Homework Assignment 6

1. Consider a hydrogen atom in its ground state

$$\phi_{100}(r,\theta,\phi) = \ exp(-r/a_0)/\sqrt{\pi {a_0}^3}.$$

(a) Find the most probable distance between the proton and the electron.

(b) Find the average distance between the proton and the electron.

(c) Calculate the rms width Δr of the probability distribution versus r.

(d) Calculate the probability that the electron will be found at a greater distance from the nucleus than would be allowed classically.

2. Consider a muonic atom which consists of a nucleus with positive charge Ze with a negative muon moving around it. The muon's charge is $-e$ and the muons mass is 207 times the electron mass. For a muonic atom with $Z = 6$ calculate:

(a) the radius of the first Bohr orbit;

(b) the energies of the first three bound states (i.e., the ground state, and the first and second excited states;

(c) the frequencies associated with the following transitions: $n_i = 2 \rightarrow n_f = 1$, $n_i = 3 \rightarrow n_f = 1$, and $n_i = 3 \rightarrow n_f = 2$.

3. Consider a hydrogen atom in the $n = 4$, $l = 3$, and $m = 3$ energy eigenstate.

(a) What is the magnitude of the angular momentum of the electron around the proton?

(b) What is the angle between the angular momentum vector and the z-axis? Can this angle be changed by changing n or m if l is held constant? What is the physical significance of this result?

(c) Sketch the probability distribution for finding the electron a distance r from the proton.

4. Consider a hydrogen atom in the $n = 2$, $l = 1$, $m = -1$ energy eigenstate

$$\phi_{21-1}(r,\theta,\phi) = N\ r\ \exp(-r/2a_0)\ Y_{1-1}(\theta,\phi).$$

(a) Calculate the normalization constant N.

(b) What is the probability per unit volume of finding the electron at $r = a_0$, $\theta = 45°$, and $\phi = 60°$?

(c) What is the probability per unit radial distance dr of finding the electron at $r = 2a_0$? *N.B.*, you must average over θ and ϕ.

(d) If you measure the energy, what are the possiblities and what are the probabilities?

(e) If you measure L^2, what are the possiblities and what are the probabilities?

(f) If you measure L_z, what are the possiblities and what are the probabilities?

5. Consider a hydrogen atom which is in the following superposition of its energy eigenstates ϕ_{nlm} at $t = 0$

$$\psi = N\,[\sqrt{3}\,\phi_{100} + \sqrt{2}\,\phi_{211} - \phi_{21-1} + \sqrt{5}\,\phi_{322} - \sqrt{3}\,\phi_{320} - \sqrt{2}\,\phi_{43-3}].$$

(a) Calculate the normalization constant N.

(b) Write down the time-dependent wavefunction.

(c) If you measure the energy, what are the possibilities and what are the probabilities?

(d) Calculate the expectation value of the energy.

(e) If you measure L^2, what are the possibilities and what are the probabilities?

(f) Calculate the expectation value of L^2.

(g) If you measure L_z, what are the possibilities and what are the probabilities?

(h) Calculate the expectation value of L_z.