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The space–time approach to quantum electrodynamics

14.1 Introduction

The basic principles and techniques of Feynman's new approach to quantum electrodynamics were described in his article on the 'Space–time approach to quantum electrodynamics'.[1] This article was written practically simultaneously with the preceding one on 'The theory of positrons'[2] (see Section 13.6), which was cited as I; the present article was a direct continuation of it. In this paper, Feynman considered the entire quantum electrodynamical description of the electromagnetic interaction between charged particles and the photon field, including the interaction between the charges themselves, which was neglected in I. This paper was received by the *Physical Review* on 9 May 1949, shortly after the Oldstone Conference, and published just twenty pages after 'The theory of positrons'.

Feynman introduced his new article by saying: 'In this paper two things are done: (1) It is shown that a considerable simplification can be attained in writing down the matrix elements for complex processes in electrodynamics. Further, a physical point of view is available which permits them to be written down directly for any physical problem. Being simply a restatement of conventional electrodynamics, however, the matrix elements diverge for complex processes. (2) Electrodynamics is modified by altering the interaction of electrons at short distances. All matrix elements are now finite, with the exception of those relating to problems of vacuum polarization. The latter are evaluated in a manner suggested by Pauli and Bethe, which gives finite results for these matrices also. The only effects sensitive to the modification are changes in the mass and charge of the electrons. Such changes could not be directly observed. Phenomena directly observable are insensitive to the details of the modification used (except at extreme energies). For such phenomena, a limit can be taken as the range of modification goes to zero. The results then agree with those of Schwinger. A complete, unambiguous, and presumably consistent method is therefore available for the calculation of all processes involving electrons and photons.

The simplification in writing the expressions results from an emphasis on the overall space–time view resulting from a study of the solution of equations of electrodynamics. The relation of this to the more conventional Hamiltonian point of view is discussed. It would be very difficult to make the modification which is proposed if one insisted on having the equation in Hamiltonian form.

'The methods apply as well to charges obeying the Klein–Gordon equation, and to the various meson theories of nuclear forces. Illustrative examples are given. Although a modification like that used in electrodynamics can make all matrices finite for all of the meson theories it is no longer true that all directly observable phenomena are insensitive to the details of the modification used.'[3]

In this fundamental article, Feynman explained his new perturbation theory, in which the matrix elements were worked out as expansions in powers of the dimensionless coupling constant α $(=e^2/\hbar c)$. Considerable simplification in writing down these elements was achieved for complex processes mainly from the fact that the old methods unnecessarily separated into individual terms closely related processes such as the effects of longitudinal and transverse waves, etc. This separation was made on a nonrelativistic basis. In Feynman's approach, the related processes were combined in a completely relativistic manner, and the results looked quite simple.

Feynman said this about the genesis of his space–time theoretical view of quantum electrodynamics: 'The conventional electrodynamics was expressed in the Lagrangian form of quantum mechanics.[4] The motion of the field oscillators could be integrated out, the result being an expression of the delayed interaction of the particles. Next the modification of the delta-function could be made directly from the analogy to the classical case.[5] This was still not complete because the Lagrangian method had been worked out in detail only for particles obeying the nonrelativistic Schrödinger equation. It was then modified in accordance with the requirements of the Dirac equation and the phenomenon of pair creation. This was made easier by the reinterpretation of the theory of holes (I). Finally, for practical calculations the expressions were developed in a power series in $e^2/\hbar c$. It was apparent that each term in the series had a simple physical interpretation. Since the result was easier to understand than the derivation, it was thought best to publish the results first in this paper. Considerable time has been spent to make these first two papers as complete and as physically plausible without relying on the Lagrangian method [that is, Feynman's path-integral method] because it is not generally familiar. It is realized that such description cannot carry the conviction of truth which would accompany the derivation. On the other hand, in the interest of keeping simple things simple the derivation will appear in a separate paper.'[6]

Feynman's prescription of the complex processes was to deal directly with the solutions of the time evolution equations, which Feynman had called Hamiltonian equations, rather than by investigating these equations themselves. He looked at the whole space–time evolution of a given system at once, rather than tracing this evolution in detail at every instant of time. He stressed that electrodynamics can be expressed in two equivalent and complementary forms. In the more usual Hamiltonian form one has the description of the *field* by Maxwell's equations. The other form is a description of a direct action-at-a-distance (albeit delayed in time) (see Sections 5.1 and 5.2). Both the forms have some advantages and shortcomings. The action-at-a-distance form is a more impractical point of view, since it is based on the interaction of the sources with absorbers. In it many different kinds of causes may produce the same kinds of effects. In the field picture all possible processes are described as a simple emission and absorption of light. But this point of view is less practical when one considers the close collisions of charged particles, where the source and the absorber are not well distinguishable. Therefore, Feynman arrived at the conclusion that 'the field point of view is most practical for problems involving real quanta, while the interaction view is best for the discussion of the virtual quanta involved'.[7]

Here one can see an important change in the evolution of Feynman's belief about the two forms of electrodynamics. He did not insist anymore on the action-at-a-distance theory as the only right form. Instead of this he tried to use both the forms in a most practical way. In addition, his earlier experience with the action-at-a-distance theory was very useful in developing the new ideas that were needed for his theory of quantum electrodynamics.

The Hamiltonian method is not well adapted to describe the direct action between charges. In many typical quantum problems, as for example the close collisions between particles, we are not interested in the detailed description of the time evolution, and the Hamiltonian form is not really practical. Feynman noted: 'We shall be discussing the solutions of the equations rather than the differential equations from which they come. We shall discover that the solutions, because of the *overall space–time view* that they permit, are as easy to understand when interactions are delayed as when they are instantaneous.'[8]

This new philosophy was extremely useful in developing Feynman's new approach to quantum electrodynamics and in overcoming many difficulties, both technical and of principle, of the old theory. In the Hamiltonian form of the equations, one has to follow the time evolution and, therefore, to use the nonrelativistic notion of separate time and three-dimensional space coordinates. The temporal analysis of different observers will lead to different pictures in their Hamiltonian prescription of the processes, which are irrelevant because the solution is the same in space–time form. Hence Feynman's important conclusion that, 'by forsaking the Hamiltonian method, the wedding of relativity and quantum mechanics can be accomplished most naturally.'[8]

14.2 The interaction between charges

First, Feynman derived the relativistic form of the interaction between charged particles with one-half spin in quantum electrodynamics. His starting point was the amplitude $K_0(3, 4; 1, 2)$ for the case that the particle a at point x_1 at the time instant t_1 will get to the point x_3 at the time instant t_3, and the particle b at the point x_2 at the time instant t_2 will get to the point x_4 at the time instant t_4. If the particles do not interact, this amplitude is the simple product of the separate amplitudes $K_{0a}(3, 1)$ and $K_{0b}(4, 2)$, the first being the amplitude that the particle a at point x_1 at instant t_1 will get to point x_3 at time instant t_3 when the particle b does not exist, and the second being the amplitude that the particle b at point x_2 at instant t_2 will get to point x_4 at instant t_4 when the particle a does not exist; thus

$$K_0(3, 4; 1, 2) = K_{0a}(3, 1)K_{0b}(4, 2). \tag{14.1}$$

This formula was discussed in detail in paper I (Ref. 2) for noninteracting particles (see Section 13.6, equation (13.27)). For interacting charged particles the quantity $K_0(3, 4; 1, 2)$ may be defined precisely if the interaction vanishes between the instants t_1 and t_2, and also between t_3 and t_4. For practical problems this means that we have to choose such long time intervals $t_3 - t_1$ and $t_4 - t_2$ that the interaction near the end points will give relatively small effects on the final result.

Feynman guessed the form of the relativistic invariant amplitude $K(3, 4; 1, 2)$ for spin-$\frac{1}{2}$ interacting particles by using the analogy with the nonrelativistic spinless case. He first considered the interaction by a Coulomb potential e^2/r, where r is the distance between two particles. The nonrelativistic Coulomb potential does not act instantaneously. We can represent $K(3, 4; 1, 2)$ formally as an integral over all t_5 and t_6, i.e. as if the potential were on at all times. But then we must include a delta-function $\delta(t_5 - t_6)$, to ensure a contribution only when $t_5 = t_6$, in order to take into account the instantaneous character of the nonrelativistic Coulomb interaction. As a result, using the notation $r_{56} = r_5 - r_6$ and $t_{56} = t_5 - t_6$, Feynman wrote the following formula:

$$K^1(3, 4; 1, 2)$$
$$= -ie^2 \iint K_{0a}(3, 5)K_{0b}(4, 6)r_{56}^{-1}\,\delta(t_{56})K_{0a}(5, 1)K_{0b}(6, 2)\,d\tau_5\,d\tau_6. \tag{14.2}$$

But, as we know, in relativistic classical electrodynamics the Coulomb interaction is delayed by a time r_{56}, taking the speed of light as unity. To take into account this effect of the finite speed of light we must replace the expression $r_{56}{}^{-1}\,\delta(t_{56})$ in equation (14.2) by something like $r_{56}{}^{-1}\,\delta(t_{56} - r_{56})$. This turned out to be not completely right, because the Fourier transform of $\delta(t_{56} - r_{56})$ contains the frequencies of both signs. As we know, the interaction, when represented by photons, must include only quanta with positive energies, i.e. only positive frequencies in the Fourier transform of the interaction potential are admissible. This means that the delta-function $\delta(t_{56} - r_{56})$ must be replaced by the positive frequency delta-function

$\delta_+(t_{56}-r_{56})$ (see Section 12.3, equation (12.12)). This result must be averaged with the analogous term $r_{56}{}^{-1}\ \delta_+(-t_{56}-r_{56})$, which corresponds to the emission of a quantum which b receives when $t_5<t_6$. This average value is

$$\tfrac{1}{2}[r_{56}^{-1}\ \delta_+(t_{56}-r_{56})+r_{56}^{-1}\ \delta_+(-t_{56}-r_{56})]=\delta_+(t_{56}^2-r_{56}^2)=\delta_+(s_{56}^2).$$

Hence, in the final formula, $r_{56}^{-1}\ \delta(t_{56})$ must be replaced by $\delta(s_{56}^2)$. But there still remains one problem. The formulas (13.20) and (13.22) are written for the scalar potential. Since, in classical electrodynamics, interaction is through the vector potential, instead of $\delta(s_{56}^2)$, we should have $(1-v_5v_6)\ \delta(s_{56}^2)$ for the classical particles (see Section 12.3, equation (12.14), and the following explanation), and

$$(1-\alpha_a\alpha_b)\ \delta(s_{56}^2)=\beta_a\beta_b\gamma_{a\mu}\gamma_{b\mu}\ \delta(s_{56}^2)$$

for spin-$\frac{1}{2}$ quantum relativistic particles. Hence, for the particles obeying the Dirac equation, the final form of the first-order interaction is given by

$$K^{(1)}(3,4;1,2)=-ie^2\iint K_{+a}(3,5)K_{+b}(4,6)\gamma_{a\mu}\gamma_{b\mu}\ \delta_+(s_{56}^2)$$
$$\times K_{+a}(5,1)K_{+b}(6,2)\ d\tau_5\ d\tau_6. \qquad (14.3)$$

Here γ_μ and β are the corresponding Dirac matrices.

Equation (14.3) is the fundamental equation in Feynman's approach to quantum electrodynamics. After the above intuitive derivation of this equation, Feynman wrote: '. . . It describes the effect of exchange of one quantum (therefore first order in e^2) between electrons. It will serve as a prototype enabling us to write down the corresponding quanties involving the exchange of two or more quanta between two electrons or the interaction of an electron with itself. It is a consequence of conventional electrodynamics. Relativistic invariance is clear. Since one sums over μ it contains the effects of both longitudinal and transverse waves in a relativistically symmetrical way.

'We shall interpret equation [(14.3)] in a manner which will permit us to write down the higher-order terms. It can be understood [see Fig. 14.1] as saying that the amplitude for "a" to go from 2 to 4 is altered to first order because they can exchange a quantum. Thus "a" can go to 5 (amplitude $K_+(5,1)$), emit a quantum (longitudinal, transverse, or scalar $\gamma_{a\mu}$), and then proceed to 3 [$K_+(3,5)$]. Meantime "b" goes to 6 [$K_+(6,2)$], absorbs the quantum ($\gamma_{b\mu}$) and proceeds to 4 [$K_+(4,6)$]. The quantum meanwhile proceeds from 5 to 6, which it does with amplitude $\delta(s_{56}^2)$. We must sum over all the possible quantum polarizations and positions and times of emission 5, and of absorption 6. Actually, if $t_5>t_6$, it would be better to say that "a" absorbs and "b" emits but no attention need be paid to these matters, as all such alternatives are contained in equation [(14.3)].'[9]

Figure 14.1 shows the simplest Feynman diagram of the first order (in $e^2/\hbar c$)

process, which describes the exchange of one photon between two electrons. According to Feynman's rules, given above, the expression (14.3) corresponds to this process, and it gives the corresponding quantum amplitude for the described process in the coordinate representation. This was indeed one of Feynman's most important inventions, namely, the visualization of fundamental processes. This achievement of Feynman's was extremely useful in understanding and developing quantum field theory in simple and natural terms, rather than only in abstract mathematical ones.

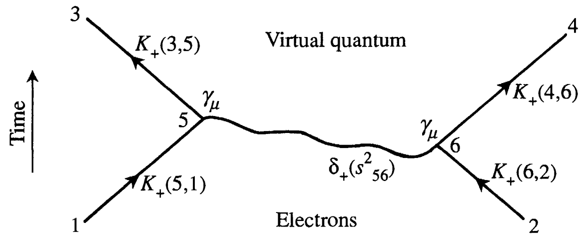


Fig. 14.1. The fundamental interaction equation (14.3). Exchange of one quantum between two electrons.

Feynman proceeded to explain the significance of equation (14.3) further: 'Although in the expression stemming from [(14.3)] the quanta are virtual, this is not actually a theoretical limitation. One way to deduce the correct rules for real quanta from [(14.3)] is to note that in a closed system all quanta can be considered as virtual (i.e. they have known sources and are eventually absorbed) so that in such a system the present description is complete and equivalent to the conventional one. In particular, the relation of Einstein's A and B coefficients can be deduced. A more practical direct deduction of the expressions for real quanta will be given in a subsequent paper. It might be noted that [(14.3)] can be rewrtten as describing the action on a,

$$K^{(1)}(3, 1)=i\int K_+(3, 5)A(5)K_+(5, 1)\, d\tau_5, \tag{14.4}$$

of the potential

$$A_\mu(5)=e^2\int K_+(4, 6)\, \delta_+(s_{56}^2)\gamma_\mu K_+(6, 2)\, d\tau_4 \tag{14.5}$$

arising from Maxwell's equations $-\Box^2 A_\mu=4\pi j_\mu$ from a "current"

$$j_\mu(6)=e^2K_+(4, 6)\gamma_\mu K_+(6, 2)$$

produced by particle b in going from 2 to 4. This is by virtue of the fact that $-\Box\delta_+$ satisfies $-\Box^2\,\delta_+(s_{21})=4\pi\,\delta(2, 1)$.'[10]

The last form of the fundamental interaction (equations (14.4) and (14.5)), given by Feynman as a footnote, is more general than the form (14.3), and allows one to describe the interaction of the Dirac particles also with external electromagnetic fields.

In this section Feynman also discussed the exclusion principle, which turned out to work exactly as in the case of noninteracting particles described in I (Ref. 2) (see Section 13.5), as well as the influence of the Bose statistics of the quanta on the results.

14.3 The self-energy problem

Feynman first used his new technique to calculate the self-energy of the electron to the first-order in perturbation theory in powers of the constant $e^2/\hbar c$. He wrote: 'Having the term representing the mutual interaction of a pair of charges, we must include similar terms to represent the interaction of a charge with itself. For under some circumstances what appears to be two distinct electrons may, according to I, be viewed also as a single electron (namely in case one electron was created in a pair with a positron destined to annihilate the other electron). Thus to the interaction between such electrons must correspond the possibility of the action of an electron on itself.'[10]

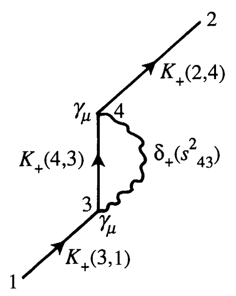


Fig. 14.2. Interaction of an electron with itself.

The corresponding first-order diagram in configuration space is shown in Fig. 14.2. To this diagram corresponds, according to the Feynman rules, the following expression for the amplitude in the first order of perturbation theory:

$$K^{(1)}(2, 1) = -ie^2 \iint K_+(2, 4)\gamma_\mu K_+(4, 3)\gamma_\mu K_+(3, 1)\, d\tau_3\, d\tau_4\, \delta_+(s_{43}^2). \quad (14.6)$$

We can connect this expression with the self-energy of the free electron if we calculate the diagonal matrix element of this amplitude between the electron states $f = g$, which are positive energy solutions of the Dirac equation for the free electron; hence, they may be represented as $u \exp(-ip \cdot x)$, where p is the energy (p_4) and momentum ($\mathbf{p}$) of the electron ($p^2 = m^2$), and u is a constant 4-index symbol (a spinor, in modern terminology). If the wave functions are normalized to unit volume, the matrix element of the amplitude (14.6) between such states gives the first-order correction, $i(\Delta E)(t_2 - t_1)$, to the factor $\exp[-i(\Delta E)(t_2 - t_1)]$ in the amplitude for arrival in state $f(2)$ at time instant t_2, starting from the state $f(1)$ at time instant t_1. Thus the whole effect is equivalent to a change of the energy ΔE, given by the expression

$$\Delta E = e^2 \int (\bar{u}\gamma_\mu K_+(4, 3)\gamma_\mu u) \exp(ip \cdot x_{43})\, \delta_+(s_{43}^2)\, d\tau_4. \tag{14.7}$$

Similarly, one can obtain an expression for the energy shift in the hydrogen atom. For this purpose, one needs to replace the amplitude for the free electron K_+ in formula (14.7) with the corresponding amplitude K_+ of the electron in the potential $V = \beta e^2/r$ of the atom, and the free state f by a wave function (of space and time) for an atomic state. The real evaluation of expressions like (14.7) can be performed more easily by the technique which Feynman described next.

14.4 Expression in momentum and energy space: the Feynman diagrams

The calculation of matrix elements is most simple in the energy and momentum representation. The reason is quite simple. In this representation all Hamiltonian equations for free particles, like the Dirac equation, the Klein–Gordon equation, and Maxwell's equations, transform into algebraic equations which have very simple solutions. Taking into account the formula (13.32) for the Fourier transform of the K_+ function, the formula (13.33) for the Fourier transform of the electromagnetic four-potential, and the formula for the Fourier transform of the δ_+-function,

$$-\delta_+(s_{21}^2) = \pi^{-1} \int \exp(-ik \cdot x_{21}) k^{-2}\, d^4k, \tag{14.8}$$

we can rewrite all the expressions for the quantum amplitude, or its matrix elements, in the momentum and energy representation. The resulting formulas need to be justfied by some rules for giving unambiguous meaning to the corresponding expressions. For example, in equation (14.8) we have to understand the term k^{-2} as the limit $\varepsilon \to +0$ of $(k_\mu k_\mu + i\varepsilon)^{-1}$,[11] i.e. certain rules for going around the poles of the corresponding singular functions in the Fourier integrals are needed. Already in paper I,[2] Feynman had proved that such rules are equivalent to the proper choice of the solution of the quantum dynamical equations. For example, in equation (11.32), one can understand the function $(p-m)^{-1}$ as $(p-m+i\varepsilon)^{-1}$, with $\varepsilon\to+0$, to obtain just the Feynman propagator K_+.[12] All such rules can be expressed in a general rule, according to which the masses of all particles and quanta have infinitesimal negative imaginary parts.

Using these rules, Feynman represented the self-energy, equation (14.8), as a matrix between $\bar{u}$ and u of the matrix

$$(e^2/\pi i)\int \gamma_\mu(p-k-m)^{-1}\gamma_\mu k^{-2}\,d^4k, \tag{14.9}$$

which obviously has quite a simple form, and wrote: 'The equation [(14.9)] can be understood by imagining [see Fig. 14.3] that the electron of momentum p emits (γ_μ) a quantum of momentum k, and makes its way now with momentum $p-k$ to the next event [factor $(p-k-m)^{-1}$] which is to absorb the quantum [another (γ_μ)]. The amplitude of propagation of quanta is k^{-2}. (There is a factor $e^2/\pi i$ for each virtual quantum.) One integrates over all quanta. The reason an electron of momentum p propagates as $1/(p-m)$ is that this operator is reciprocal to the Dirac equation operator, and we are simply solving this equation. Likewise light goes as $1/k^2$, for this is the reciprocal Dalembertian operator of the wave equation of light. The first γ_μ represents the current which generates the vector potential, while the second is the velocity operator by which this potential is multiplied in the Dirac equation when an external field acts on an electron.'[12]

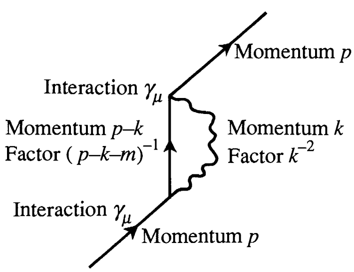


Fig. 14.3. Interaction of an electron with itself. Momentum space, equation (14.9).

These rules permit one to describe via the corresponding diagrams every process in quantum electrodynamics and to write down the matrix elements of the amplitude of this process. Today, these rules are the most well known part of Feynman's approach to quantum electrodynamics and, in general, to field theory and statistical physics. The practical usefulness of the Feynman rules and diagrams made them one of the most essential elements of the scientific training of every theoretical physicist.

As Feynman recalled many years later: '. . . The diagrams were intended to represent physical processes and the mathematical expressions used to describe them. Each diagram signified a mathematical expression. In these diagrams I was seeing things that happened in space and time. Mathematical quantities were associated with points in space and time. I would see electrons going along, being scattered at one point, then going over to another point and getting scattered there, emitting a photon and the photon goes over there. I would make little pictures of all that was going on; these were physical pictures involving the mathematical terms. These pictures evolved only gradually in my mind. There were some old pictures that were quite similar, but not as clean and as final as the diagrams I was drawing; they became a shorthand for the processes I was trying to describe physically and mathematically.

'The diagrams became very important as I began to treat more and more of these problems. They became pictorial representations of the more and more abstract things I was trying to describe. I remember that when I was at the Telluride House (at Cornell) and was working on the self-energy of the electron, there were many terms which I was trying to visualize, when it occurred to me that these pictures looked very funny. In ancient Egypt and Greece the priests and oracles used to look at the veins in sheep's livers to forecast the future, and that's the kind of pictures I was drawing to describe physical phenomena. I thought that if they really turn out to be useful it would be fun to see them in the pages of the *Physical Review*. I was conscious of the thought that it would be amusing to see these funny-looking pictures in the *Physical Review*.'[13]

The Feynman rules for the matrix elements in spinor electrodynamics in modern form are summarized in Table 14.1.[14] This is just Feynman's 'handbook on how to do quantum electrodynamics'.[15]

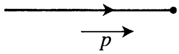
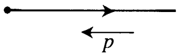
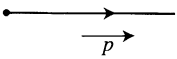
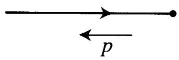
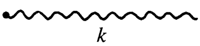
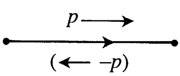
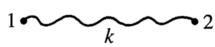
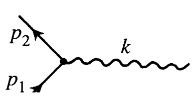
By making use of these rules, Feynman wrote down the matrix elements for a different kind of process in quantum electrodynamics. For example, the total matrix element for the Compton scattering of an electron in second-order perturbation theory is then

$$e_2(p_1+q_1-m)^{-1}e_1+e_1(p_1+q_2-m)^{-1}e_2. \qquad (14.10)$$

According to Feynman's rules this expression corresponds to two possible diagrams for Compton scattering as shown in Fig. 14.4. One has to take the matrix elements of the expression (14.10) to obtain the Klein–Nishina formula.

For the radiative corrections to the scattering of the electron in the lowest order of perturbation theory, Feynman gave three diagrams, which are shown in Fig. 14.5.[16] The three diagrams differ in the ordering of the processes of scattering of the electron, and of the emission and absorption of the photon. Feynman remarked that the expressions so obtained for various processes 'are, as has been indicated, no more than the re-expression of conventional quantum electrodynamics. As a consequence, many of them are meaningless. For example, the self-energy expression [(14.7)] or [(14.9)] gives an infinite

Table 14.1. Feynman's Rules

	Element of the Feynman's diagram	Factor in the matrix element
1	Electron in the initial state with momentum p 	$(2\pi^{-3/2}u^{s,-}(p)$
2	Positron in the initial state with momentum p 	$(2\pi^{-3/2}\bar{u}^{s,-}(p)$
3	Electron in the final state with momentum p 	$(2\pi^{-3/2}\bar{u}^{s,+}(p)$
4	Positron in the final state with momentum p 	$(2\pi^{-3/2}u^{s,+}(p)$
5	Photon in the initial or final state with polarization e_ν and momentum k 	$\frac{e_\mu^\nu}{(2\pi)^{3/2}\sqrt{2k_0}}$ $(\nu \neq 0)$
6	Motion of an electron from 1 to 2 (or of a positron from 2 to 1) 	$\frac{1}{(2\pi)^4 i}\int d^4p\,\frac{m+p}{m^2-p^2-i\varepsilon}$
7	Motion of a photon between vertices with summation indices μ and ν 	$\frac{q^{\mu\nu}}{(2\pi)^4 i}\int d^4k\,\frac{1}{k^2+i\varepsilon}$
8	Vertex with summation index ν with electron line p_1 and photon line k incoming, and electron p_2 outgoing 	i.e. $\gamma^\nu(2\pi)^4\delta(p_2-p_1-k)$

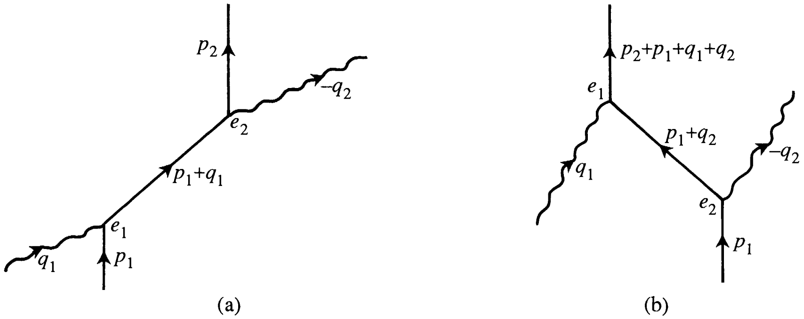


Fig. 14.4. Compton scattering, equation (14.10).

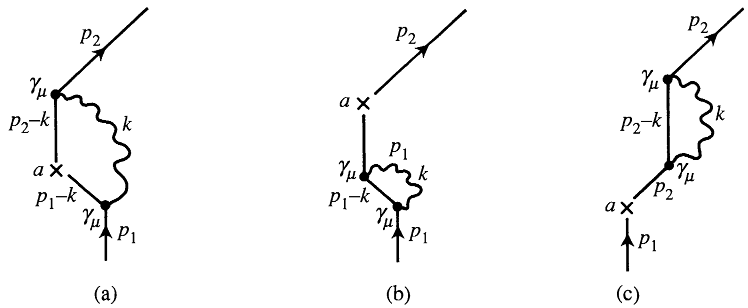


Fig. 14.5. Radiative correction to scattering, momentum space.

result when calculated. The infinity arises, apparently, from the coincidence of the δ-function singularities in $K_+(4,3)$ and $\delta(s_{43}^2)$. Only at this point is it necessary to make a real departure from conventional electrodynamics, a departure other than simply rewriting expressions in a simpler form.'[16]

In order to overcome the difficulties with the divergences connected with virtual quanta, Feynman made use of the regularization procedure which he had invented earlier[5, 17] (see Sections 13.1 and 13.2). Using this procedure with a little modification, he obtained for the expression (14.10), which gives the self-energy of the free electron, the result,

$$(e^2/2\pi)\{4m[\ln(\lambda/m)+\tfrac{1}{2}]-p[\ln(\lambda/m)+\tfrac{5}{4}]\}, \qquad (14.11)$$

up to terms in higher order of the ratio λ/m of Feynman's cut-off parameter λ and the mass m of the electron. When applied to the state of the free electron with momentum p, satisfying the Dirac equation, the equation (14.7) gives the change of the mass,

$$\Delta m = m(e^2/2\pi)\,[3(\ln(\lambda/m)+\tfrac{3}{4}]. \qquad (14.12)$$

For the radiation corrections to the scattering, the sum of three terms (which corresponds to the three Feynman diagrams in Fig. 14.5) leads [for small momentum transfer q ($\sqrt{q^2}=2m \sin \theta$, θ being the scattering angle)] to the result:

$$(e^2/2\pi)\left\{\frac{1}{2m}(qa-aq)+\frac{4q^2}{3m^2}a\left[\ln\left(\frac{m}{\lambda_{\min}}\right)-\frac{3}{8}\right]\right\}. \qquad (14.13)$$

Here a is the amplitude of the four-dimensional electromagnetic potential $A=A_\mu\gamma_\mu$, which is supposed to be of the form $a \exp(-iqx)$, and $\lambda_{\min}$ is the small mass of the proton: $\lambda_{\min}<m<\lambda$, which has to be introduced to avoid infrared divergences, according to Bloch and Nordsieck (see Section 11.4). This formula shows the change of the magnetic moment of the electron in accordance with Schwinger's result, and the Lamb shift as was interpreted in greater detail by Feynman previously[17] (see Section 13.2). It is remarkable that the result (14.13) does not depend on the regularization procedure at all, assuming that the mass m is the experimental mass of the electron.

Feynman then discussed some of the difficulties of his regularization procedure, which is good enough only in the limit when λ goes to infinity, after mass renormalization. He concluded: 'I have no proof of the mathematical consistency of this procedure, but the presumption is very strong that it is satisfactory.'[18]

14.5 The problem of vacuum polarization

In his article on the 'Space–time approach to quantum electrodynamics', Feynman published for the first time his point of view on the problem of vacuum polarization. The problem arises, for instance, in the analysis of the radiative corrections to scattering, where one type of term was not considered until now. The potential, on which the electron scatters, was assumed to vary as $a_\mu \exp(ip\cdot x)$, and the direct scattering on this external potential—in the lowest order of perturbation theory—corresponds to the diagram shown in Fig. 14.6(a). This diagram is a part of the diagrams shown in Fig. 14.5. But the same potential may create an electron–positron pair (see Fig. 14.6(b)) which then reannihilates, emitting a quantum with momentum $q=p_a-p_b$. This quantum scatters the original electron from state 1 to state 2.

The matrix element connected with the diagram for the process indicated in Fig. 14.6(b) is given by the expression

$$-(e^2/\pi i)\,\bar{u}_2\gamma_\mu u_1)\int S_p[(p_a+q-m)^{-1}\gamma_\nu(p_a-m)^{-1}\gamma_\mu]q^{-2}a_\nu\, d^4p, \qquad (14.14)$$

where no regularization has been made. One can imagine that the closed loop of the electron–positron pair is equivalent to the current

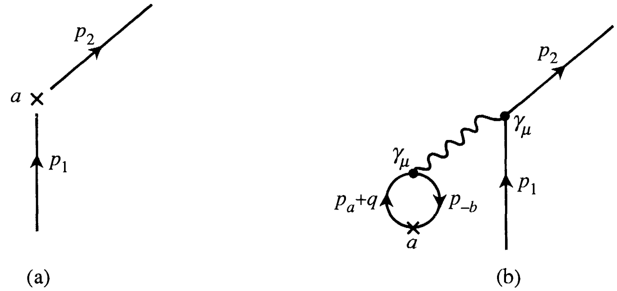


Fig. 14.6. (a) Direct scattering of the electron; (b) vacuum polarization effect on scattering, equation (14.14).

$$4\pi j_\mu = J_{\mu\nu} a_\nu, \tag{14.15}$$

which produces the quantum acting on the electron. The quantity $J_{\mu\nu}$, characteristic of the vacuum polarization problem in the lowest order of perturbation theory, is then given by

$$J_{\mu\nu} = -(e^2/\pi i)\,(\bar{u}_2\gamma_\mu u_1)\int S_\mathrm{p}[(p_a+q-m)^{-1}\gamma_\nu(p_a-m)^{-1}\gamma_\mu]\,d^4p. \tag{14.16}$$

The integral in equation (14.16) is badly divergent. The use of the Feynman regularization was not able to lead to a convergent result here, because in equation (14.16) there is no photon propagator which could be modified by such a procedure. In his 1948 paper,[17] Feynman had proposed an analogous modification of the electron propagator. Although it does lead to the convergence of the expression (14.16), this procedure has another difficulty: it spoils the gauge invariance and, thereby, the corresponding fundamental law of conservation of the electric charge and the current j_μ. In the momentum and energy representation the conservation of the electric current leads to the condition

$$q_\mu J_{\mu\nu} = 0 \tag{14.17}$$

for the symmetric quantity $J_{\mu\nu} = J_{\nu\mu}$, which turned out to be broken under the Wataghin-like renormalization of the electron propagator, when Feynman had first tried to overcome the vacuum polarization divergence (see Section 13.2).

Now, however, this problem could be treated because ‘A method of making [(14.16)] convergent without spoiling the gauge invariance has been found by Bethe and by Pauli. The convergence factor for light can be looked upon as the result of superposition of the effects of quanta of various masses (some contributing negatively). . .. (If) the quantity [(14.14)], integrated over some finite range of p, is called $J_{\mu\nu}(m^2)$ and the corresponding quantity over the same range of p, but with m replaced by $(m^2+\lambda^2)^{1/2}$ is $J_{\mu\nu}(m^2+\lambda^2)$ we should calculate

$$J^p_{\mu\nu} = \int_0^\infty [J_{\mu\nu}(m^2) - J_{\mu\nu}(m^2+\lambda^2)]G(\lambda)\, d\lambda, \tag{14.18}$$

the function $G(\lambda)$ satisfying the conditions $\int_0^\infty G(\lambda)\, d\lambda = 1$ and $\int_0^+ G(\lambda)\lambda^2\, d\lambda = 0$. Then in the expression for $J^p_{\mu\nu}$ the range of the p integration can be extended to infinity as the integral now converges. The result of the integration using this method is the integral on $d\lambda$ over $G(\lambda)$ of

$$J^p_{\mu\nu} = \frac{e^2}{\pi}(q_\mu q_\nu - \delta_{\mu\nu} q^2)\left\{\tfrac{1}{3}\ln\left(\frac{\lambda^2}{m^2}\right) - \left[\frac{4m^2+2q^2}{3q^2}\left(1 - \frac{\theta}{\tan\theta}\right) - \tfrac{1}{9}\right]\right\}, \tag{14.19}$$

with $q^2 = 4m^2 \sin\theta$.

'The gauge invariance is clear, since $q_\mu(q_\mu q_\nu - q^2\,\delta_{\mu\nu}) = 0$. Operating (as it always will) on a potential of zero divergence, the (quantity) $(q_\mu q_\nu - q^2\,\delta_{\mu\nu})a_\nu$ is simply $-q^2 a_\mu$, the Dalembertian of the potential. The term $-\frac{1}{3}[\ln(\lambda^2/m^2)]\ (q_\mu q_\nu - q^2\,\delta_{\mu\nu})$ therefore gives a current proportional to the current producing the potential. This would have the same effect as a *change in a charge*, so that we would have a difference $\Delta(e^2)$ between e^2 and the experimentally observed $e^2+\Delta(e^2)$, analogous to the difference between m and observed mass. This charge depends logarithmically on the cut-off, $\Delta(e^2)/e^2 = -(2e^2/3\pi)\ln(\lambda/m)$. After this renormalization of charge is made, no effects will be sensitive to the cut-off.

'After this is done the final term remaining in [(14.12)] contains the usual effects[19, 20] of polarization of the vacuum. . ..

'Closed loops containing a number of quanta of potential interactions larger than two produce no trouble. Any loop with an odd number of interactions gives zero (as noted in I). Four or more potential interactions give integrals which are convergent even without a convergence factor, as is well known. The situation is analogous to that for self-energy. Once the simple problem of a single closed loop is solved there are no further divergence difficulties for more complex processes.'[21]

This was the complete resolution of the renormalization problem. In Dyson's paper[22] it was proved that, in higher-order terms no new divergences occurred, and all divergences may be connected with either the mass or the charge renormalization. Thus the full renormalization program had been realized.

While discussing the problem of higher-order terms dealt with in the paper on the 'Space–time approach to quantum electrodynamics', Feynman looked for the following remark he had made in it: 'The methods of calculation given in this paper are deceptively simple when applied to lower order processes. For processes of increasingly higher orders the complexity and difficulty increases rapidly and these methods soon become impractical in their present form.'[1]

'An honest man, you know! I'm proud of doing things in a clean and honest fashion. I think you should always point out where your thing is good and where it stops and is no good.'[13]

14.6 Some other results and concluding remarks

In the following sections of the article, Feynman gave several new results. First, he gave a discussion of the unique treatment of longitudinal and transverse waves in the general case in quantum electrodynamics. The relativistic invariance of the notion of 'unpolarized light' was derived on this basis, and new rules for calculating the cross sections for such light were given. In the next section, Feynman considered scalar charged particles obeying the Klein–Gordon equation and extended the application of the new methods to these particles. The self-energy of such particles of momentum P_μ was written in the form

$$(e^2/2\pi im)\int[2p-k)_\mu((p-k)-m)^{-1}(2p-k)_\mu-\delta_{\mu\mu}]k^{-2}C(k)\,d^4k \qquad (14.20)$$

with some regularization factor $C(k)$. The integral without this factor is quadratically divergent, and therefore $C(k)$ must satisfy stronger conditions than in the case of Dirac particles.

Feynman showed that the lowest order of contribution of a closed loop to vacuum polarization, which, for particles obeying the Bose statistics has an opposite sign, gives

$$J^p_{\mu\nu}=\frac{e^2}{\pi}(q_\mu q_\nu-\delta_{\mu\nu}q^2)\left\{\tfrac{1}{6}\ln\left(\frac{\lambda^2}{m^2}\right)-\left[\frac{4m^2-q^2}{3q^2}\left(1-\frac{\theta}{\tan\theta}\right)+\tfrac{1}{9}\right]\right\}, \qquad (14.21)$$

using the same notation as in equation (14.19).

The radiative corrections to scattering after mass renormalization are sensitive to the cut-off just as for Dirac particles.

In the last section, dealing with the meson theories, Feynman wrote: 'The theories which have been developed to describe mesons and the interaction of nucleons can be easily expressed in the language used here. Calculations, to the lowest order in the interactions can be made very easily for the various theories, but agreement with experimental results is not obtained. Most likely all of our present formulations are quantitatively unsatisfactory. We shall content ourselves therefore with a brief summary of the methods which can be used.'[23]

Feynman then described different processes with nucleons and mesons of different kind (scalar mesons, pseudoscalar and pseudovector coupling, etc.). In several appendices at the end of the article, Feynman demonstrated the details of the calculation of integrals that occurred, the calculation of the self-energy of the electron, and more complex processes of higher order in quantum electrodynamics: higher-order corrections to the Møller scattering, to the Compton scattering, and the interaction of the neutron with an electromagnetic field by virtue of the fact that the neutron may emit virtual negative mesons. Here Feynman described the results which he had obtained during the 1949 APS meeting in connection with Slotnick's calculations (see Section 13.3).

Feynman's paper on the 'Space–time approach to quantum electrodynamics' was truly fundamental in that it had a great influence on the development of quantum electrodynamics, quantum field theory, and statistical physics—in which the same methods were adopted later, and on physical thinking as a whole in the postwar period. The initial reactions of the physicists to this paper were described by Feynman himself as follows:

'And I went out into the world; [the paper] was published. At that time, Schwinger had invented a method. Now he could explain his method better because it was closer to the normal, it was very ingenious and wonderful, but it was a little closer to the conventional ways of thinking, so he had it easier. Great difficulties, but [still] he had it easier [in] explaining where it came from. So at the beginning, when people would write down problems, things on quantum electrodynamics, it would be an eight-page paper on some problem. And somewhere around the sixth or seventh page of grinding formalism, they would write something down and [then] they would say this is what you would write immediately according to the intuitive methods of Feynman. That's all they would say. And then they would go and solve the problem and there would be integral tricks, and so on. But to get to that they went through a lot of pages of algebraic shenanigans with psi's and psi-bars, and they would sort of always seem to be surprised that somehow I was already there . . . . These methods were not intuitive; they were hard work. It's just that I hadn't published. So, it was a bit unfortunate that [until then] I had not published where it came from, but still in a way fortunate because it was a long ways around. This went on for a while. And then [came my] first paper. When you wrote a paper you said, "We want to study this problem. According to Feynman's principles, the formula for this is so and so." After that, more people had the courage to start right out trusting that it would work. Dyson proved that [all three theories of Feynman, Tomonaga, and Schwinger] were equivalent.' [24]

At the end of November 1948, David M. Dennison had invited Richard Feynman to give a series of nine lectures, beginning 11 July 1949 (three lectures per week) on his space–time formulation of quantum mechanics and quantum electrodynamics at the Symposium on Modern Physics at the University of Michigan summer school.[25] Feynman gladly accepted this invitation and gave lectures on his 'work and experience in the field of quantum electrodynamics', which Dennison wanted him to do. By the summer of 1949, Feynman's work in this field had been completed and he gave a full report on it at the symposium. The other speakers at the symposium in Ann Arbor were Gregor Wentzel, Bruno Rossi, Frederick Seitz, and Louis Alvarez. In the summer of 1948 Julian Schwinger had given a series of lectures on his formulation of the theory of quantum electrodynamics.

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