

50:50 SUPERPOSITION STATE

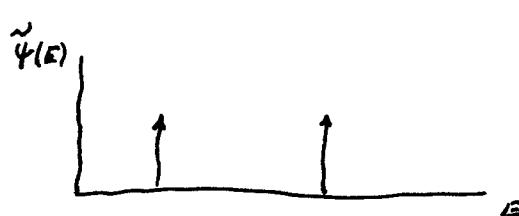
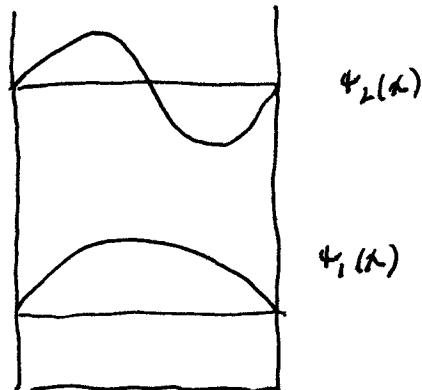
$$|\psi(0)\rangle = \frac{1}{\sqrt{2}} |E_1\rangle + \frac{1}{\sqrt{2}} |E_2\rangle$$

$$|\psi(t)\rangle = \frac{1}{\sqrt{2}} |E_1\rangle e^{-iE_1 t/\hbar} + \frac{1}{\sqrt{2}} |E_2\rangle e^{-iE_2 t/\hbar}$$

$\langle x | \psi(x, t) = \frac{1}{\sqrt{2}} \psi_1(x) e^{-iE_1 t/\hbar} + \frac{1}{\sqrt{2}} \psi_2(x) e^{-iE_2 t/\hbar}$

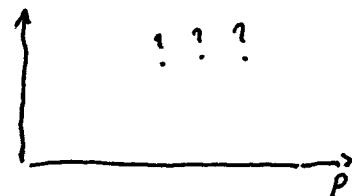
$\langle p | \hat{\psi}(p, t) = \frac{1}{\sqrt{2}} \hat{\psi}_1(p) e^{-iE_1 t/\hbar} + \frac{1}{\sqrt{2}} \hat{\psi}_2(p) e^{-iE_2 t/\hbar}$

$\langle E | \tilde{\psi}(E, t) = \frac{1}{\sqrt{2}} \delta(E - E_1) e^{-iE_1 t/\hbar} + \frac{1}{\sqrt{2}} \delta(E - E_2) e^{-iE_2 t/\hbar}$



TWO QUESTIONS:

$$\hat{\psi}(p)$$



$$\langle \hat{x}(t) \rangle$$

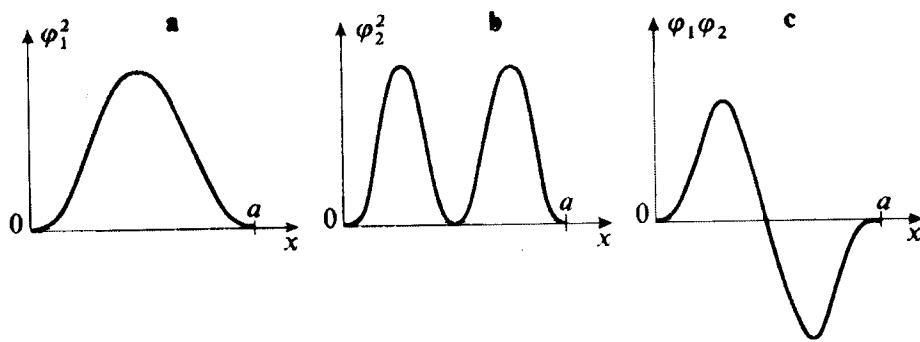


FIGURE 4

Graphical representation of the functions φ_1^2 (the probability density of the particle in the ground state), φ_2^2 (the probability density of the particle in the first excited state) and $\varphi_1\varphi_2$ (the cross term responsible for the evolution of the shape of the wave packet).

Using these figures and relation (18), it is not difficult to represent graphically the variation in time of the shape of the wave packet (cf. fig. 5): we see that the wave packet oscillates between the two walls of the well.

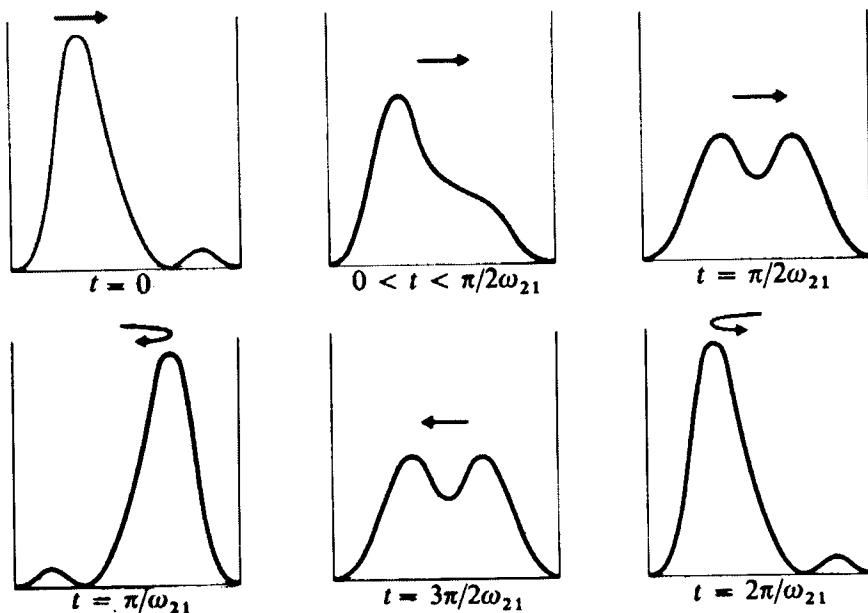


FIGURE 5

Periodic motion of a wave packet obtained by superposing the ground state and the first excited state of a particle in an infinite well. The frequency of the motion is the Bohr frequency $\omega_{21}/2\pi$.

POSITION: $\langle x \rangle$ $\langle x^2 \rangle$ Δx

$$[x, H] = x \left(\frac{\hbar^2}{2m} \frac{d^2}{dx^2} \right) - \left(\frac{\hbar^2}{2m} \frac{d^2}{dx^2} \right) x$$

IN GENERAL

$$[x, H] \neq 0$$

$$[x^2, H] \neq 0$$

BUT FOR THE STATIONARY STATES

$$\begin{aligned} \langle E_n | (xH - Hx) | E_n \rangle &= \langle E_n | (-\hbar x + x\hbar) | E_n \rangle \\ &= \langle E_n | (-E_n x + x E_n) | E_n \rangle \\ &= 0 \end{aligned}$$

$$\langle x(t) \rangle = \langle \psi(t) | x | \psi(t) \rangle$$

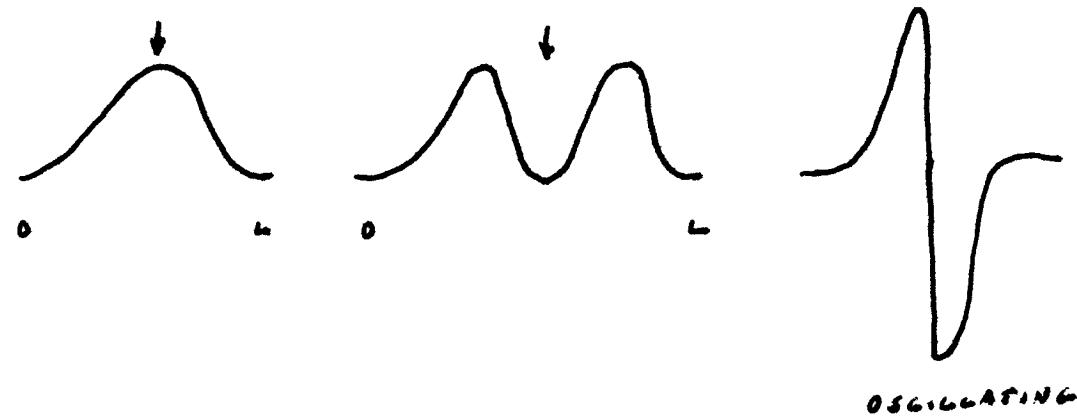
$$= \frac{i}{\sqrt{2}} \left(\langle E_1 | e^{+iE_1 t/\hbar} + \langle E_2 | e^{+iE_2 t/\hbar} \right) x$$

$$\frac{i}{\sqrt{2}} \left(|E_1\rangle e^{-iE_1 t/\hbar} + |E_2\rangle e^{-iE_2 t/\hbar} \right)$$

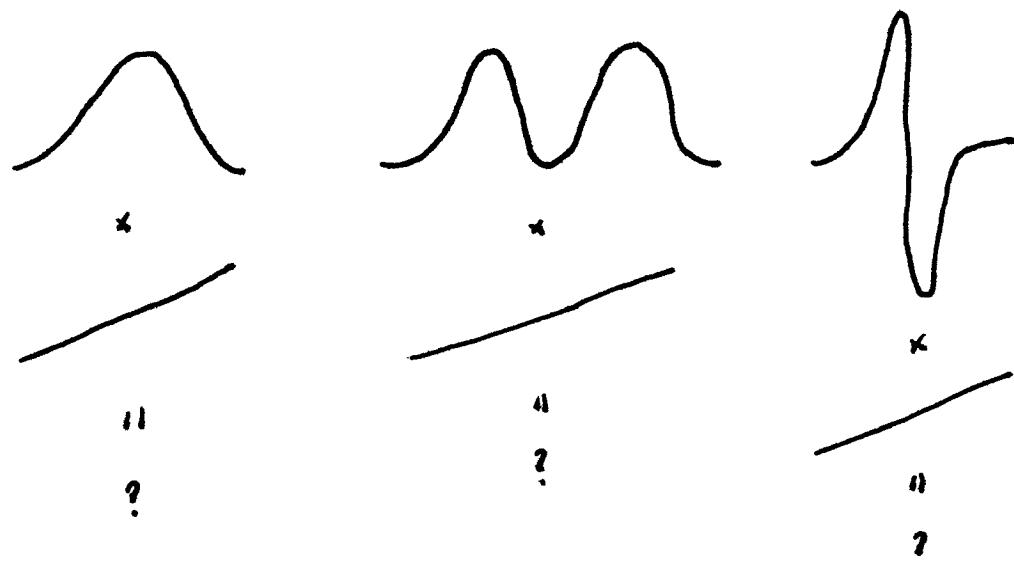
$$\begin{aligned} &= \frac{1}{2} \int x dx \left(\psi_1^*(x) e^{i\omega_1 t} + \psi_2^*(x) e^{i\omega_2 t} \right) \\ &\quad \left(\psi_1(x) e^{-i\omega_1 t} + \psi_2(x) e^{-i\omega_2 t} \right) \end{aligned}$$

$$\langle x(t) \rangle = \int_0^L x \, dx \left[\frac{1}{2} |\psi_1(x)|^2 + \frac{1}{2} |\psi_2(x)|^2 + \psi_1(x) \psi_2(x) \cos(\Omega t) \right]$$

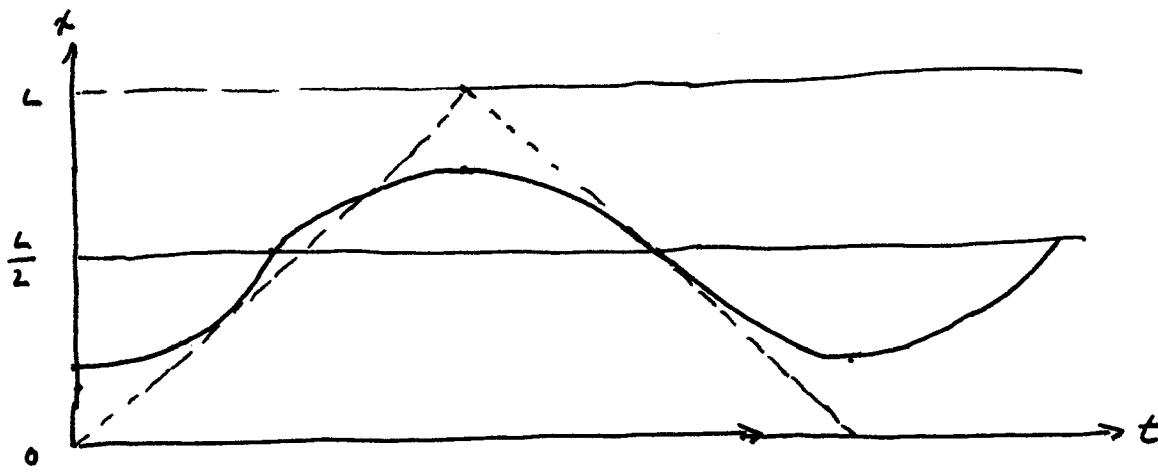
$$\Omega = \omega_2 - \omega_1 = (\epsilon_2 - \epsilon_1)/\hbar = 3\epsilon_1/\hbar = 3\omega_1$$



DOING THE INTEGRALS



$$\langle \psi(t) | x | \psi(t) \rangle = \langle x | \psi(t) \rangle = \frac{L}{2} - \frac{16L}{9\pi^2} \cos(\Omega t)$$



quantum particle feels the walls
sooner than the classical point
particle

$$\frac{2}{L} \int_0^L \sin\left(\frac{\pi}{L}x\right) x \sin\left(\frac{2\pi}{L}x\right) dx = -\frac{16L}{9\pi^2}$$

EXPECTATION VALUES IN MOMENTUM SPACE

in general $[p, H] \neq 0$

for stationary states $[p, H] = 0$

$$\langle E_m | p H - H p | E_m \rangle$$

$$\langle E_m | -H p + p H | E_m \rangle$$

$$\langle E_n | -E_m p + p E_n | E_n \rangle$$

0

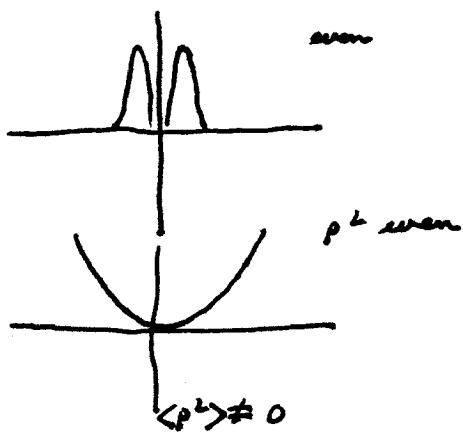
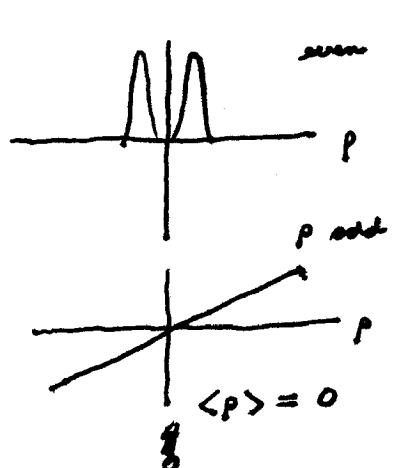
FOR THE ENERGY EIGENSTATES

$$\langle p \rangle = 0$$

$$\langle p^2 \rangle = \left(\frac{\hbar^2 \pi^2}{a^2} \right) m^2$$

$$\Delta p = \left(\frac{\hbar \pi}{a} \right) n$$

FROM COMPUTER SIMULATION



QM BABY RATTLE

$$|\psi(0)\rangle = \frac{1}{\sqrt{2}} (|E_1\rangle + |E_2\rangle)$$

$$\langle p(0) \rangle = \langle \psi(0) | p | \psi(0) \rangle$$

$$= \frac{1}{\sqrt{2}} (\langle E_1 | + \langle E_2 |) p \frac{1}{\sqrt{2}} (| E_1 \rangle + | E_2 \rangle)$$

in x -space

$$= \frac{1}{2} \int [\hat{\psi}_1^*(x) + \hat{\psi}_2^*(x)] - i \hbar \frac{d}{dx} [\hat{\psi}_1(x) + \hat{\psi}_2(x)] dx$$

in p -space

$$= \frac{1}{2} \int [\hat{\psi}_1^*(p) + \hat{\psi}_2^*(p)] p [\hat{\psi}_1(p) + \hat{\psi}_2(p)] dp$$

$$\langle p(t) \rangle = \langle \psi(t) | p | \psi(t) \rangle$$

similar to $\langle x(t) \rangle$

$$\int p dp \frac{1}{2} (| \hat{\psi}_1(p) |^2 + | \hat{\psi}_2(p) |^2) + [\hat{\psi}_1^*(p) \hat{\psi}_2(p) + \hat{\psi}_1(p) \hat{\psi}_2^*(p)] = \sin(2t)$$

WAVEFUNCTIONS IN MOMENTUM SPACE

$$\psi_m(x) = \sqrt{\frac{2}{a}} \sin\left(\frac{m\pi}{a}x\right)$$

$$\begin{aligned}\hat{\psi}_m(p) &= FT(\psi_m(x)) \\ &= \sqrt{\frac{1}{2\pi\hbar}} \int_0^a \sqrt{\frac{2}{a}} \sin\left(\frac{m\pi}{a}x\right) e^{-ipx/\hbar} dx \\ &\quad \downarrow \\ &= \frac{1}{2i} \left[e^{i\left(\frac{m\pi}{a} - \frac{p}{\hbar}\right)x} - e^{-i\left(\frac{m\pi}{a} + \frac{p}{\hbar}\right)x} \right] \end{aligned}$$

$$\frac{\exp\left(\frac{m\pi}{a} - \frac{p}{\hbar}\right)x - 1}{i\left(\frac{m\pi}{a} - \frac{p}{\hbar}\right)} - \frac{\exp\left(\frac{m\pi}{a} + \frac{p}{\hbar}\right)x - 1}{-i\left(\frac{m\pi}{a} + \frac{p}{\hbar}\right)}$$

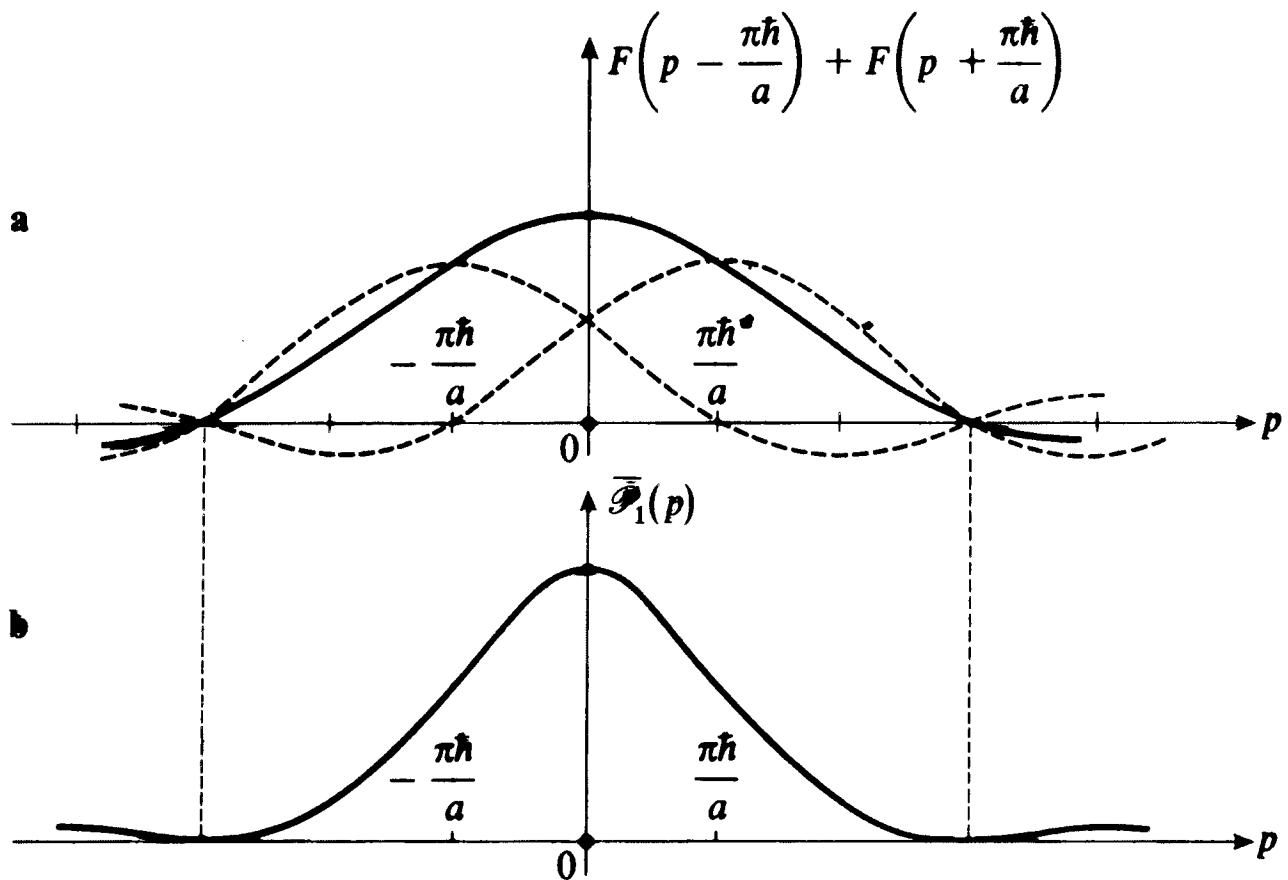
$$\hat{\psi}_m(p) = \frac{1}{2i} \sqrt{\frac{a}{\pi \hbar}} e^{i \left(\frac{m\pi}{2} - \frac{pa}{2\hbar} \right)}$$

$$= \left[F\left(p - \frac{m\pi\hbar}{a}\right) + (-1)^{m+1} F\left(p + \frac{m\pi\hbar}{a}\right) \right]$$

$$F(p) = \frac{\sin(pa/2\hbar)}{(pa/2\hbar)}$$

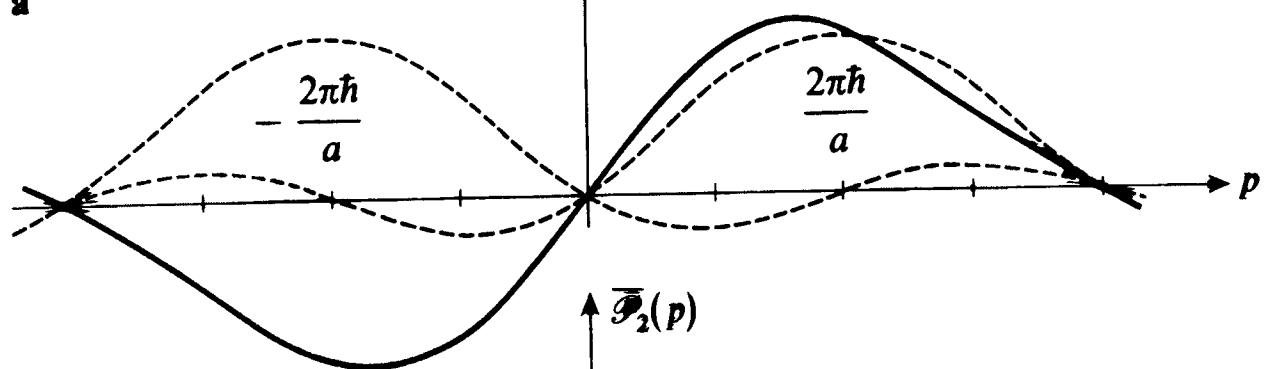
$$= \frac{\sin x}{x}$$

$$= \text{sinc}(x)$$

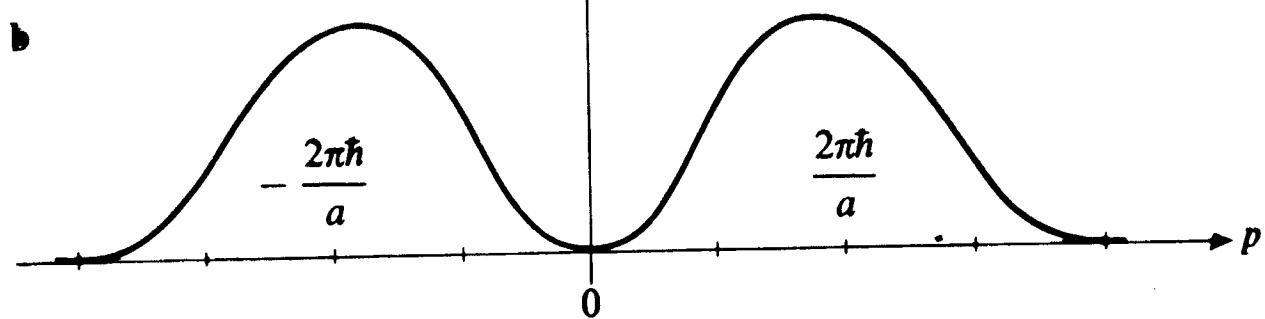


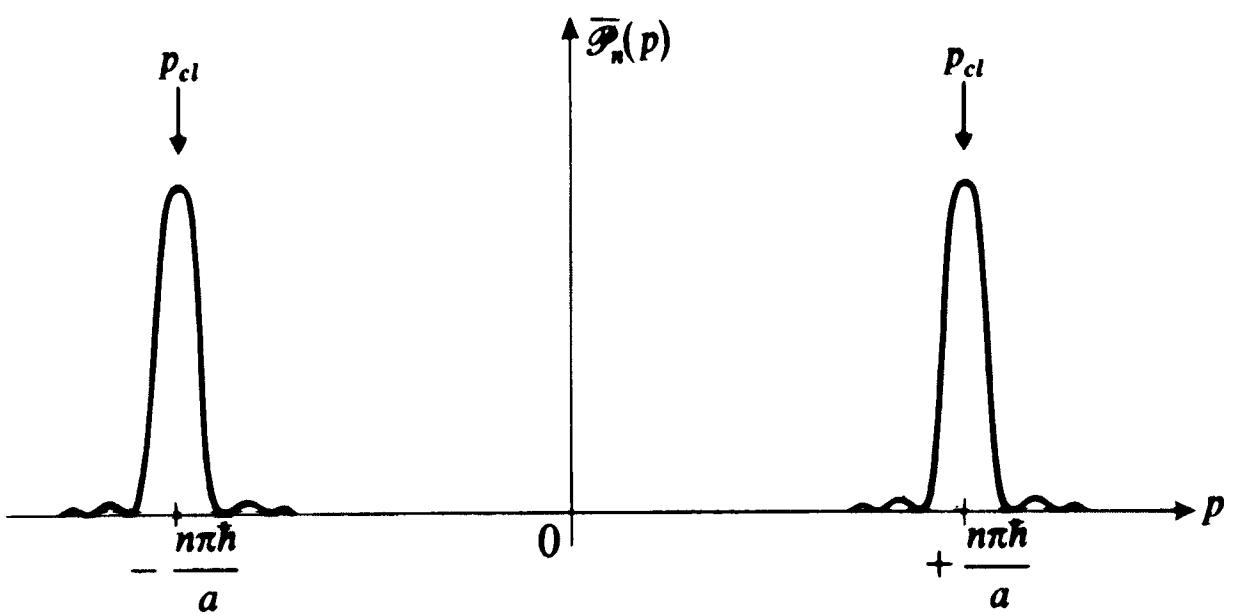
$$F\left(p - \frac{2\pi\hbar}{a}\right) - F\left(p + \frac{2\pi\hbar}{a}\right)$$

a

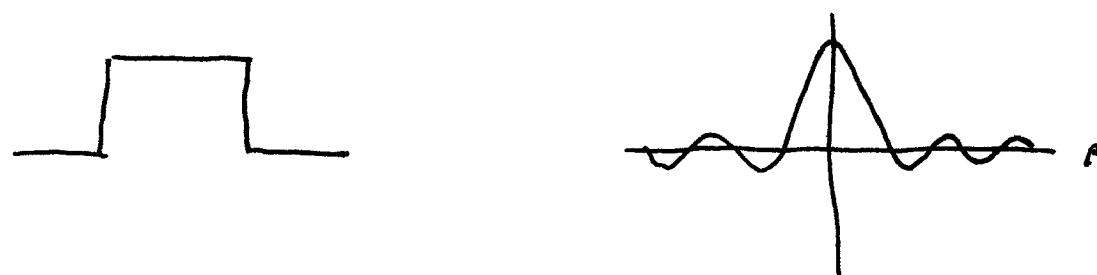
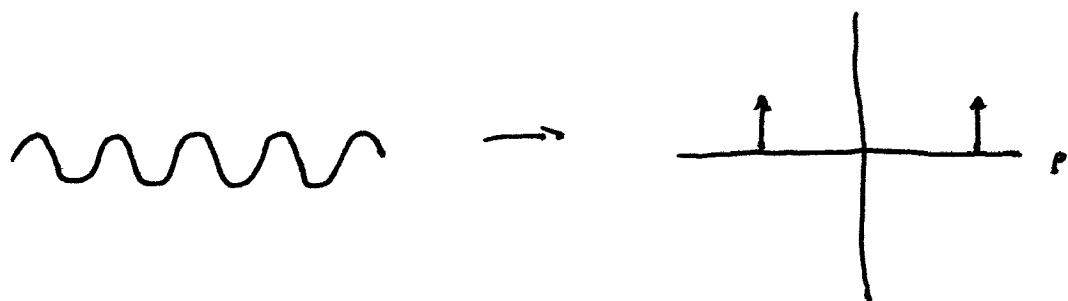


b

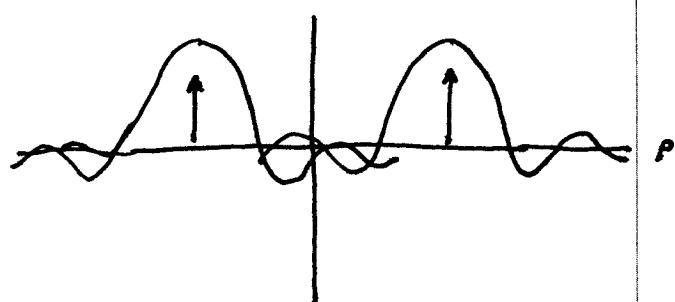




CARTOON VERSION



II



$$A \cdot B = C \iff \hat{A} \otimes \hat{B} = \hat{C}$$

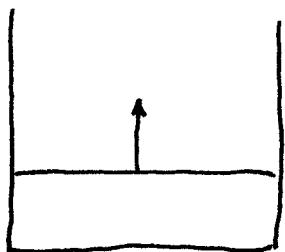
CONVOLUTION THEOREM

as $n \uparrow$, more and more like a ^{single} standing wave



MEASURE X

FIND IN CENTER OF WELL



BY SYMMETRY \Rightarrow ONLY ODD n



$$P(E_n) = \frac{2}{a} n \text{ odd}$$

$$= 0 n \text{ even}$$

$\Rightarrow \infty$ ENERGY

FINITE X RESOLUTION

