We here summarize the rules for adding quantized angular momenta, which are treated in more detail elsewhere.[6] Because angular momentum is quantized, j is restricted to integer and half-integer values. This means that if you add two angular momenta, the resultant magnitude of this vector sum is also restricted to integer or half-integer values. However, the "third component" of angular momentum, with quantum number M_j, adds arithmetically. As an illustration, suppose we start with two angular momenta, each with $j = 1$. Associated with each $j = 1$ there are the three M_j values of 1, 0, and −1. What are the possible M_j values of the resultant? Each possible resultant M_j is the arithmetic sum of the possible individual M_j values. So with 3 possible M_j's in one group and 3 in the other there are nine possible sums. They are 2, 1, 0, 1, 0, −1, 0, −1, −2. Or rearranging them more suggestively:

$$\begin{array}{ccccc} 2 & 1 & 0 & -1 & -2 \\ & 1 & 0 & -1 & \\ & & 0 & & \end{array}$$

These are the possible M_j values of the resultant angular momentum. The upper line contains all the possible M_j values for $j = 2$, the middle

[6] See "Quantized Angular Momenta" MISN-0-251.

one represents $j = 1$, and the last $j = 0$. This then suggests that if you add a $j = 1$ to a $j = 1$ the possible values of the resultant are $j = 2$, 1, and 0, and this, indeed, is the case. Instead of going through the above analysis each time you add two angular momenta, you can remember the rule that if you add two angular momenta whose quantum numbers are j_1 and j_2, the resultant angular momentum quantum number will have possible values that occur in the interval from $j_1 + j_2$ to $|j_1 - j_2|$, with all possible values in between occurring in integer steps.

The addition of two isospins follows the addition of two angular momenta exactly. The example

$$\pi^- + \mathrm{p} \Rightarrow \Lambda^0 + \mathrm{K}^0\,,$$

does conserve I-spin. The right side has I-spin equal to 1/2 ($I = 0$ and $I = 1/2$ have only one possible sum). The left side is the resultant of $I = 1$ and $I = 1/2$. Therefore, the reaction can go via the SI because isotopic spin is conserved: both the left side and the right side can have $I = 1/2$. Note: It is very important that you know the distinction between I and I_3: adding two I's is done differently than adding two I_3's.

Thus for an odd number of particles in a multiplet, with each particle in a multiplet having its own I_3 value, the following list shows the I_3 values for members of quintuplets, trios, and singlets:

$$\begin{array}{ccccc} 2 & 1 & 0 & -1 & -2 \\ & 1 & 0 & -1 & \\ & & 0 & & \end{array}$$

while for an even number of particles the smallest three multiplets have particles with these I_3 values:

$$\begin{array}{cccccc} 5/2 & 3/2 & 1/2 & -1/2 & -3/2 & -5/2 \\ & 3/2 & 1/2 & -1/2 & -3/2 & \\ & & 1/2 & -1/2 & & \end{array}$$

Acknowledgments

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PROBLEM SUPPLEMENT

Lepton Numbers	
	Electron No.
e^-	+1
e^+	−1
ν_e	+1
$\bar{\nu}_e$	−1
	Muon No.
μ^-	+1
μ^+	−1
ν_μ	+1
$\bar{\nu}_\mu$	−1

Hadron Numbers			**I_3**				
B	S	I	−1	−1/2	0	1/2	+1
1	0	1/2		n		p	
1	−1	0			Λ^0		
0	0	1	π^-		π^0		π^+
0	+1	1/2		K^0		K^+	
0	−1	1/2		K^-		$\overline{K}^0$	
1	−1	1	Σ^-		Σ^0		Σ^+
1	−2	1/2		Ξ^-		Ξ^0	
1	−3	0			Ω^-		
0	0	0			η		

$$\frac{Q}{e} = \frac{B}{2} + \frac{S}{2} + I_3$$

1. The Ω^- baryon is a singlet energy state whose electric charge is given by: $Q/e = -1$. The Ω^- is a hadron which eventually decays to the proton in a three stage decay process.
   a. What are (I, I_3) for the Ω^-? *Help: [S-3]*
   b. What is the strangeness of the Ω^-? *Help: [S-8]*
   c. What is the hypercharge of Ω^-? *Help: [S-4]*
   d. The Ω^- is observed to decay to $\Xi^- + \pi^0$. The Ξ^- has strangeness $S = -2$. Does this decay go via the SI or WI? *Help: [S-1]*
   e. What is I_3 for the Ξ^-? *Help: [S-5]*
   f. Ξ^- is one of the members of a doublet. What is the electric charge of the other particle in this doublet? *Help: [S-2]*
2. A multiplet of particles consists of three baryons with strangeness $S = +1$. What is the charge of each member of this three particle multiplet? *Answer: 1*
3. The μ^+ (positive muon) decays to an e^+ and two other particles. Using lepton conservation laws identify the other two particles. *Answer: 17*

4. In the process $A + B \Rightarrow C + D$ particles C and D belong to isospin zero multiplets. Particles A and B each belong to an isospin 1/2 multiplet. If I_3 for A is + 1/2 what is it for B? Can this reaction go via the strong interaction? *Answer: 2*
5. Suppose in the above reaction particle A belonged to an isospin zero multiplet and B was the $I_3 = 0$ component of an $I = 1$ multiplet. Would the reaction go via the SI? *Answer: 24*
6. Consider the following reactions. Determine whether or not each one is allowed via the strong interaction. If not allowed by the strong interaction determine whether or not it's allowed by the weak interaction or whether it's absolutely forbidden. If forbidden, state which conservation law forbids it.
   a. $\Xi^- \Rightarrow \Sigma^- + \pi^0$ *Answer: 3*
   b. $K^- + p \Rightarrow \Lambda^0 + \pi^- + \pi^+$ *Answer: 19*
   c. $n + p \Rightarrow n^- + p^-$ *Answer: 4*
   d. $K^- + \Xi^0 \Rightarrow \Omega^- + \pi^0$ *Answer: 7*
   e. $\Lambda^0 \Rightarrow n^0 + \pi^0$ *Answer: 22*
7. Determine the quantum numbers (I, I_3, S, B, Y, Q) for the antiparticle of the particle Δ^0 (one of the Δ^{++}, Δ^+, Δ^0, Δ^- multiplet). *Answer: 20*
8. What are the possible values for the isospin of the deuteron? *Answer: 5* (The deuteron is a nucleus consisting of a neutron and a proton).
9. A fictitious particle called Z is produced via the strong interaction in the process

$$K^+ + p \Rightarrow Z + \Lambda^0$$

   For this particle determine these quantum numbers:
   a. Charge *Answer: 21*
   b. Strangeness *Answer: 16*
   c. Baryon number *Answer: 9*
   d. I_3 *Answer: 11*
   e. With this value of I_3 what is the smallest value of I itself that the particle Z may have? *Answer: 8*

f. For this smallest value of I, how many companion particles are there for Z? *Answer: 15*

g. What are the charges of those companion particles? *Answer: 25*

10. Assuming that Z has enough mass for the following decays to be energetically possible, state which of the following might occur and which will be forbidden. If forbidden, state why. (Supply appropriate charges to balance the reaction).

a. $Z \Rightarrow \mathrm{p} + \pi$ *Answer: 12*

b. $Z \Rightarrow \mathrm{K} + \mathrm{K}$ *Answer: 18*

c. $Z \Rightarrow \mathrm{p} + \overline{\mathrm{p}} + \pi + \pi$ *Answer: 13*

d. $Z \Rightarrow \mu + \mu + \nu_\mu + \nu_\mu$ *Answer: 6*

11. Suppose (hypothetically) that strong interaction processes could go if the reactants were in the $I = 3/2$ state, but were forbidden in the $I = 1/2$ state. Which of the following processes would be allowed and which forbidden?

a. $\Sigma^+ + \mathrm{p} \Rightarrow \Lambda^0 + \mathrm{p} + \pi^+$ *Answer: 10*

b. $\pi^- + \mathrm{p} \Rightarrow \mathrm{n} + \pi^0$ *Answer: 23*

c. $\mathrm{p} + \Lambda^0 \Rightarrow \Sigma^+ + \pi^0$ *Answer: 26*

Brief Answers:

1. 2,1,0
2. $-1/2$; yes
3. Weak (Strangeness not conserved)
4. Forbidden (baryon no. and charge)
5. 1 or 0 (actually is zero)
6. Weak (if at all), if both μ's are μ^+ and neutrinos are ν_μ.
7. Strong.
8. +1

9. Zero
10. Allowed
11. +1
12. Forbidden (baryon conservation)
13. Weak if at all (strangeness)
14. $Q = I_3 + Y/2$
15. 2
16. +2
17. $\bar{\nu}_\mu$, ν_e
18. Allowed, strong
19. Strong
20. $I = 3/2$, $Q = 0$; $I_3 = 1/2$; $S = 0$, $B = -1$, $Y = -1$.
21. +2
22. Weak (strangeness)
23. Allowed
24. No
25. 1, 0
26. Forbidden

SPECIAL ASSISTANCE SUPPLEMENT

S-1 *(from PS, problem 1d)*

Weak

S-2 *(from PS, problem 1f)*

Zero

S-3 *(from PS, problem 1a)*

(0,0)

S-4 *(from PS, problem 1c)*

-2

S-5 *(from PS, problem 1e)*

$-1/2$

S-6 *(from TX-4b)*

Λ^0 ($S = -1$); π ($S = 0$); p, n ($S = 0$); K^+ ($S = 1$)

S-7 *(from TX-3b)*

$\bar{\nu}_\mu$

S-8 *(from PS, problem 1b)*

-3

S-9 *(from TX-5d)*

Left to right and top to bottom: -1, $+1$, 0, 0; -1, 1/2, 0, $-1/2$; 0, 0, -1, $+1$; 0, 1, 1, 0; 1, 1/2, 0, 1/2.

S-10 *(from TX-5b)*

The Σ occurs as a charge triplet with Σ^+, Σ^0, and Σ^-. It exists as three particles that differ only in charge. Then the isospin assignment is $I = 1$ with $I_3 = 1, 0, -1$ for the three charge states.

S-11 *(from TX-5b)*

The Δ occurs as a charge quartet with Δ^{++}, Δ^+, Δ^0, and Δ^-. Then the isospin assignment is $I = 3/2$ with $I_3 = 3/2, 1/2, -1/2$ and $-3/2$ for the four charge states.

MODEL EXAM

Lepton Numbers	
	Electron No.
e^-	$+1$
e^+	-1
ν_e	$+1$
$\bar{\nu}_e$	-1
	Muon No.
μ^-	$+1$
μ^+	-1
ν_μ	$+1$
$\bar{\nu}_\mu$	-1

Hadron Numbers			**I_3**				
B	S	I	-1	$-1/2$	0	$1/2$	$+1$
1	0	1/2		n		p	
1	-1	0			Λ^0		
0	0	1	π^-		π^0		π^+
0	$+1$	1/2		K^0		K^+	
0	-1	1/2		K^-		$\overline{K}^0$	
1	-1	1	Σ^-		Σ^0		Σ^+
1	-2	1/2		Ξ^-		Ξ^0	
1	-3	0			Ω^-		
0	0	0			η		

$$\frac{Q}{e} = \frac{B}{2} + \frac{S}{2} + I_3$$

1. For each of the following reactions and decays (assuming for the reactions that there is enough kinetic energy to start with) state whether it is allowed or forbidden. If it is allowed, explain by which kind of interaction it goes. If it is forbidden, explain why.

   a. $\mu^+ \Rightarrow e^+ + \nu_e$

   b. $\Omega^- \Rightarrow n + n + \pi^-$

   c. $K^+ \Rightarrow \pi^+ + \pi^0$

   d. $\Sigma^0 \Rightarrow \Lambda^0 + \gamma$

   e. $\pi^- + p \Rightarrow n + \pi^0$

2. Three fictitious particles, x_1, x_2, and x_3 all have approximately the same rest mass. They are observed to be created in strong interaction processes of which these are typical:

$$\begin{aligned} \pi^- + p &\Rightarrow x_1 + \overline{K}^0 \\ p + p &\Rightarrow x_2 + K^- + p \\ K^+ + n &\Rightarrow x_3 + \pi^0 + \pi^0 \end{aligned}$$

   a. What is the baryon number of x_1, of x_2, and of x_3?

   b. What is the charge of x_1, of x_2, and of x_3?

   c. What is the strangeness of x_1, of x_2, and of x_3?

   d. What is the third component of isospin of x_1, of x_2, and of x_3?

   e. These three x-particles are all observed to decay via the weak interaction. Write down a possible weak decay mode for particle x_2.

Brief Answers:

1. a. forbidden; muon and electron lepton numbers are not conserved.

   b. forbidden; baryon number not conserved.

   c. allowed; by weak interaction but not by strong because strangeness not conserved.

   d. allowed; by electromagnetic but not by weak.

   e. allowed; by strong.

2. a. $B = 1$ for each.

   b. $Q_1/e = 0$; $Q_2/e = 2$; $Q_3/e = 1$

   c. $S = +1$ for each.

   d. x_1 has $I_3 = -1$; x_2 has $I_3 = 1$; x_3 has $I_3 = 0$.

   e. In weak decay, S is not conserved but B and Q are. Therefore, a possible decay of x_2 is:

$$x_2 \Rightarrow p + \pi^+$$

PHYSNET

MISN-0-255

FUNDAMENTAL FORCES AND ELEMENTARY PARTICLE CLASSIFICATION

neutron photon tau's neutrino eta

electron's neutrino

omega electron lambda muon

proton xi Kaon pion muon's neutrino

Project PHYSNET Physics Bldg. Michigan State University East Lansing, MI

FUNDAMENTAL FORCES AND ELEMENTARY PARTICLE CLASSIFICATION

by

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Title: **Fundamental Forces and Elementary Particle Classification**

Author: J. S. Kovacs and William C. Lane, Dept. of Physics, Mich. State Univ

Version: 11/13/2001 Evaluation: Stage 1

Length: 1 hr; 16 pages

Input Skills:

1. Vocabulary: conservation of energy (MISN-0-21) or (MISN-0-416); momentum (MISN-0-15) or (MISN-0-413); charge, Coulomb force (MISN-0-114) or (MISN-0-419); gravitation (MISN-0-101); energy levels (MISN-0-215); photons, quanta (MISN-0-212).

Output Skills (Knowledge):

K1. Vocabulary: antiparticle, baryon, electromagnetic interaction, elementary particle, fundamental forces, hadron, lepton, muon, meson, neutrino, nucleon, pair annihilation, pair production, strong interaction, weak interaction.

K2. State the four fundamental forces of nature.

K3. List seven known elementary particles that do not take part in the strong interaction.

Output Skills (Problem Solving):

S1. Determine, from the nature of the particles involved in a given reaction, whether the reaction goes via the strong, electromagnetic, or weak interaction.

Post-Options:

1. "Conservation Laws for Elementary Particles" (MISN-0-256).
2. "Elementary Particle Interaction Times, Ranges, Cross Sections" (MISN-0-266).

FUNDAMENTAL FORCES AND ELEMENTARY PARTICLE CLASSIFICATION

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1. Introduction

1a. The Fundamental Constituents of Matter. The fundamental constituents of matter are called "elementary particles." What do we mean when we refer to a particle as an elementary particle? In the 19th century you would have meant the atoms of the chemical elements. These were thought to be the "basic building blocks," the ultimate indivisible subdivision of matter. But, as the studies of the early 20th century revealed, the atom is divisible, it does have internal structure, therefore it's not an elementary entity. These revelations led to the modified view that the fundamental constituents of matter were those things of which the atoms were made: the various kinds of nuclei (of which there was a different one for each different atom), the electron, and the photon.

1b. The Photon is a Constituent of Matter. The photon is the quantum of electromagnetic field and must be included among the elementary particles. This particle is observed when the constituents of an atom, arrayed in one of the excited atomic states, "rearrange themselves" into a state of lower total energy. The force field which determines which "arrangements" are possible in a given atom is the electromagnetic field. Upon rearrangement, when the system undergoes a transition from an excited state to a state of lower energy, a quantum of the field, a photon, carries away the excess energy (see Fig. 1).

1c. The Nucleus Has Internal Structure. The picture became both simplified and complicated around 1930 when it was discovered that the nucleus was not elementary; it has internal structure. It became simpler because it seemed that the number of elementary particles was reduced. All nuclei are composed of various combinations of neutrons and protons, collectively called "nucleons." So instead of there being hundreds of "fundamental nuclei," there are only two nucleons which were the fundamental constituents of nuclei.

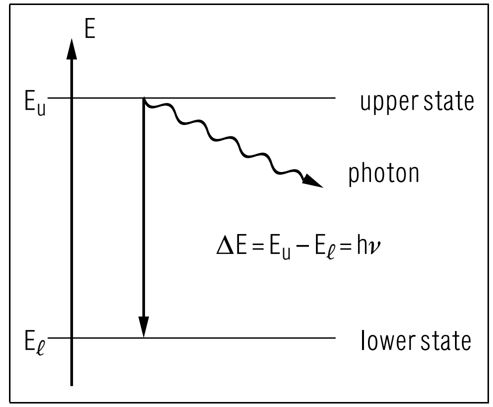

Figure 1. The transition of an atom from an excited state to the ground state, with the accompanying production of a photon.

2. The Fundemental Forces

2a. The Electromagnetic Interaction. The single basic force responsible for the structure of the atom is the "electromagnetic interaction." This single force field, together with the laws of quantum mechanics which, through the Schrödinger equation and the Pauli principle, tell you how the particles behave under the action of this force field, determines completely all chemical and biological properties of all matter. The prevailing viewpoint before the 1920's is illustrated in Table 1.

Table 1. The pre-1920 view of elementary particles and fundamental forces.

Elementary Particles	Interaction Forces
Nuclei (proton, helium, iron ...) electron photon	Electromagnetic

2b. The Strong Interaction. With the discovery of the internal structure of the nucleus, however, the situation became complicated because a new fundamental force field, the "strong interaction,"[1] hitherto unobserved in any macroscopic experience, was needed to explain how the nucleons were held together in a nucleus. The strong interaction is so-called because it apparently is stronger at short distances than the

[1] The current view of the basic strong interaction goes another stage beyond what is discussed here. There is now strong evidence that the proton, for example, has a substructure. The particles that make up the proton, called quarks, are held together by a more basic strong interaction, called "quantum chromodynamics," with mediating quanta called "gluons" [see "SU(3) and the Quark Model," MISN-0-282].

electromagnetic interaction. After all, it keeps protons together inside the nucleus in spite of the repulsive Coulomb force. To explain theoretically the properties of the nucleus you need to deal not only with the formidable Schrodinger equation (which is impossible to solve exactly for a many-particle system even if you do know exactly the properties of the force field involved, as you do in atomic problems) but you also need to deal with a force field whose detailed properties are unknown. Much of nuclear physics deals with the reverse problem: how to infer, from the properties of the myriad nuclei, what is the nature of the force field that is responsible for holding these systems together.

2c. The Weak Interaction. In addition to the strong interaction that is necessary to hold together the constituents of the nucleus, there is a "weak interaction," different from both the strong interaction and the electromagnetic interaction, which causes certain elementary particles to radioactively decay to other elementary particles. As an example of a decay via the weak interaction, the neutron decays to a proton with the emission of an electron and a massless particle called a "neutrino."

2d. The Gravitational Interaction. The fourth and weakest fundamental force is the "gravitational interaction." All of the elementary particles, including the massless photon and the neutrino, take part in the gravitational interaction. However, on the elementary particles scale the gravitational interaction is negligible compared with the other forces.

3. Elementary Particles

3a. Elementary Particles Circa 1930. By the 1930's physicists recognized four elementary particles and four fundamental forces (see Table 2). The interactions that the particles take part in are shown after the particle name. Note that only the neutron and proton take part in the strong interaction.[2] Thus, one might have ventured to say in the 1930's that the number of elementary particles was remarkably few. Furthermore these particles, interacting via the four fundamental forces were the components of all matter in the universe and that all phenomena in the universe could, in principle, be explained in terms of these particles and interactions. However, there were large gaps of knowledge to be filled in this picture before full understanding was achieved. First of all, the

[2]The neutron takes part in the electromagnetic interaction because it has a magnetic dipole moment.

"quanta" of the strong interaction needed to be identified.[3]

Table 2. Elementary particles and the fields with which they interact, as viewed in the 1930's. The fields are denoted: S = Strong, E = Electromagnetic, W = Weak, and G = Gravitational.

Elementary Particles	**Interacting Fields**
Neutron	S, E, W, G
Proton	S, E, W, G
Electron	E, W, G
Photon	E, G

3b. Baryons, Mesons, and Leptons. When, in the late 1940's, what was thought to be these quanta were discovered, it turned out to be only the beginning. Over the years, not only did a complex spectrum of quanta of various masses appear, called "mesons," there also was discovered a spectrum of heavier strongly interacting particles, called "baryons." The "ground state" of this baryon spectrum is the familiar proton, and the first excited state is the neutron. The particles in these two spectra, all of which take part in the strong interaction, are collectively called "hadrons." The study of the properties of these hadrons and the search for what these properties tell about the nature of the strong interaction, occupies a significant part of what is called "high energy physics." An important step in unraveling the mysteries of hadronic physics is devising intelligent schemes for classifying these large numbers of hadrons.[4] A third spectrum of particles, called "leptons," or "light particles," do not participate in the strong interaction. The members of this class includes the electron, the muon (a heavy version of the electron) and neutrinos corresponding to the electron and the muon.

3c. Intrinsic Properties of the Elementary Particles. The intrinsic properties of the elementary particles are their mass, charge, spin, and magnetic moments. These properties are listed in Table 3 for the elementary particles we will encounter.

[3]Just as the quantum of the electromagnetic interaction is the photon, there was thought to be a corresponding quantum associated with the strong interaction.

[4]You may recall from chemistry that the classification of the elements in the periodic table of the elements played an important role in uncovering the structure of atoms.

Table 3. Intrinsic Properties of Elementary Particles. Mass in MeV/c^2, charge in units of e, spin in units of $\hbar$.

Family	Particle	Symbol	Mass	Charge	Spin
	photon	γ	0	0	1
LEPTONS	electron's neutrino	ν_e	0	0	1/2
	muon's neutrino	ν_μ	0	0	1/2
	tau's neutrino	ν_τ	0	0	1/2
	electron	e	0.511	−1	1/2
	muon	μ	105.66	−1	1/2
	tau	τ	1784.2	−1	1/2
HADRONS:					
mesons	pion	π^0	134.96	0	0
		π^+	139.57	+1	0
		π^-	139.57	−1	0
	Kaon	K^+	493.8	+1	0
		K^-	493.8	−1	0
		K^0	493.8	0	0
	eta	η	548.8	0	0
baryons	proton	p	938.26	+1	1/2
	neutron	n	939.55	0	1/2
	lambda	Λ^0	1115.6	0	1/2
	sigma	Σ^+	1189.4	+1	1/2
		Σ^0	1192.5	0	1/2
		Σ^-	1197.4	−1	1/2
	xi	Ξ^0	1315	0	1/2
		Ξ^-	1321.3	−1	1/2
	omega	Ω^-	1673	−1	3/2

3d. Pair Production and Annihilation. For every particle listed in Table 3 there is an "antiparticle" with the same mass but opposite charge. An antiparticle is symbolized by the particle symbol with a bar over it e.g. $\bar{p}$ is an antiproton. Exceptions to this nomenclature rule include the chargeless photon and π° meson, which are their own antiparticles, and the electron, whose antiparticle is given the special name of "positron" with the symbol e^+. A particle and its antiparticle may be created spontaneously out of energy in a process called "pair production." To conserve momentum, such a particle-antiparticle pair must have equal and opposite momenta. The energy required for such a process is equal to the rest

energy of both particles plus their total kinetic energy. When a particle and an antiparticle combine, they destroy each other in a process called "pair annihilation." The total energy of the pair (rest energy plus kinetic) is converted into two or more photons of equivalent energy. At least two photons must be created to conserve momentum in the process.

3e. Particle Decay. The majority of the particles in Table 3 have finite lifetimes and decay radioactively to some other state or group of particles with a characteristic mean life.[5] Most particles that decay have more than one final state, or "decay mode," available, although frequently one decay mode is dominant. The only particles which appear to be stable against decay are the photon, the electron, both neutrinos (ν_e and ν_μ) and the proton.[6]

4. Elementary Particle Reactions

4a. Introduction. What factors determine which one of the three fundamental interactions is responsible for a given reaction among elementary particles? Why, for example, does the Δ^+ always decay via the strong interaction, the π^+ always via the weak interaction, and the π^0 always via the electromagnetic interaction? Why does the reaction

$$\pi^- + \mathrm{p} \Rightarrow \pi^0 + \mathrm{n}$$

(where p and n are the proton and neutron) go via the strong interaction and not the weak interaction? The answer to all of these questions is that it depends upon which particles, on both sides of the reaction equation, are involved in the reaction. Some reactions involve more than one interaction. An example of this is

$$\gamma + \mathrm{p} \Rightarrow \pi^+ + \mathrm{n}$$

which requires the intervention of both the electromagnetic and strong interactions for it to take place. Most simple reactions, however, take place via a single interaction, and we'll restrict our concerns to one-interaction reactions.

4b. Conservation Laws Restrict Reactions. Conservation laws have been deduced for various classes of reactions and form a means of determining which interaction is responsible for any particular reaction.

[5] See "Exponential Decay: Observation, Derivation," MISN-0-311.
[6] See "Conservation Laws for Elementary Particles Reactions" (MISN-0-256).

If you consider a reaction and write down the reaction equation, the reaction will go if all appropriate conservation laws are satisfied. Energy, momentum, and charge must be conserved in any reaction.

4c. The Strong Interaction Takes Precedence. Any reaction will go via the strong interaction (hereafter denoted SI) unless at least one of the particles involved does not take part in the SI. Most of the 200 or so elementary particles, are hadrons − particles which interact via SI. A reaction involving hadrons such as

$$\pi^- + \mathrm{p} \Rightarrow \pi^0 + \mathrm{n}$$

may go via the weak interaction (hereafter denoted WI) as well as the SI. However, the SI is so much more likely (by a factor of 10^{13}) that the reaction has never been observed to go via the WI. In fact, there are only nine particles which do not interact strongly: these are the electron and its antiparticle, the positron, the muons, (μ and $\bar{\mu}$), the electron's neutrinos (ν_{e} and $\bar{\nu}_{\mathrm{e}}$), the muon's neutrinos (ν_μ and $\bar{\nu}_\mu$), and the photon.

4d. Leptons Indicate Weak Interaction. The first eight non-hadronic particles are the leptons, and their presence in a reaction signals the participation of the weak interaction. Note that these consist of 4 particles and their antiparticles. For example, consider the pion decay reaction:

$$\pi^- \Rightarrow \mu^- + \bar{\nu}_\mu$$

where $\bar{\nu}_\mu$ is the muon's antineutrino. The pion is a hadron but the reaction products are leptons, so the reaction can not go via SI. Furthermore, the presence of one of the four neutrinos or antineutrinos assures that the reaction goes via WI and not the electromagnetic interaction. The neutrinos are unique in that they only interact via WI.

4e. Photons Indicate Electromagnetic Interaction. Similarly, if you notice that a photon is involved in an reaction, then you can be assured that the reaction involves the electromagnetic interaction. Note also, that all of the particles currently known take part in the WI except the photon. More about the classification of these particles can be found in the text and assigned readings.

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Glossary

- **antiparticle**: a counterpart of a given particle with the same mass but opposite charge and magnetic moment.
- **baryon**: a family, or spectrum of heavy particles, the ground state of which is the proton.
- **electromagnetic interaction**: an interaction between elementary particles mediated by the exchange of a photon.
- **elementary particle**: a fundamental constituent of matter.
- **fundamental forces**: four basic physical interactions between elementary particles which include strong, electromagnetic, weak, and gravitational interactions.
- **hadron**: any particle which may participate in the strong interaction, i.e., a baryon or a meson.
- **lepton**: a family of "light particles" which as a class, together with the photon, does not interact strongly. Members of the lepton family include the electron and its neutrino, the muon and its neutrino, and the antiparticles of each of the above particles.
- **meson**: a family of intermediate mass particles which mediate the strong interaction between baryons.
- **muon**: a lepton which is identical to an electron, except that it is roughly 200 times more massive.
- **neutrino**: a virtually massless lepton which comes in two varieties (an electron's neutrino and a muon's neutrino).
- **nucleon**: a constituent of the nucleus, i.e., a proton or a neutron.
- **pair annihilation**: the conversion of a particle-antiparticle pair into two or more photons.
- **pair production**: the creation of a particle-antiparticle pair out of energy.

- **strong interaction**: an interaction between elementary particles mediated by the exchange of a meson between two baryons or two mesons.
- **weak interaction**: a fundamental interaction between elementary particles that is weaker than the strong and electromagnetic interaction. This interaction is responsible for the radioactive decay of many of the elementary particles.

MODEL EXAM

1. See Output Skills K1-K3 in this module's *ID Sheet*. One or more of these skills may be on the actual exam.
2. An antiproton, $\overline{\text{p}}$, and a proton, p, may "annihilate" via the strong interaction. The products resulting from the annihilation must carry off the energy, net charge, momentum, etc., that the p and $\overline{\text{p}}$ had to begin with. Many possible final states of reaction products exist.
   a. For the three possible annihilation reactions considered below, which interaction is responsible for each reaction?
      i) $\text{p} + \overline{\text{p}} \Rightarrow \nu_e + \bar{\nu}_e$
      ii) $\text{p} + \overline{\text{p}} \Rightarrow \pi^- + \pi^+$
      iii) $\text{p} + \overline{\text{p}} \Rightarrow \gamma + \gamma$
   b. Which one of these reactions is most likely to happen and which is least likely?

Brief Answers:

1. See this module's *text*.
2. a. i) weak interaction
      ii) strong interaction
      iii) electromagnetic interaction
   b. most likely: $\text{p} + \overline{\text{p}} \Rightarrow \pi^- + \pi^+$
      least likely: $\text{p} + \overline{\text{p}} \Rightarrow \nu_e + \bar{\nu}_e$