LECTURE 10 7-8-09

TIME EVOLUTION

(1) EXPAND $|\psi(0)\rangle$ IN ENERGY EIGENKETS

$$|\psi(0)\rangle = I\,|\psi(0)\rangle$$

$$= \left(\sum_i |E_i\rangle\langle E_i|\right)|\psi(0)\rangle$$

$$= \sum_i |E_i\rangle \underbrace{\langle E_i|\psi(0)\rangle}_{a_i}$$

$$= \sum_i a_i\,|E_i\rangle$$

(2) ADD THE TIME DEPENDENCE

$$|\psi(t)\rangle = \sum_i a_i\,|E_i\rangle\, e^{-iE_it/\hbar}$$

TIME-DEP
PHASE FACTORS

WORKS FOR ALL t

WHERE DOES THE SIMPLE TIME DEPENDENCE COME FROM?

THE $e\vec{v}$'s OF H

OBEY TDSE $\quad H|E_i\rangle = i\hbar \frac{d}{dt}|E_i\rangle$

OBEY TISE $\quad H|E_i\rangle = E_i|E_i\rangle$

$$|E_i(t)\rangle = e^{-iE_it/\hbar}|E_i(0)\rangle$$

PHASE EVOLUTION OF ENERGY EIGENKETS

ONLY EIGENKETS OF H ARE STATIONARY

LOOKED AT

$$|P=3\rangle \longrightarrow \frac{1}{\sqrt{2}}\begin{pmatrix}0\\1\\1\end{pmatrix}$$

$$|P=3\rangle = \sum_i |E_i\rangle\langle E_i|P=3\rangle$$

$$= |E_1\rangle\langle E_1|P=3\rangle$$

$$+ |E_2\rangle\langle E_2|P=3\rangle$$

$$+ |E_3\rangle\langle E_3|P=3\rangle$$

$$= 0\,|E_1\rangle + \frac{1}{\sqrt{2}}|E_2\rangle + \frac{1}{\sqrt{2}}|E_3\rangle$$

$$|P=3,t\rangle = 0\,|E_1\rangle\, e^{-iE_1t/\hbar}$$

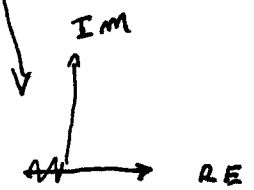
$$+ \frac{1}{\sqrt{2}}|E_2\rangle\, e^{-iE_2t/\hbar}$$

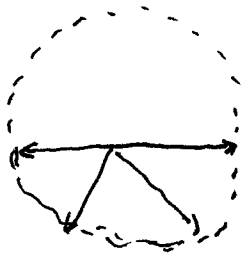
$$+ \frac{1}{\sqrt{2}}|E_3\rangle\, e^{-iE_3t/\hbar}$$

$$\frac{1}{\sqrt{2}}\begin{pmatrix}0\\1\\0\end{pmatrix} e^{-i4t/\hbar} + \frac{1}{\sqrt{2}}\begin{pmatrix}0\\0\\1\end{pmatrix} e^{-i5t/\hbar}$$

$$\frac{1}{\sqrt{2}} e^{-i4t/\hbar}\left[\begin{pmatrix}0\\1\\0\end{pmatrix} + e^{-i(1)t/\hbar}\begin{pmatrix}0\\0\\1\end{pmatrix}\right]$$

COEFF'S





$$\begin{pmatrix}0\\1\\0\end{pmatrix} + 1\begin{pmatrix}0\\0\\1\end{pmatrix} = \begin{pmatrix}0\\1\\1\end{pmatrix}$$

$$\begin{pmatrix}0\\1\\0\end{pmatrix} + (-1)\begin{pmatrix}0\\0\\1\end{pmatrix} = \begin{pmatrix}0\\1\\-1\end{pmatrix}$$

$$\begin{pmatrix}0\\1\\e^{-it/\hbar}\end{pmatrix}$$

DEGENERATE OPERATORS

$$H' = \begin{pmatrix} 3 & & \\ & 4 & \\ & & 4 \end{pmatrix}$$

$$\frac{1}{\sqrt{14}} \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

POSSIBLE ENERGIES	3	4	4
RESULTING STATE	$\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$	?	?

PROB $= \frac{1}{14}$ TOGETHER $\frac{13}{14}$

NEED TO (BOLDLY) GO BEYOND

$$\text{PROB}(E_i) = |\langle E_i | \psi \rangle|^2 \longrightarrow \text{STATE} = |E_i\rangle$$

GENERAL CASE

$$\text{PROB}(E_i) = \langle \psi | \mathbb{P}(E_i) | \psi \rangle$$

$$= \left| \mathbb{P}(E_i) | \psi \rangle \right|^2$$

WHAT IS $\mathbb{P}(E_i)$

$$\mathbb{P} = |2\rangle\langle 2| + |3\rangle\langle 3|$$

$$= \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \begin{pmatrix} 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\mathbb{P} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\text{PROB}(E=4) = \left| \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \frac{1}{\sqrt{14}} \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \right|^2$$

$$= \frac{2^2 + 3^2}{14} = \frac{13}{14}$$

WHAT IS THE STATE?

$$|\psi(0^+)\rangle = \mathbb{P}\,|\psi(0^-)\rangle$$

$$= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \frac{1}{\sqrt{14}} \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

$$= \frac{1}{\sqrt{14}} \begin{pmatrix} 0 \\ 2 \\ 3 \end{pmatrix}$$ NOT NORMALIZED

$$= \frac{1}{\sqrt{13}} \begin{pmatrix} 0 \\ 2 \\ 3 \end{pmatrix}$$

MEASURE E AGAIN ALWAYS FIND $E = 4$

STAY IN SAME STATE

The Addition of Phasors

http://ptolemy.eecs.berkeley.edu/eecs20/berkeley/phasors/demo/phasors.html

Falstad's One-Dimensional QM

http://www.falstad.com/qm1d/

More about Phasors

http://www.jhu.edu/signals/phasorlecture2/indexphasorlect2.htm