Physics 441 Exam 1
Nominally Due February 7, 2011

Physics is about much more than equations---it is about ideas !!! Mathematics is the language that allows us to express those ideas in a compact, precise form. Some ideas require both the language of mathematics and the language of people to express them---quantum mechanics certainly does. Other ideas can be expressed using only words---because people intuitively understand the math. However, before some training, most people do not intuitively understand the mathematics used in quantum mechanics.

I want you to understand quantum mechanics, which to me means being able to express it in words, in pictures, and in equations. Rutherford said that if you really understand something you should be able to explain it to your grandmother. I am not asking you to explain quantum mechanics in a way that your grandmother would understand it, because your grandmother might not understand linear algebra and its infinite dimensional generalization called Hilbert space.

For each of the topics on the next page, write clear, concise, physical descriptions that demonstrate you really understand the important qualitative aspects of quantum mechanics. You should be able to do this in a few sentences to a paragraph for each topic.

Explain the physics for each topic in your own words. You do not have to write a perfect essay on each topic, but do write enough to convince me that you really do understand the topic. Make sure to include any important pictures, graphs, and equations.

Write down or draw at least four important things for each topic.

	A
1	Linear vector spaces and Hilbert spaces
2	Dirac notation, bras, and kets
3	Changing basis using Dirac notation
4	Complex conjugation and the adjoint operation
5	Inner products, outer products, and projection operators
6	Hermitian operators and physical observables
7	A complete set of states (continuous and discrete)
8	Resolutions of the identity (continuous and discrete)
9	The canonical commutation relations
10	Degenerate Hermitian operators
11	Simultaneous diagonalization of Hermitian operators
12	A complete set of commuting observables
13	Unitary operators, time evolution, and changing bases
14	The propagator
15	Compatible, partially compatible, and completely incompatible operators
16	The position, momentum, and energy bases
17	Eigenvalues, eigenvectors, and eigenfunctions
18	The analogy between classical normal modes and quantum stationary states
19	Measurement possibilities and measurement probabilities
20	The time-independent Schrodinger equation
21	Solving the TISE by finding the stationary states
22	The time-dependent Schrodinger equation
23	Solving the TDSE by expanding the initial state in terms of the stationary states
24	Time-dependent and time-independent expectation values
25	Time-dependent and time-independent uncertainties
26	The first postulate of quantum mechanics
27	The second postulate of quantum mechanics
28	The third postulate of quantum mechanics
29	The fourth postulate of quantum mechanics
30	Measurement and the collapse of the wavefunction in the double slit experiment

Consider a system described by the Hamiltonian operator

$$H = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} .$$

(1) Find the eigenvalues of the Hamiltonian.

(2) Find the normalized eigenvectors of the Hamiltonian.

(3) However, you solve this problem—using your own brain (gasp!), using a computer, divination using the I Ching or the crack patterns in turtle shells, reading tea leaves, Tarot cards, ouwiga boards—show that your eigenvalues and eigenvectors work!!!

At $t = 0$ this system is in the initial state

$$\mid \psi(t=0) > = N \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} .$$

(4) If you were to measure the energy at time $t = 0$, what results could you obtain, and with what probabilities would you obtain them?

(5) Calculate the expectation value of the energy of this system at $t = 0$, and show that your result agrees with the possibilities and probabilities you obtained above, *i.e.*, show that

$$< \psi(t=0) \mid H \mid \psi(t=0) > = \sum_i E_i \, P(E_i) \, .$$

(6) Calculate the standard deviation of the energy of this system at $t = 0$, and show that your result agrees with the possibilities and probabilities you obtained above, *i.e.*, show that

$$\Delta H = \sqrt{\sum_i P(E_i) \, (E_i - < H >)^2} \, .$$

(7) Plot your calculated probabilities $P(E_i)$ versus E. Show your expectation value and standard deviation on your plot. Discuss what this plot shows, *i.e.*, explain how the expectation value and the standard deviation are related to the outcome of a series of energy measurements.

(8) Now expand the initial state $\mid \psi(0)>$ in the energy eigenbasis, and calculate the full time-dependent state vector $\mid \psi(t)>$.

(9) Calculate the full time-dependent possibilities and probabilities of energy measurements at time t.

Finally, consider a second observable operator Ω

$$\Omega = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix}.$$

(10) Find the eigenvalues of the Omega operator.

(11) Find the normalized eigenvectors of the Omega operator.

(12) Again, however you solve this problem show that your eigenvalues and eigenvectors work.

(13) Calculate the commutator $A = [H, \Omega]$. Are H and Ω completely compatible, partially compatibly, or completely incompatible? If you were to make Ω measurements would they be time-dependent? Explain why or why not.

(14) If you were to measure the omega-ness at time t, calculate the full-time dependent probability that you will obtain each of the three eigenvalues of the Omega operator, ω_1, ω_2, and ω_3.

(15) Calculate the outer product for each the three eigenvectors of Ω to obtain the corresponding projection operators. Show that the sum of the three projection operators is equal to the identity matrix. Warning: this will not work unless you use the normalized eigenvectors.