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## Resource Letter QEDV-1: The QED vacuum

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(Received 17 July 2007; accepted 9 November 2007)

This Resource Letter provides a guide to the literature on vacuum structures and their effects in Quantum Electrodynamics. References to books and journal articles that deal with perturbative vacuum effects as well as nonperturbative processes in extremely strong external fields like spontaneous positron production in strong Coulomb fields are provided. © 2008 American Association of Physics Teachers.

[DOI: 10.1119/1.2820395]

### I. INTRODUCTION

The history of quantum electrodynamics (QED) is an impressive success story of theoretical and experimental physics. Since its development beginning in the 1920s by the leading theorists of that time, QED has achieved incredibly accurate experimental verification as exemplified by the agreement of theory and experiment in the electronic ( $g-2$ ) experiments, measuring the deviation of the magnetic moment of the electron from the value that follows from the Dirac equation, which have achieved an experimental and theoretical accuracy of about one part in 200 million.

As the extension of the quantum-mechanical description of a relativistic single-particle system to a many-body state in terms of a quantized field theory, quantum electrodynamics has developed through various steps and formulations with crucial work by Dirac, Heisenberg, Pauli, Jordan, and others (see Refs. 9–20). Following the first formulations, the discussion on how to regulate the occurring divergences followed throughout the 1930s and 1940s, including seminal papers by Weisskopf, Heisenberg, Dirac, Bethe, Tomonaga, Schwinger, and Feynman.

The richness of phenomena arising in relativistic quantum theory can be anticipated already in Dirac's equation of a relativistic spin-1/2 particle (Refs. 9, 10). The existence of positive and negative-energy eigenvalues extends directly to the many-particle concept of two continua of particles and their corresponding antiparticles. The formal solution for the multitude of available states with arbitrarily large (negative) energy came when Dirac postulated that in the vacuum state the lower continuum is filled with particles (Ref. 17) (al-

though originally postulating the proton, instead of the later-discovered positron, as the lower-continuum counterpart to the electron). A year earlier, in 1929, Klein calculated the scattering of an electron from a potential barrier with a height that exceeds twice the electron's rest mass (Ref. 16). He found large reflexion and penetrability factors (the so-called Klein paradox), which originate from the direct coupling of the positive and negative-energy continua in such an external potential.

In a general sense, most of the activities related to the QED vacuum can be derived from these basic concepts and discoveries.

The vacuum of noninteracting particles is free of excitations, consisting of, in Dirac's picture, an empty positive-energy continuum and a filled lower continuum. Switching on the interaction, the state is perturbed, leading to virtual excitations of this state. Here the basic process is the vacuum polarization, creating a virtual electron-positron pair by exciting an electron from the lower to the upper continuum with subsequent deexcitation. These perturbations affect all quantities calculated in QED, like the well-measured leptonic anomalous magnetic moment and atomic Lamb shift (the energy difference between atomic  $2s_{1/2}$  and  $2p_{1/2}$  states, which vanishes when one does not take into account vacuum fluctuations).

Integrating out the fermions in QED, the vacuum-polarization effects lead to effective interaction terms that are nonlinear in the electric and magnetic fields, generating among other phenomena light-by-light scattering (Refs. 12, 13). Vacuum-polarization effects can generate not only virtual but real particles, which is the many-body consequence of the Klein paradox. This can happen in extremely strong electromagnetic fields that might even be experimentally ac-

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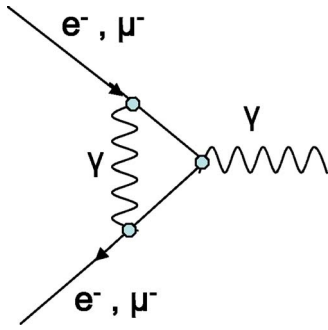


Fig. 1. Biggest contribution to the leptonic anomalous magnetic moment. A virtual photon is exchanged across the vertex that couples the electromagnetic field to the lepton.

cessible in future high-power laser facilities (Refs. 99–102) as well as in the strong Coulomb fields during a heavy-ion collision leading to so-called spontaneous positron production, or in other hypothetical external fields.

In principle, instead of investigating strong external fields, on a theoretical level one can artificially increase the fine-structure constant  $\alpha$  to study QED at large coupling constants, which can lead to complex nonperturbative vacuum structures of QED. Such studies can be performed using so-called Schwinger-Dyson-type equations or numerical simulations of QED on the lattice.

Another way to consider the influence of the vacuum is to change boundary conditions, which affects the QED vacuum properties as had been shown originally and very elegantly by Casimir, leading to the eponymous effect.

## JOURNALS

Vacuum effects in QED are investigated over a wide range of fields in physics, encompassing such diverse topics as state-of-the-art laser research and the study of basic quantum field theory. Therefore the articles spread over quite a range of journals. However, most of them, in fact the majority of the ones listed in the bibliography below, can be found in the following journals:

*Journal of Physics G*  
*Nuclear Physics B*  
*Physics Letters B*  
*Physics Reports*  
*Physical Review A*  
*Physical Review C*  
*Physical Review D*  
*Physical Review Letters*

## II. VACUUM FLUCTUATIONS IN PERTURBATIVE QED

The coupling of vacuum fluctuations to the interaction of an electron with an external static magnetic field yields a shift of the standard value of the leptonic gyromagnetic factor  $g=2$  as obtained in the corresponding single-particle Dirac equation. The theoretical and experimental determination of these fluctuations, that is, of the anomalous magnetic moment of the electron,  $a_e = \frac{g_e - 2}{2}$ , have become a test case for

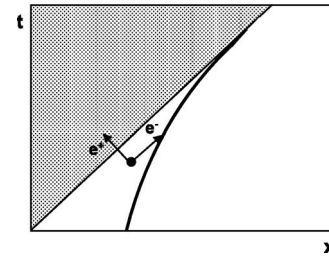


Fig. 2. A constantly accelerating object travels on a hyperbolic world line. Signals from the darkened area cannot reach an observer traveling with the object. A slight energy transfer from the object to vacuum fluctuations can generate, for instance, an electron-positron pair that is emitted in opposite directions. One of them will enter the forbidden region, whereas the other one might interact with the object generating the heat bath with the Unruh temperature—see (Ref. 78) for a more extended version of the argument.

the understanding of perturbative QED contributions, starting from the analysis of the first measurements that showed a deviation of the electron's  $g$  factor from its Dirac value of 2 in 1947 (Ref. 30). The first correction to this value comes from the exchange of a virtual photon across its vertex (see Fig. 1), followed by more and more complex diagrams including leptonic—and to a much suppressed degree—hadronic vacuum polarizations. The techniques of calculating the value of  $g-2$  to the maximum accuracy have been perfected over the decades to an art form by Kinoshita and his group (Ref. 37). Their most recent publications show the determination of the electronic  $g-2$  by calculating the remarkable number of 891 Feynman diagrams up to order  $\alpha^4$ ; work on the next order involving more than 12,000 diagrams is underway. Similarly, the experimental efforts have recently improved significantly using a Penning trap and one-electron cyclotron techniques to study the electron's magnetic moment (Ref. 53). The analogous value for the muon, owing to its larger mass, contains a much larger contribution from hadronic virtual states beyond pure QED. The current value for  $a_\mu$  shows a difference of 2.6 standard deviations to the theoretical value. Whether this discrepancy originates from new physics remains an open question. Relevant articles are listed in (Refs. 29–59).

## III. THE UNRUH EFFECT, NONLINEAR ELECTRODYNAMICS, AND HIGH-INTENSITY ELECTROMAGNETIC FIELDS

A significant effect of the QED vacuum can be studied simply by considering an accelerated observer (Refs. 60–78). As was worked out by Unruh in 1976 (Ref. 60), owing to acceleration through the vacuum (in contrast to a constant velocity that can be removed by switching to the appropriate Lorentz frame), an accelerated observer experiences a heat bath of particles (the vacuum) with a thermal distribution at a temperature  $kT = \frac{\hbar g}{2\pi c}$ , where  $g$  is the acceleration of the object in the proper frame (see Fig. 2). This so-called Unruh radiation is directly related to the Hawking radiation that is emitted from a black hole with temperature  $kT = \frac{\hbar c^3}{8\pi GM}$ , where now, following the correspondence of a gravitational field with an accelerated observer, the gravitational acceleration of the black hole replaces the Unruh acceleration. In fact, the Unruh effect was described in an analysis of Hawking's original paper (Ref. 63). This effect is one of the most intriguing results of the motion through a complex vacuum.

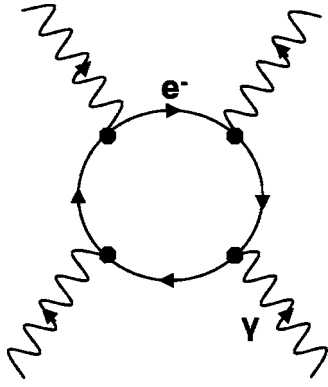


Fig. 3. Photons can interact with each other through vacuum polarization.

The effect is small, though. For normal free-fall acceleration of  $g \sim 9.8 \text{ m/s}^2$ , the temperature of the vacuum experienced is about  $4 \cdot 10^{-20} \text{ K}$ ! However, thanks again to improving experimental techniques, proposals have been developed to measure the effect (Ref. 64) using ultrastrong lasers for accelerating electrons and observing the corresponding photon emission in the laboratory frame, which has to be distinguished from the background of classical Larmor radiation.

In classical electrodynamics there is no photon-photon coupling term. However, owing to the coupling of the photon field to the polarization of the vacuum, in lowest order an electron-positron loop, photons can interact with each other on a quantum level. This was recognized early and was already worked out by Sauter, Heisenberg, Euler, and Kockel in the case of constant electromagnetic fields. The so-called 1-loop effective interaction Lagrangian, generated by processes as shown in Fig. 3, reads (Refs. 12–14, 16, 21–25)

$$\mathcal{L}_{vac} = \frac{2\alpha^2}{45m_e^4} [(B^2 - E^2)^2 + 7(\vec{E} \cdot \vec{B})^2 + \dots]. \quad (1)$$

Many works have extended this calculation to higher orders in the loop expansion and including varying fields.

Following this result, one intriguing approach to study the QED vacuum, and hypothetical physics beyond that, is to consider the vacuum as a complex medium in which light propagates. One vacuum effect arises when photons travel through the vacuum when perturbed by an external magnetic field. In such a situation the vacuum acts as a birefringent medium. The index of refraction depends upon the polarization of the light, such that a linearly polarized photon beam will acquire a small elliptical polarization after passing through the region with the magnetic field. Again, electron-positron loops coupling to the incoming and outgoing photon as well as to the photons of the magnetic field are responsible for this effect. Current experimental efforts by the PVLAS

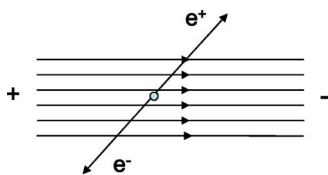


Fig. 4. A constant electric field generates a linear potential for charged particles. A sufficiently strong and extended field can generate real lepton pairs from the vacuum, see also Fig. 5.

collaboration using linearly polarized light traveling through a strong magnetic field not only show ellipticities but also dichroism (rotation of the polarization) that is beyond Eq. (1) and point, if correct, to physics beyond pure QED (Ref. 79). Several theoretical efforts exist to reconcile the results with so far unsuccessful axion searches that in principle could be the source of the dichroism. Improved experiments that search for the original QED effect are under discussion. Literature is given in (Refs. 79–91).

#### IV. THE QED VACUUM IN A STRONG EXTERNAL FIELD

The case of a constant electric field not only contains the virtual effect of the electron-positron vacuum polarization and the induced photon-photon coupling; it also can generate  $e^+e^-$  pairs based upon Klein’s original result (Fig. 4). The pair-production rate per volume  $w(E)$  for a constant field strength  $E$ , following the result in (Ref. 23) reads

$$w(E) = eE \int \frac{d^2k_{\perp}}{(2\pi)^2} \sum_{n=1}^{\infty} \frac{1}{n} \exp\left[-\frac{\pi n(m_e^2 + k_{\perp}^2)}{|eE|}\right] \quad (2)$$

integrating over the transverse momenta  $k_{\perp}$  of the produced particles. One can read off the “critical” value of the electric field,  $E_{cr} = \pi m_e^2 / e$ , of about  $4 \cdot 10^{18} \text{ V/m}$ , which marks the onset of large pair-production rates. A number of calculations improve on this ansatz by including the back reaction of the spontaneously produced particles on the external field (Refs. 103–111). Various astrophysical scenarios exist, in which such field values might be reached at the surface of quark stars and in the formation of a black hole. For example, pair creation occurs in the so-called Kerr–Newmann geometry (Refs. 153–155), the pair-producing layer around a black hole being called dyadosphere (Ref. 156), which might be the source of the famous and still not really understood gamma-ray bursts (Refs. 158–160). The effects of similarly strong electric fields on the surface of the core of a neutron star were analyzed in (Ref. 161). Note that calculations exist that question the stability of these strong fields (Ref. 162).

One fascinating approach to make the concept of the Dirac sea visible is to study the QED vacuum in the vicinity of a strong localized electric field that can be supplied by the Coulomb field of a heavy nucleus. Using the standard formula for the lowest energy eigenvalue of the electron in a hydrogen-like atom,

$$E_{1s_{1/2}} = m_e c^2 \sqrt{1 - Z^2 \alpha^2}, \quad (3)$$

the expression for the binding energy formally becomes imaginary at  $Z > 1/\alpha$ , signaling the collapse of the electronic wavefunction at small distances. In a more realistic approach one has to take into account the finite extension of the nuclear-charge distribution. In such a calculation the electronic binding energy of the  $1s$  state will exceed  $2m_e c^2$  for a hypothetical nucleus with charge  $Z > Z_{cr} \sim 173$  (Refs. 112–116). If the  $1s$  state is not occupied the state will be filled by an electron from the lower continuum, leading to the emission of a (monoenergetic), so-called spontaneous positron. This state is also called charged vacuum although, of course, charge is not generated but the negative charge is bound to the ion. This process is closely connected to the Klein paradox and can be essentially understood by solving the single-particle Dirac equation as in Klein’s case. A charged ion with

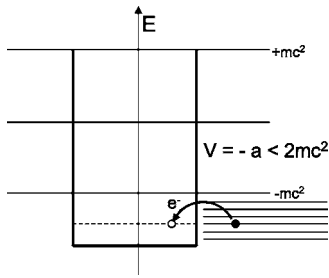


Fig. 5. Energy diagram for an electron in a simple square-well vector potential with a depth of more than two times the electron's rest mass. This allows for empty states to be filled spontaneously by electrons in the Dirac sea, leaving holes in the lower continuum that can be detected as so-called spontaneous positrons.

some large  $Z$  beyond the critical value would screen itself by production of spontaneous positrons until its total charge has dropped below  $Z=173$  (Ref. 124). An experimentally feasible way to produce an overcritical charge is to study heavy-ion collisions with a combined charge of  $Z_T+Z_P>173$ . Choosing beam energies close to the Coulomb barrier, for the case that the projectile-target system forms a long-lived ( $>10^{-20}$  s) nuclear quasi-molecule, the spontaneous positrons should appear as a distinct line structure in the measured positron spectrum in similar manner as was indicated in Fig. 5 for a simplified potential. Several groups investigated this scenario for scattering systems up to uranium on curium ( $Z=188$ ). After detecting erroneously identified positron and correlated electron-positron lines, the final conclusion was that there was no statistically significant positron-line structure related to supercriticality (Refs. 135–137). However, owing to some intermediate excitement about possible newly identified particles from the  $e^+e^-$  pairs, the original idea of finding the supercritical QED states became side-tracked. Still, the total yield of measured positrons neatly fits with calculations including the effect of positron production by filling the empty supercritical electronic states. At this point a renewed experimental effort should be started to clearly identify this exciting QED prediction experimentally by measuring the  $e^+$  line structures. Currently, several calculations show new approaches to reach the necessary long-lived nuclear quasi-molecular states (Refs. 144–146). For more information on supercritical electric fields, see (Refs. 112–152).

A number of calculations have been performed studying vacuum instabilities in very strong magnetic fields. The combination of Coulomb and magnetic forces can lead to supercritical states and dynamical chiral symmetry breaking in the vacuum as well as to the instability of positronium (Refs. 163–171).

Owing to enormous progress in the field of high-power laser facilities and the projected continued improvements, there are various efforts not only to study the photon-by-photon scattering using laser light following improved calculations based upon Eq. (1), but also reaching field strengths of the order of  $E_{cr}$  where there is real electron-positron pair production in the over-critical laser field. A number of theoretical calculations and experimental proposals exist for studying the feasibility of reaching this goal in the near future, which are listed in (Refs. 92–102).

## V. THE CASIMIR EFFECT

One striking manifestation of the quantum vacuum is the change in the vacuum energy when introducing or varying specific boundary conditions. The pioneering work in this field was done by Casimir in his papers of 1948 (Refs. 193, 194). He showed that two parallel perfectly conducting plates exhibit an attractive force:

$$F = -\frac{\pi^2 \hbar c}{240a^4}, \quad (4)$$

where  $a$  is the distance between the plates. He demonstrated how to derive this result by considering the change in the zero-point fluctuations by varying their boundary conditions. Analogous calculations have been performed for many different geometries, sometimes leading to—at first sight—surprising results of repulsive or attractive effects. A calculation of the Casimir effect for a perfectly conducting spherical shell gave a repulsive force. Radiative corrections have been considered in some detail in (Ref. 199). Others studied the modification of the Casimir effect in systems with nonvanishing temperature. An intriguing study considered the drag of the vacuum on rotating objects, that is, by considering time-dependent boundary conditions (Refs. 202–204). Since the concept of the dependence of vacuum energies on boundary conditions is such a general one, there are of course a huge number of applications of this idea beyond QED, in gravitational physics, strong-interaction physics, and many other fields.

One should note that there is also an alternative interpretation of the Casimir effect, arising from giving the boundary conditions a physical meaning, such that the two conducting plates are not viewed as idealized boundaries but the attraction between them originates from the interaction of the system of those charge distributions without the need to connect the effect to vacuum modes (Ref. 205). Thus, the physical interpretation of the Casimir effect is an ongoing topic (Ref. 217). In addition to all this theoretical work there are many experiments focusing on the Casimir effect (Ref. 198).

The definition of what is counted as genuine Casimir-effect measurements and calculations is constantly extending and beyond this survey, see for instance (Ref. 206) for a discussion. Major papers of interest are listed in (Refs. 193–217).

## VI. QED CALCULATIONS ON THE LATTICE

In contrast to the impressive progress of numerical simulations of strong interactions on a space-time lattice (“Lattice QCD”), a direct simulation of QED is very difficult owing to its small coupling and long-range Coulomb forces. However, to better understand the general behavior of the theory one can adopt the coupling strength as a variable parameter and explore QED behavior and the structure of the vacuum at strong coupling strengths (SCQED, that is, strongly coupled QED). There are a number of numerical studies of QED based upon different lattice formulations, the compact and noncompact lattice QED, which in a naive continuum limit recover the original QED but introduce subtle topological differences. Note, however, that in the pure photonic theory without any electrons or other charged particles present, the system will be a free gas of photons in the limit of an infinitely fine lattice. Compactification can lead to interesting vacuum structures like the occurrence of a condensate of

magnetic monopoles (Ref. 186). Coupling external fields to the calculation, the phase structure of lattice QED was studied early in (Ref. 176) showing the onset of chiral symmetry breaking similar to the Dyson-Schwinger approaches discussed earlier in (Ref. 174). There were some attempts to study the interplay of spontaneous-positron creation in a strong external field with the chiral condensate for large fine-structure constants, showing that one effect hinders the onset of the other one (Ref. 176). Since the U(1) structure of QED also shows up in other theoretical approaches, like various unified theories, these results also are of interest beyond the general understanding of possible phases of the QED vacuum. Additionally, lattice QED calculations investigated the question of whether the theory is a “trivial theory” because of the Landau pole (Ref. 190) of the renormalized charge at high momenta that is obtained in a re-summed perturbative calculation. The divergence at this pole, if it exists, then yields a vanishing renormalized coupling constant for any finite bare coupling strength of the theory. The reason for this is that in QED at small distances the vacuum polarization gets stronger and stronger so that any charge gets screened by the polarization cloud and the charge effectively vanishes. This intriguing but largely theoretical question—since the Landau pole (taking into account electrons only) occurs at energies of about  $10^{227}$  MeV, far beyond any unification scale and thus outside of the realm of pure QED—has been studied on the lattice in various calculations. So far the results seem to point to the originally assumed “triviality” of QED, leaving it as a well-defined effective theory below a certain cutoff. Literature is given in (Refs. 172–192).

## VII. SUMMARY

The study of QED and its vacuum structures has many great theoretical and experimental successes. Since theory and experiment are often pushed to the limit, deviations of measured and calculated results show up from time to time as in the search for vacuum birefringence and the anomalous magnetic moment of the muon, pointing perhaps either to new physics or to inadequately understood experimental setups or theoretical corrections. So far no really clear signal can be identified that contradicts the theory. The study of QED and vacuum instabilities under extreme conditions is still an exciting and developing field. As has been discussed for the case of the over-critical Coulomb fields generated in heavy-ion collisions, increased experimental commitment is needed to confirm and directly observe the beauty of the concept of the QED vacuum and its decay in strong fields.

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Slide Projector. Before the advent of the convenient 35 mm slide, the standard size for American lantern slides was 3.25 in. by 4 in. The substrate was glass, and another sheet of thin glass was held to the surface by black gummed tape. The Greenslade Collection has about 600 of these glass slides, and this 1920-vintage slide projector, made by the Smith Animatograph Company of Davenport, Iowa, is used to project them. This device, with its 500 W bulb, was designed for auditorium work; when used in smaller rooms the slide to lens distance must be greatly increased. (Photograph and Notes by Thomas B. Greenslade, Jr., Kenyon College)