Total Angular Momentum: HW2

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1. Show that, when you add spin $\frac{1}{2}$ and spin 1, you obtain six states, namely: a quartet of four states with total angular momentum $\frac{3}{2}$, and a doublet of states with total angular momentum $\frac{1}{2}$. Show this by constructing the multiparticle raising and lowering operators S_+ and S_-, and the multiparticle S^2 operator, from the single-particle operators and showing how/why/that it works.

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2. Consider an electron in the ψ_{311} state in a hydrogen atom. If the electron is spin up and if you measure the total angular momentum squared of the electron alone, what are the possible values that you could obtain, and with what probabilities would you obtain them? If the electron is spin down and if you measure the total angular momentum squared of the electron alone, what are the possible values that you could obtain, and with what probabilities would you obtain them?

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3. Consider an electron in a hydrogen atom which is in the combined spin and position state

$$\left(\sqrt{\frac{1}{5}}\ R_{32}\ Y_{2-1}\ \chi_+ \ +\ \sqrt{\frac{4}{5}}\ R_{43}\ Y_{3-2}\ \chi_- \right).$$

If you measure the following quantitites, what are the possible values that you could obtain, and with what probabilities would you obtain them?

(a) L^2

(b) L_z

(c) S^2

(d) S_z

(e) J^2

(f) J_z

(g) the energy

Now consider two of the continuous probability distributions:

(h) If you measure the position of the electron without determining its spin, what is the probability density as a function of $r,\ \theta,$ and ϕ?

(i) If you measure both the z component of the spin and the distance from the origin, what is the probability density for finding the electron with spin up at radius r?
