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1	Why study angular momentum?
2	The angular momentum ladder operators in Hilbert space
3	The angular momentum ladder operators in their matrix representation
4	The angular momentum ladder operators in position space
5	The totally antisymmetric tensor and vector cross products
6	The angular momentum commutation relations in vector and component form
7	The L2 and Lz operators, their eigenkets, and eigenvalues in the Hilbert space
8	The L2 and Lz operators, their eigenkets, and eigenvalues in position space
9	The L2 and Lz operators, their eigenkets, and eigenvalues in their matrix representation
10	Obtaining the eigenvalues and the eigenkets using the ladder operators
11	Obtaining the eigenvalues and the eigenfunctions by solving the differential equation
12	Sketch and explain the semi-classical vector model for angular momentum for l=3
13	The uncertainty relations for Lx, Ly, and Lz
14	The spherical harmonics
15	Adrien-Marie Legendre, the Legendre equation, and the Legendre polynomials
16	Why study the hydrogen atom?
17	The asymptotic decay of the spatial wavefunctions for hydrogen
18	The n, l, and m quantum numbers for the hydrogen atom
19	Edmond Laguerre, the Laguerre equation, and the Laguerre polynomials
20	What is your favorite topic in quantum mechanics and why?
21	What bothers you about quantum mechanics and why?

Consider the paradigmatic hydrogen atom.

(a) Make a plot of the energy levels of the hydrogen atom. Plot the energy values E_n in the vertical direction for $n = 1, 2, 3, 4, 5$. Plot the the orbital angular momentum quantum number in the horizontal direction for $l = 0, 1, 2, 3, 4$. For each n, show every allowed value of l. Label every energy level spectroscopically (1s, 2s, 2p, ...). Indicate the m degeneracy of each l level. Show that the total degeneracy of each E_n is n^2.

(b) Make a composite sketch showing the $l = 0, 1, 2$ effective potentials. Add the locations of the $n = 1, 2, 3$ energy levels to your sketch, and explain how the levels in the different wells are lined up.

(c) Sketch just the $l = 0$ effective potential. Add the locations of the $n = 1, 2, 3$ energy levels to your sketch. Sketch the radial wavefunctions for each of these energy levels and label each wavefunction with the appropriate R_{nl} designation. Sketch the corresponding probability distributions per unit volume $|\psi(r)|^2\ dV$. Sketch the corresponding radial probability distributions $4\pi r^2\ |\psi(r)|^2\ dr$.

(d) Sketch just the $l = 1$ effective potential. Add the locations of the $n = 1, 2, 3$ energy levels to your sketch. Sketch the radial wavefunctions for each of these energy levels and label each wavefunction with the appropriate R_{nl} designation. Sketch the corresponding per unit volume and radial probability distributions.

(e) Sketch just the $l = 2$ effective potential. Add the locations of the $n = 1, 2, 3$ energy levels to your sketch. Sketch the radial wavefunctions for each of these energy levels and label each wavefunction with the appropriate R_{nl} designation. Sketch the corresponding per unit volume and radial probability distributions.

(f) Sketch the three-dimensional probability distributions for $n = 1, 2, 3$. For each n, show all allowed values of l and m.

Consider a hydrogen atom that is in the following superposition of its energy eigenkets $|nlm>$ at $t = 0$

$$|\psi(t=0)>= N\left[\ |211> + \sqrt{2}\,|320> + \sqrt{3}\,|432>\ \right].$$

(a) Calculate the normalization constant N and write down the normalized time-dependent state vector$|\psi(t)>$.

(b) If you measure the energy at time t, what are the possibilities and what are the probabilities? Calculate the expectation value of the energy $< E >$. Calculate the uncertainty in the energy ΔE.

(c) If you measure L^2 at time t, what are the possibilities and what are the probabilities? Calculate the expectation value $< L^2 >$. Calculate the uncertainty ΔL^2.

(d) If you measure L_z at time t, what are the possibilities and what are the probabilities? Calculate the expectation value $< L_z >$. Calculate the uncertainty ΔL_z.

Consider the 2d simple harmonic oscillator.

In 2d Cartesian coordinates, the Hamiltonian is given by

$$H = -\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) + \frac{1}{2}m\omega^2(x^2 + y^2).$$

(a) Write down the time-dependent Schrodinger equation as a differential equation in Cartesian coordinates. Separate the time and space variables to obtain the time-independent Schrodinger equation. Separate the x and y variables.

(b) For the Cartesian case, show that the eigenenergies are given by

$$E = (n_x + n_y + 1)\hbar\omega.$$

In 2d polar coordinates, the Hamiltonian is given by

$$H = -\frac{\hbar^2}{2m}\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2}{\partial \phi^2}\right] + \frac{1}{2}m\omega^2 r^2.$$

(c) Write down the time-dependent Schrodinger equation as a differential equation in polar coordinates. Separate the time and space variables to obtain the time-independent Schrodinger equation. Separate the r and ϕ variables.

(d) For the polar case, show that the eigenenergies are given by

$$E = (n_r + 1)\hbar\omega.$$

(e) The number of degenerate energy eigenstates for each energy is given by $n_r + 1$. The number of degenerate eigenstates is the same in the two coordinate systems:

There is 1 ground state:
Cartesian ($n_x = 0$ and $n_y = 0$)
Polar ($n_r = 0$)

There are 2 first excited states:
Cartesian-1 ($n_x = 1$ and $n_y = 0$)
Cartesian-2 ($n_x = 0$ and $n_y = 1$)
Polar-1 ($n_r = 1$ and $L_z = +1$)
Polar-2 ($n_r = 1$ and $L_z = -1$)

Continue this list up to $n_r = 5$.