8-18-09

NR PI → FEYNMAN DIAGRAMS

FIRST GENERALIZED MOMENTUM

$$\pi = \vec{p} - \frac{e}{c}\vec{A}$$

PARTICLE MOMENTUM

FIELD MOMENTUM

$$KE = \frac{\vec{\pi}^2}{2m} = \frac{\vec{p}^2 + \frac{e}{c}\vec{p}\cdot\vec{A} + \frac{e}{c}\vec{A}\cdot\vec{p} + \frac{e^2}{c^2}\vec{A}^2}{2m}$$

PHOTON EMISSION

$\vec{p}\cdot\vec{A}$

PHOTON ABSORPTION

$\vec{A}\cdot\vec{p}$

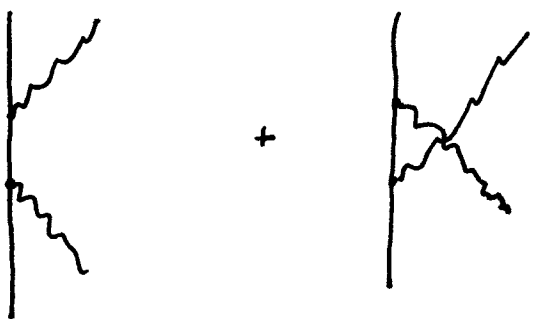
PHOTON SCATTERING THOMSON SCATTERING

A^2

$\langle f \mid \vec{p} \cdot \vec{A} \mid i \rangle$ PROB AMP TO EMIT A PHOTON

SEAGULL DIAGRAM IS NOT COVARIANT

$\Leftarrow$ SAME TIME NO SIMULTANEITY



COMPTON SCATTERING

ALL POSSIBLE TIMES

ALL POSSIBLE PLACES

EVERY POINT IN SPACE TIME

$\Rightarrow$ MANIFESTLY COVARIANT

Thus we can get the correct answer for the probability of partial reflection by imagining (falsely) that all reflection comes only from the front and back surfaces. In this intuitively easy analysis, the front surface and back surface arrows are mathematical constructions that give us the right answer, whereas the analysis we just did---with the spacetime drawing and the arrows forming a part of a circle---is a more accurate representation of what is really going on: partial reflection is the scattering of light by electrons **inside** the glass.

Now what about the light that goes **thru** the layer of glass? First there is an amplitude that the photon goes straight thru the glass without hitting any electrons. This is the most important arrow in terms of length. But there are six other ways that a photon could reach the detector below the glass: a photon could hit X1 and scatter down to the detector, a photon could hit X2, X3, X4, X5, X6 ... These arrows have the same length as the arrows that formed the circle in the previous example

The same effect would appear if photons went slower thru glass than thru air: there would be extra turnings of the final arrow. That's why I said earlier that light appears to go slower thru glass (or water) than thru air. In reality the "slowing' of the light is extra turning caused by the atom in the glass (or water) scattering the light. The degree to which there is extra turning of the final arrow as the light goes thru a given material is called its index of refraction.

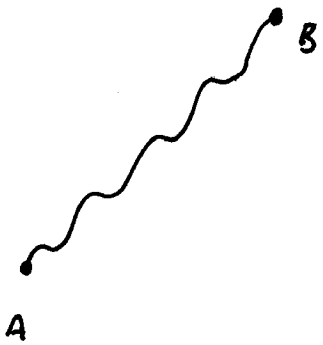
It is hard to believe that the vast apparent variety of Nature results from the monotony of repeatedly combining just these three basic actions. But it does.

Throughout these lectures I have delighted in showing you that the price of gaining such an accurate theory has been the erosion of our common sense. We must accept some very bizarre behavior: the amplification and suppression of probabilities, light reflecting from all parts of a mirror, light travelling in paths other than a straight line, photons going faster or slower than the conventional speed of light, electrons going backwards in time, photons suddenly disintrigrating into a positron-electron pair, and so on. That we must do, in order to appreciate what Nature is really doing underneath nearly all of the phenomena we see in the world.

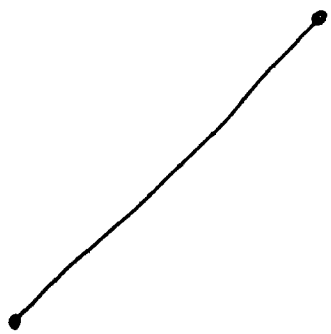
With the exception of technical details of polarization, I have described to you the framework by which we understand all of these phenomena. We draw **amplitudes** for every way an event can happen and add them when we would expect to add probabilities under normal circumstances; we multiply amplitudes when we would have expected to multiply probabilities. Thinking of everything in terms of amplitudes may cause difficulty at first because of their abstraction, but after a while, one gets used to this strange language. Underneath so many of the phenomena we see every day are only three basic actions: one is described by the simple coupling number j; the other two by functions---P(a to b) and E(a to b)---both of which are closely related. That's all there is to it, and from it all of the rest of the laws of physics come.

However before I finish this lecture, I would like to make a few additional remarks. One can understand the spirit of quantum electrodynamics without including this technical detail of polarization. But I'm sure that you will all feel uncomfortable unless I say something about what I've been leaving out. Photons it turns out come in four diferent varieties, called polarizations, that are related geometrically to the directions of space and time. Thus there are photons polarized in the X, Y, Z, and T directions. Perhaps you have heard that light comes in only two states of polarization---for example, a photon going in the Z direction can be polarized at right angles, either in the X or the Y direction. Well, you guessed it: in situations where the photon goes a long distance and appears to go at the speed of light, the amplitudes for the Z and the T terms exactly cancel out. But for virtual photons going between a proton and an

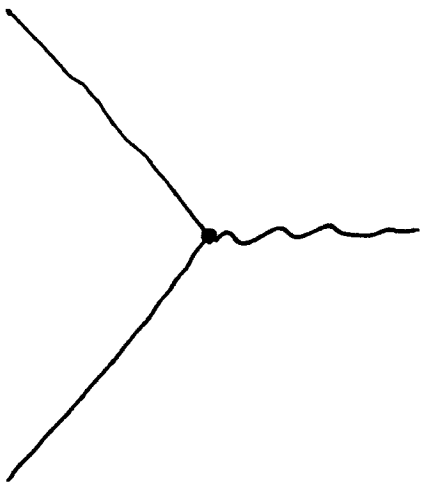
THREE ACTIONS FOR QED "A PARTICLE PICTURE"



PHOTON $P(A \to B)$



ELECTRON $E(A \to B)$

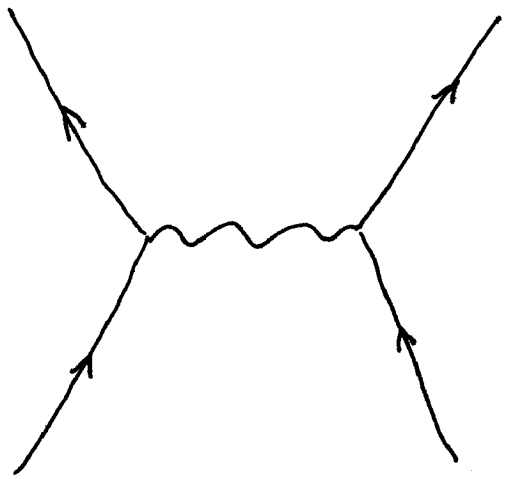


ELECTRON EMITS OR ABSORBS A PHOTON

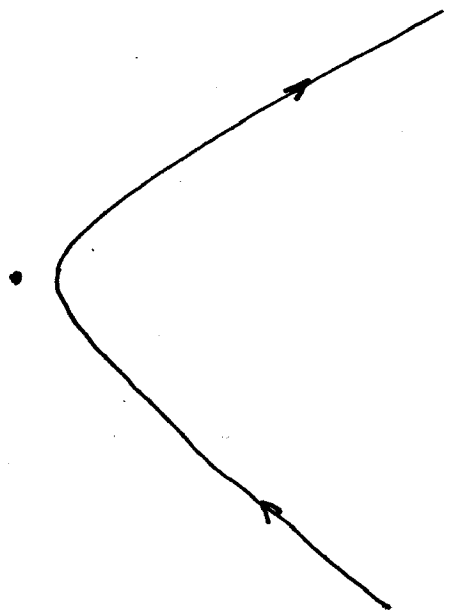
$j \approx -0.1$

NO ELECTRIC FIELDS

NO MAGNETIC FIELDS



VIRTUAL PHOTON EXCHANGE



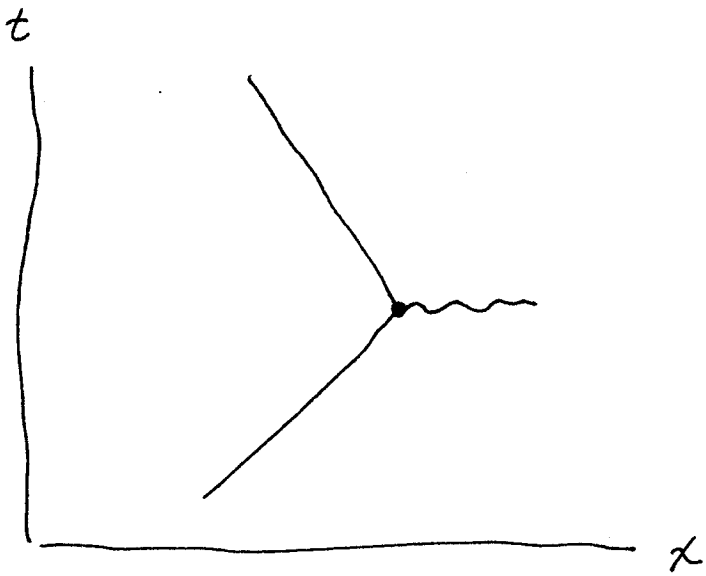
COULOMB

POTENTIAL

$\Delta E \, \Delta t \sim \hbar/2$.

BREMSSTRALUNG

ACTION 3 : AN ELECTRON EMITS OR ABSORBS A PHOTON



COUPLING $j \approx -0.1$

SAME TO EMIT

OR ABSORB

FOR THE PHOTON

$$ds^2 = (c\Delta t)^2 - (\Delta x^2 + \Delta y^2 + \Delta z^2) = 0$$

OVER LARGE DISTANCES $v = c$ REAL PHOTONS

BUT OVER SHORT DISTANCES

$$\left.\begin{array}{c} v > c \\ \text{and} \\ v < c \end{array}\right\} \text{BOTH CONTRIBUTE}$$

VIRTUAL PHOTONS

over large distances these average to zero.

FOR THE ELECTRON $E(a \to b)$

$E(a \to b)$ IS SIMILAR TO $P(a \to b)$

$$E(a \to b) = \underbrace{P(a \to b)}_{\text{NO HOPS}} + \underbrace{P(a \to c)\, m^2\, P(c \to b)}_{\text{1 HOP}}$$

$$+ \underbrace{P(a \to d)\, m^2\, P(d \to e)\, m^2\, P(e \to b)}_{\text{2 HOPS}}$$

$$+ \; 3, 4, 5, \ldots \text{ HOPS}$$

if $m = 0$ $\quad E(a \to b) = P(a \to b)$

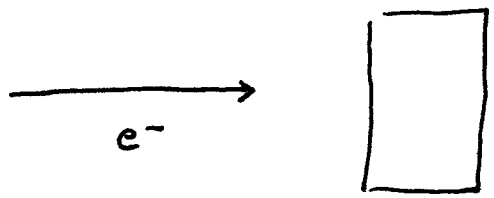
AT LARGE DISTANCES, ONLY REAL ELECTRONS

$$E^2 = p^2 c^2 + m_e c^4$$

AT SMALL DISTANCES, VIRTUAL ELECTRONS

$$E^2 \neq p^2 c^2 + m_e c^4$$

BREMSSTRAHLUNG

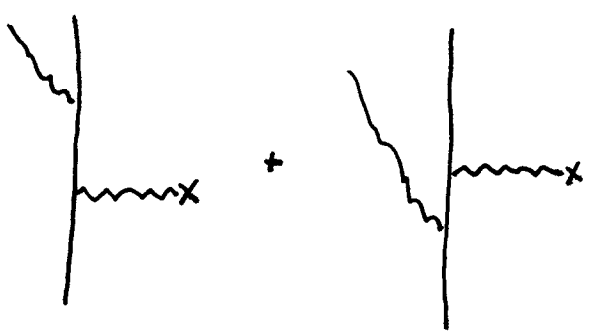


SUDDEN STOP

LARGE ACCELERATION

LOTS OF RADIATION

QED



SAME FINAL STATE

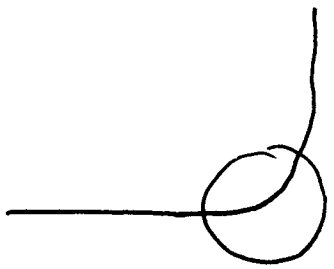
NUCLEUS SCATTERING WITH VIRTUAL ~~ELECTRONS~~ PHOTONS

COLLISION WITH ELECTRON MAKES ONE REAL

ONLY REAL PHOTONS TRAVEL TO ∞

PAIR PRODUCTION NEAR NUCLEI

SYNCHROTRON RADIATION

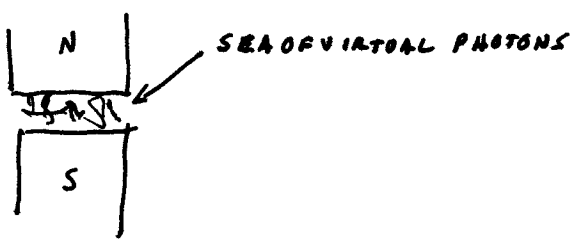


SHARP BEND

LARGE ACCELERATION

LOTS OF RADIATION

QED



COLLISION WITH e MAKES ONE REAL

PHOTON $E = pc$ $E^2 = p^2c^2$ REAL

$E \neq pc$ VIRTUAL

ELECTRONS $E^2 = p^2c^2 + m_0^2c^4$ REAL

$E^2 \neq$ VIRTUAL

OFF MASS SHELL

REAL PHOTONS LIVE FOREVER — MACROSCOPIC DISTANCE

" ELECTRONS " "

VIRTUAL PARTICLES DO NOT

$$\Delta E \, \Delta t \geq \frac{\hbar}{2}$$

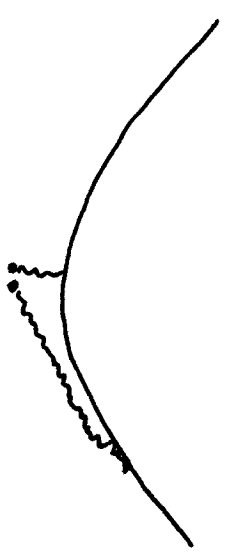
$$\Delta t \approx \frac{\hbar}{2 \Delta E}$$

$$d = v t = c \frac{\hbar}{2E}$$

LARGE ENERGY => SHORT DISTANCE

SMALL ENERGY => LONG DISTANCE

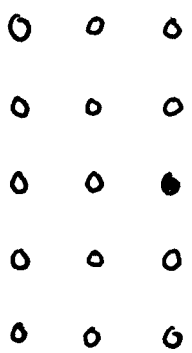
COULOMB SCATTERING



FORCE BETWEEN 2 ELECTRONS ...

BREMSSTRAHLUNG HOLOGRAPHY

CRYSTAL

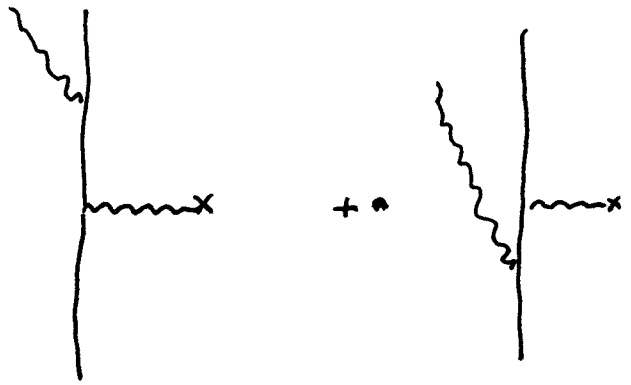


INTERFERENCE PATTERN

$\Downarrow$

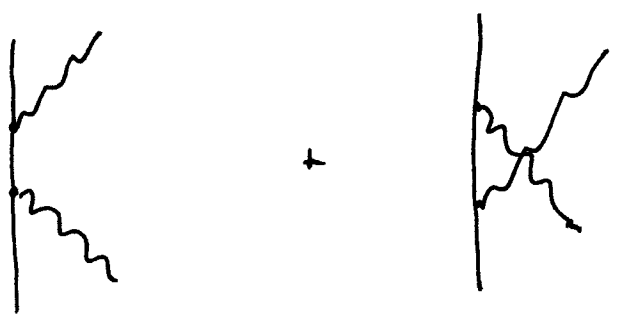
CRYSTAL STRUCTURE

BREMSSTRAHLUNG SOURCE



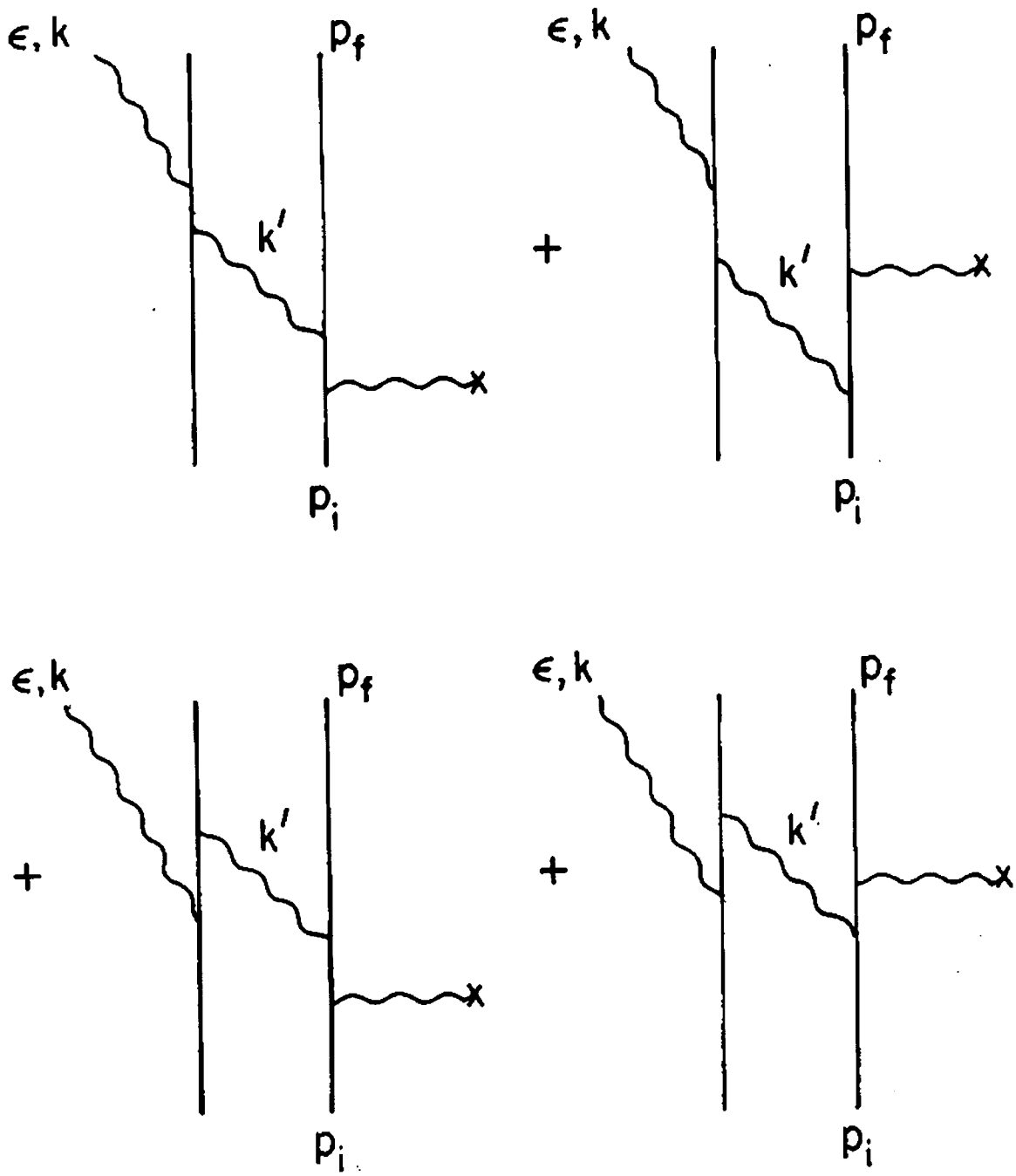
SAME FINAL STATE => INTERFERE

COMPTON SCATTERING



SAME FINAL STATE => INTERFERE

FOUR DIAGRAMS


ϵ, k
p_f
k′
x
p_i
+
ϵ, k
p_f
k′
x
p_i
+
ϵ, k
p_f
k′
x
p_i
+
ϵ, k
p_f
k′
x
p_i

$$\mathcal{O} = -Z^2 e^4 e_p \int \langle C^\mu(\epsilon, k, k') \rangle \frac{\widetilde{B}_\mu(k')}{\omega^2 - \vec{k}'^2 + i\epsilon}$$

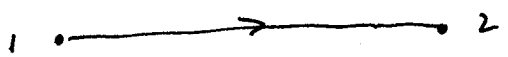
$$\times e^{-i(\vec{k}-\vec{k}')\cdot\vec{r}} \frac{d^3k'}{(2\pi)^3} \frac{[1 - F(|\vec{q}'|)]}{\vec{q}'^2 + i\epsilon},$$

where $\vec{q}' = \vec{p}_f - \vec{p}_i + \vec{k}'$. Note that $k'^2 = \omega^2 - \vec{k}'^2 \neq 0$.

JUST LIKE DIRAC NOTATION, AT SOME POINT SOME ONE MUST DO THE INTEGRALS...

PROPAGATORS

$$\int d^4 k \quad \frac{1}{k^2 + i\epsilon}$$



$$\int d^4 p \quad \frac{m + p}{m^2 - p^2 - i\epsilon}$$

PRB 56, 2399 (1997).

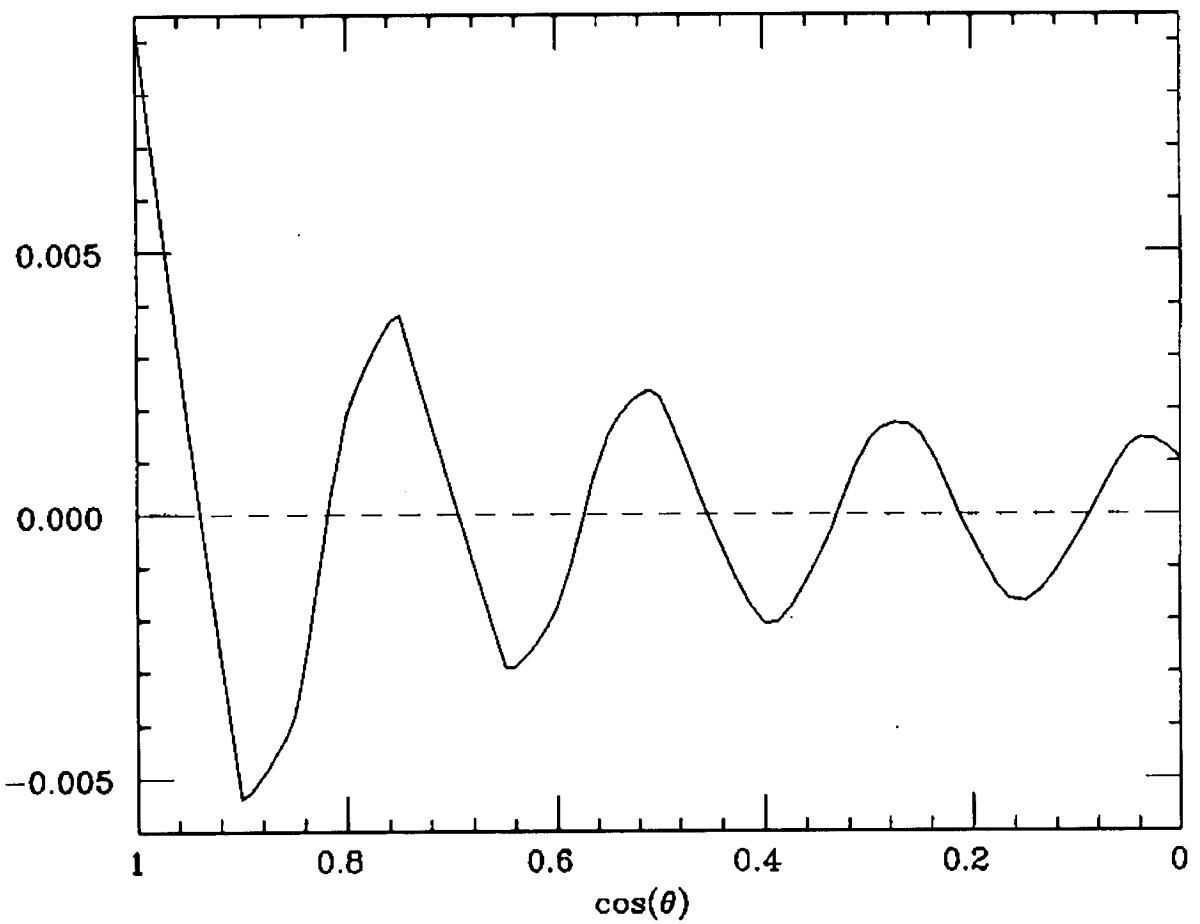


FIG. 8. The small effects of virtual electron propagation in the uncrossed graph in Fig. 2. The real part of the uncrossed correction $\delta J_{\rm on}^{\rm uncr}$ (dashed line) given by Eq. (57) is compared with the real part of the on-shell separated atom approximation $J_{\rm on}$ (solid line) given by Eq. (27). Here $\hat{k}\cdot\hat{p}_f=0.5$ and $\hat{p}_i\cdot\hat{r}=0.5$.

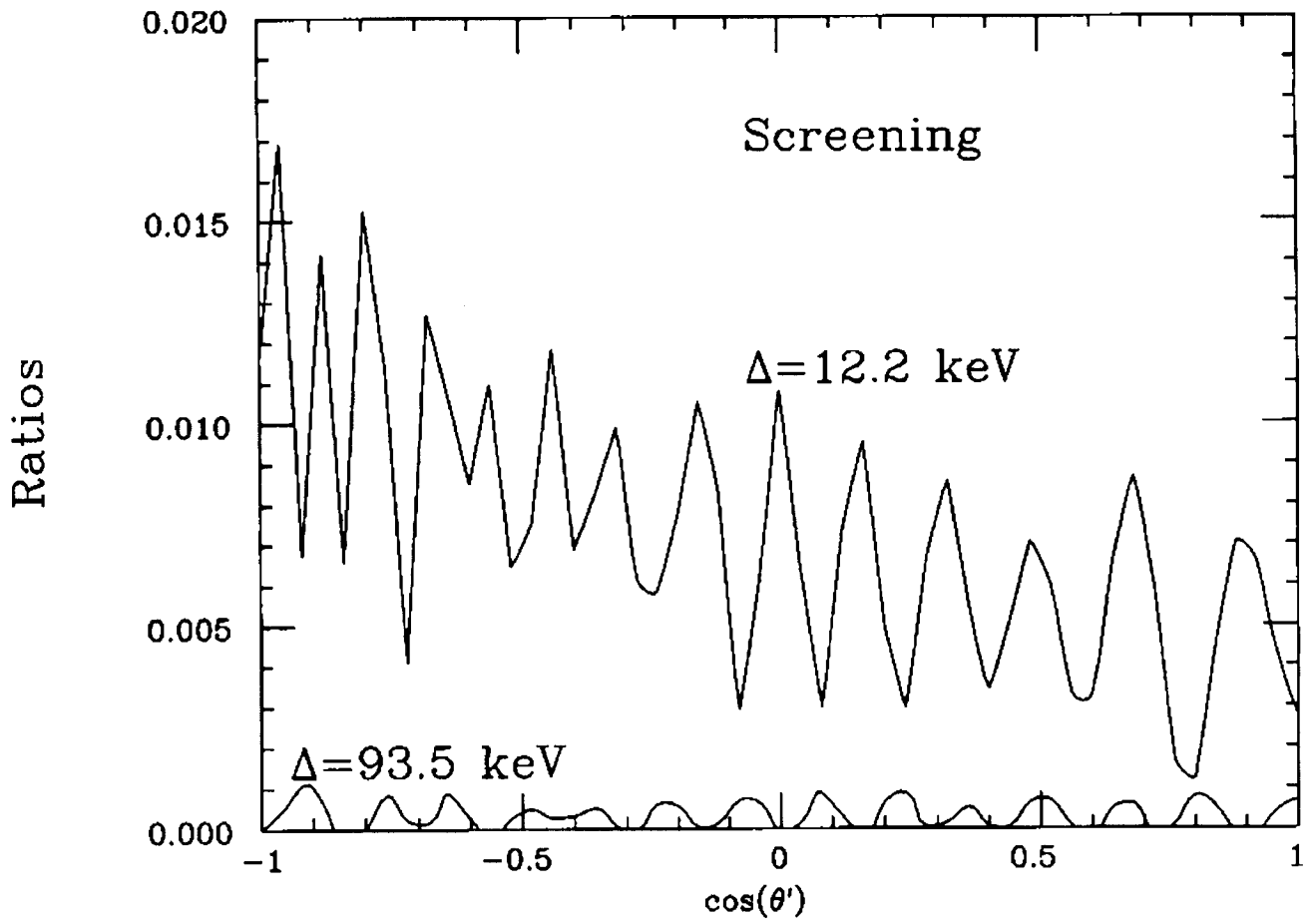


FIG. 6. The small effects of photon virtuality on the screening correction given by the ratio of the second to the first term in Eq. (40). The ratio $\mathrm{Re}\,\delta I_s/\mathrm{Re}\{[1-F(q_2)]J_{\mathrm{on}}\}$ is plotted to illustrate the size of these corrections for two typical experimental values of the momentum transfer, namely 12.2 keV and 93.5 keV. Here the momentum transfer $\vec{\Delta}\equiv\vec{p}_i-\vec{p}_f$ and the angle θ' is specified by $\cos(\theta')=\hat{\Delta}\cdot\hat{r}$.

So You Want to be a PI?

Path integral is simpler and hotter
Facebook
http://www.facebook.com/group.php?gid=29472939420

Path Integral Methods and Applications
MacKenzie
http://arxiv.org/abs/quant-ph/0004090v1

Classification of Solvable Feynman Path Integrals
Grosche and Steiner
http://arxiv.org/abs/hep-th/9302053

MIT Links to PI papers
http://web.mit.edu/redingtn/www/netadv/Xpathinteg.html

Path Integrals in Quantum Mechanics, Statistics, Polymer Physics, and Financial Markets
Kleinert
http://users.physik.fu-berlin.de/~kleinert/kleiner_re.html
http://users.physik.fu-berlin.de/~kleinert/kleinert/?p=booklist&details=11
http://users.physik.fu-berlin.de/~kleinert/cgi-bin/getaccess/nocookie/kleiner_reb3/3rded.html

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