Bra, Ket, Dirac, and all that . . .

An informal self-study guide for beginning students of quantum mechanics

Evening MS in Physics Project Report
by
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About this book: what, why, and how?

If you have flipped through the pages of this book, you have probably already noticed that it is not a typical quantum mechanics book. In this section, we will try to explain what kind of book we intended to write, why we wrote it, and how you might use it---either as a supplementary book for a first course in quantum mechanics out of a traditional textbook, or as a self-study guide in conjunction with a traditional textbook.

The three traditional textbooks that we have used in the course where we developed this book are R. Shankar's Principles of Quantum Mechanics, D. Griffiths' Introduction to Quantum Mechanics, and Cohen-Tannoudji et al's Quantum Mechanics. However, we believe that the method and the material that we have developed, and are presenting here, can be used with essentially any good quantum mechanics textbook.

In a recent seminar at the University of Washington, David Griffiths defined his working premise behind each of his three textbooks to be: "an orderly exposition designed to illuminate the logical structure of the subject". Although there are many wonderful textbooks about quantum mechanics that do present the complete logical structure (including Griffiths', of course), it is still very difficult for most beginning students of quantum mechanics to teach themselves out of one (or even many) of these textbooks. In this book, we have tried to fill this gap in the same way that we try to fill it when we teach the course, namely, we do not try to present the complete logical structure of quantum mechanics which is presented in the best textbooks, but instead we try to select a crucial subset of the topics, and to present them in ordinary conversational language (instead of in "classical textbookese"). Our goal is to illustrate this crucial subset of topics with lots of geometric, analytic, and numerical examples all designed to show how we actually do real calculations using quantum mechanics, and how we think intuitively about what these calculations mean physically. We have found that once our students understand a crucial subset of the material, that they are then able to rapidly expand---both their understanding and their ability to calculate---from this crucial subset to the rest of the nearby related logical structure. Thus, the lecture notes and examples presented in part one of this book are very similar to the lectures and examples we have developed to try to help our students learn a crucial subset.

The other very important learning tools in a quantum mechanics courses are the problem sets. We have also been experimenting with different ways to present these problems to help our students learn the subject matter more easily, and better! The approach in this book is the essentially the same as the approach that we have used in our classes, namely, first we distribute the problems in the "standard physics textbook language"....i.e., the problems are stated very tersely, and students who are essentially perfectly prepared, can do them by spending a lot of time and energy, looking things up, and putting the pieces together. This was very unsatisfactory for the students that we developed this book for, who did not have this "perfect physics preparation" assumed by the problems in most books. Our students were in the Evening Master's Degree Program at the University of Washington, which is a special 3-4 year evening program designed to train people who: 1) have at least an undergraduate degree in engineering or in another physical science (i.e., besides physics), 2) who typically work full-time and attend one class per quarter which meets for 2 two hour lectures per week for two eleven week quarters, and 3) who want and need to learn enough quantum mechanics so they can apply it in the rest of their classes in the program. Consequently, many of our students often have very little (or no) training in all the subjects we diligently teach our undergraduates to try to prepare them for our junior class. To help our students as much as possible, we wanted to develop a detailed set of motivations, hints, comments about the reasons we do the calculations the ways that we do them, and intermediate results.....so that our students could go away, and not only do the problems, but could also develop some understanding of how to take a simple quantum mechanics problem and break it down into the pieces that can be handled by the standard techniques that we have available to solve these problems. We called these detailed expositions of what needed to be done to solve the traditional very terse statements of quantum mechanics problems our "Garden Path Versions of the Problems", and we will continue to call them garden paths here. With these

garden paths, our students reported that they were able to do the homework with much less aggravation, frustration, and suffering....and with much more comprehension of what they were doing step-by-step, and why they were doing what they were doing. We continued to hold weekly homework help sessions, but it was clear that our students were understanding much more, and were asking much deeper questions about the problems (presumably, because they had a much better understanding of many more of the issues involved). Although we developed this method specifically for our Evening Master's Degree Program quantum mechanics course (over two three year periods during the past ten years), we think that many undergraduates may also find this material very useful as either a self-study guide, or as a complement/supplement to their regular junior (or senior) quantum mechanics course.

The problems presented in this book are precisely the problems that our students worked during the 1996-7 academic year. All of our students---independent of their initial preparation---were able to work essentially all of these problems (when, of course, their full-time day jobs and their families didn't take them out of commission). After they had worked our problem sets, and turned them in to be graded, we prepared very detailed solution sets that showed them the detailed step-by-step solutions to every problem assigned. We wrote these solutions for our students---and not for other professors and TA's who are clearly the intended audience for the solution sets/manuals of many textbooks. We solved the problems the way we expected our students to solve them; i.e., just as though they were done by a very conscientious student---who really liked doing all the steps!!! So, instead of leaving out many steps which are obvious to professors and to TA's, we did not leave out any steps unless we knew that our students had been sufficiently drilled in the required leaps.

If you want to use this book as a self-study guide, we think that you will get the most learning out of it if you really try to solve all of the problems that we've presented, using only your favorite traditional quantum mechanics textbook(s) and our lectures, examples, and garden paths. Our students were able to solve these problems step-by-step by using the garden paths---and working hard! If you do all of these these problems in the same diligent manner, we know that you will learn a lot of quantum mechanics. If, on the other hand, you want to use this book as a supplement to your quantum mechanics course, which probably already has some heavy duty homework (it certainly seems to be more the rule than the exception to have massive homework assignments with the junior/senior level quantum courses), then you might want to study the garden paths enough to understand how you would attack the problems, and then to work through the solutions line-by-line writing everything down and making sure that you understand every step. In this mode, our book becomes a quantum mechanics problem book, but instead of being one of the usual encyclopedic set of solutions for professors and TA's (and, of course, for you too---after you have learned how to calculate), our book is a set of problems designed to illustrate in detail how to do almost everything that you'll need to do....

If you really understand how to work our problems, and you understand how and why to use the standard methods and the standard quantum mechanical ideas to work them, then you will almost certainly be able to work the problems assigned in your course much more easily (and with a lot more understanding!!!) than you would be able to achieve otherwise.

We hope that our book will serve as a safe reliable bridge from the land of wonderful formal textbooks, across the river of quantum mechanical despair and uncertainty, to the land of quantum mechanical calculations and, eventually, to the land of quantum mechanical intuition. We would really like to hear from you as, or after, you use this book. Please tell us what worked for you, what did not, how you think we can improve things, if you find errors, etc.

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