Hints for Homework Assignment 6

1. Consider a hydrogen atom in its ground state

$$\Phi_{100}(r,\theta,\phi) = \ exp(-r/a_0)/\sqrt{\pi a_0{}^3}.$$

(a) To find the most probable distance between the proton and the electron, you need to find the point with the largest probability—which is the point with zero slope in the probability versus r curve. The probability curve $P(r)$ is given by the square of the wavefunction with r^2 weighting:

$$P(r) \ = \ |\Phi_{100}(r,\theta,\phi)|^2 \ r^2 \ dr \ d\Omega.$$

(b) To find the average distance between the proton and the electron, you need to calculate the expectation value of r for the $\Phi_{100}(r,\theta,\phi)$ state. So you need to do the r expectation value integral:

$$< r > = \ \int_0^\infty \ \Phi^*{}_{100}(r,\theta,\phi) \ r \ \Phi_{100}(r,\theta,\phi) \ r^2 \ dr \ d\Omega.$$

(c) To calculate Δr, the rms width of the probability distribution versus r, you need to do the r^2 expectation value integral:

$$< r^2 > = \ \int_0^\infty \ \Phi^*{}_{100}(r,\theta,\phi) \ r^2 \ \Phi_{100}(r,\theta,\phi) \ r^2 \ dr \ d\Omega,$$

and to combine your result with your result from part b to obtain Δr.

(d) To calculate the probability that the electron will be found at a greater distance from the nucleus than would be allowed classically: First, figure out r_*, the largest classical distance that the electron can be from the proton. Second, integrate the probability distribution from r_* to infinity to find the probability that the separation will be greater than r_*. To find r_*, the "largest possible classical distance," set the electrostatic energy equal to the binding energy and solve for r_*. Once you have r_*, do the integral

$$\int_{r_*}^\infty \ \Phi^*{}_{100}(r,\theta,\phi) \ \Phi_{100}(r,\theta,\phi) \ r^2 \ dr \ d\Omega.$$

2. Consider a muonic atom which consists of a nucleus with positive charge Ze with a negative muon moving around it. The muon's charge is $-e$ and the muons mass is 207 times the electron mass. For a muonic atom with $Z = 6$ calculate:

(a) the radius of the first Bohr orbit;

(b) the energies of the first three bound states (i.e., the ground state, and the first and second excited states;

(c) the frequencies associated with the following transitions: $n_i = 2 \rightarrow n_f = 1$, $n_i = 3 \rightarrow n_f = 1$, and $n_i = 3 \rightarrow n_f = 2$.

This problem is very straightforward: you must simply repeat the same calculations that you would do for a hydrogen-like atom with $Z = 6$—except that you must use the reduced mass of muonium instead of the reduced mass of hydrogen.

3. Consider a hydrogen atom in the $n = 4$, $l = 3$, and $m = 3$ energy eigenstate.

(a) What is the magnitude of the angular momentum of the electron around the proton?

(b) What is the angle between the angular momentum vector and the z-axis? Can this angle be changed by changing n or m if l is held constant? What is the physical significance of this result?

(c) Sketch the probability distribution for finding the electron a distance r from the proton.

This problem is also very straightforward. It is designed to help you make sure that you understand the semiclassical vector model for angular momentum, and the radial probability distributions for the hydrogen atom.

4. Consider a hydrogen atom in the $n = 2$, $l = 1$, $m = -1$ energy eigenstate

$$\phi_{21-1}(r, \theta, \phi) = N\ r\ \exp(-r/2a_0)\ Y_{1-1}(\theta, \phi).$$

(a) To calculate the normalization constant N, you must do the normalization integral:

$$\int_0^\infty N^*\ r^*\ \exp(-r/2a_0)^*\ Y_{1-1}(\theta, \phi)^*\ N\ r\ \exp(-r/2a_0)\ Y_{1-1}(\theta, \phi)\ r^2\ dr\ d\Omega = 1.$$

(b) To calculate the probability per unit volume of finding the electron at $r = a_0$, $\theta = 45°$, and $\phi = 60°$, you must calculate the probability density with r^2 weighting:

$$P(r) \ = \ |\Phi_{21-1}(r, \theta, \phi)|^2\ r^2\ dr\ d\Omega.$$

(c) What is the probability per unit radial distance dr of finding the electron at $r = 2a_0$? *N.B.*, you must average over θ and ϕ.

(d) If you measure the energy, what are the possiblities and what are the probabilities?

(e) If you measure L^2, what are the possiblities and what are the probabilities?

(f) If you measure L_z, what are the possiblities and what are the probabilities?

5. Consider a hydrogen atom which is in the following superposition of its energy eigenstates Φ_{nlm} at $t = 0$

$$\psi = \; N \; [\sqrt{3} \; \Phi_{100} + \sqrt{2} \; \Phi_{211} - \Phi_{21-1} + \sqrt{5} \; \Phi_{322} - \sqrt{3} \; \Phi_{320} - \sqrt{2} \; \Phi_{43-3}].$$

(a) To calculate the normalization constant N, you must do the normalization integral:

$$\int_0^\infty \psi^* \psi \; r^2 \; dr \; d\Omega = 1.$$

To do this integral it is much easier to use the orthonormality of the spherical harmonics and of the radial functions than it is to do a lot of hairy integrals!

(b) Write down the time-dependent wavefunction.

(c) If you measure the energy, what are the possibilities and what are the probabilities?

(d) To calculate the expectation value of the energy, you must do the expectation value integral:

$$< E >= \int_0^\infty \psi^* \; H_{op} \; \psi \; r^2 \; dr \; d\Omega.$$

Remember that you know precisely what H_{op} does to its eigenfunctions, and that you know that both the spherical harmonics and the radial functions are orthonormal.

(e) If you measure L^2, what are the possibilities and what are the probabilities?

(f) To calculate the expectation value of L^2, you must do the expectation value integral:

$$< L^2 >= \int_0^\infty \psi^* \; L^2 \; \psi \; r^2 \; dr \; d\Omega.$$

Remember that you know precisely what L^2 does to its eigenfunctions, and that you know that both the spherical harmonics and the radial functions are orthonormal.

(g) If you measure L_z, what are the possibilities and what are the probabilities?

(h) Calculate the expectation value of L_z, you must do the expectation value integral:

$$< L_z >= \int_0^\infty \psi^* \; L_z \; \psi \; r^2 \; dr \; d\Omega.$$

Remember that you know precisely what L_z does to its eigenfunctions, and that you know that both the spherical harmonics and the radial functions are orthonormal.