Spin: Homework Assignment 1

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1. Consider an electron in the spin state

$$\chi = N \begin{pmatrix} 2+3i \\ 3+4i \end{pmatrix}.$$

(a) Determine the normalization constant N.

(b) Find the expectation values of $S_x,\ S_y,$ and S_z.

(c) Find the standard deviations $\Delta S_x,\ \Delta S_y,$ and ΔS_z.

(d) Confirm that your results are consistent with all three uncertainty principles,

$$\Delta S_i \Delta S_j \geq \frac{\hbar}{2} \left| < S_k > \right|,$$

where the indices $i,\ j,$ and k represent the cyclic permutations of $x,\ y,$ and z.

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2. Consider the S_x operator for spin 1/2

$$S_x = \frac{\hbar}{2} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}.$$

(a) Find the eigenvalues and the eigenvectors of the S_x operator.

(b) If you measure S_x on a particle in the general state χ given in Solved Problem 2, what values could you obtain, and with what probabilities would you obtain them? Check that the probabilities add up to 1.

(c) If you measure S_x^2, what values could you obtain, and with what probabilities would you obtain them?

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3. Consider an electron at rest in a uniform magnetic field $\vec{B}_0 = B_0\ \hat{z}$. At $t = 0$, the spin is pointing in the $+\hat{y}$ direction, *i.e.*, $< S_y(t = 0) > = +\hbar/2$. Calculate the expectation value $< \vec{S}(t) >$ for all times t.

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4. Consider an electron at rest in the oscillating magnetic field

$$\vec{B} = B_0\ \cos(\omega t)\ \hat{z},$$

where B_0 and ω are constants.

(a) Construct the Hamiltonian matrix for this system.

(b) The electron starts out at $t = 0$ in the spin up state with respect to the y-axis. Determine $\chi(t)$ at all subsequent times by solving the TDSE.

(c) Calculate the probability of getting $-\hbar/2$ for a measurement of S_y at time t.

(d) Calculate the minimum value of B_0 required to force a complete "spin-flip" in S_y.

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5. Consider another spin 1/2 system.

(a) Find the eigenvalues and eigenvectors of the operator $O = S_x + S_y + S_z$.

(b) Suppose that a measurement of O is made, and the system is found to be in the state $\mid \alpha >$ that corresponds to the larger eigenvalue. What are the possibilities and probabilities for an immediately following measurement of S_z?

(c) Find, if possible, the direction $\mathbf{n}$ in which the spin measurement will with certainty yield the value $S_n = \hbar/2$.

See Schaum's problem 7.4.

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6. Consider a particle with spin 1/2.

(a) What are the eigenvalues and eigenvectors of S_x, S_y, and S_z?

(b) Consider a particle in the $+S_y$ eigenstate. What are the possibilities and the probabilities if we measure S_z?

(c) The particle is in a magnetic field and its Hamiltonian is $H = (eB/mc)\, S_z$. At $t = 0$ the particle is in the $-S_y$ eigenstate; find $\chi(t)$ for $t \geq 0$.

(d) If we measure S_y at $t = t_1$, what are the possibilities and probabilities? If we measure S_z at $t = t_1$, what are the possibilities and probabilities? If we measure S_x at $t = t_1$, what are the possibilities and probabilities? Explain the differences in the t_1 dependences.

(e) Calculate the expectation values of S_x, S_y, and S_z at time t_1.

See Schaum's problem 7.5.
