Problem 1. Hydrogen Essentials

Almost everything you should remember about hydrogen forever ...

(a) Make a plot of the energy levels of the hydrogen atom. Plot the energy values E_n in the vertical direction for $n = 1, 2, 3, 4, 5$. Plot the the orbital angular momentum quantum number in the horizontal direction for $l = 0, 1, 2, 3, 4$. For each n, show every allowed value of l. Label every energy level spectroscopically (1s, 2s, 2p, ...). Indicate the m degeneracy of each l level. Show that the total degeneracy of each E_n is n^2.

(b) Make a composite sketch showing the $l = 0, 1, 2$ effective potentials. Add the locations of the $n = 1, 2, 3$ energy levels to your sketch, and explain how the levels in the different wells are lined up.

(c) Sketch just the $l = 0$ effective potential. Add the locations of the $n = 1, 2, 3$ energy levels to your sketch. Sketch the radial wavefunctions for each of these energy levels and label each wavefunction with the appropriate R_{nl} designation. Sketch the corresponding probability distributions per unit volume $|\psi(r)|^2 \; dV$. Sketch the corresponding radial probability distributions $4\pi r^2 \; |\psi(r)|^2 \; dr$.

(d) Sketch just the $l = 1$ effective potential. Add the locations of the $n = 1, 2, 3$ energy levels to your sketch. Sketch the radial wavefunctions for each of these energy levels and label each wavefunction with the appropriate R_{nl} designation. Sketch the corresponding per unit volume and radial probability distributions.

(e) Sketch just the $l = 2$ effective potential. Add the locations of the $n = 1, 2, 3$ energy levels to your sketch. Sketch the radial wavefunctions for each of these energy levels and label each wavefunction with the appropriate R_{nl} designation. Sketch the corresponding per unit volume and radial probability distributions.

(f) Sketch the three-dimensional probability distributions for $n = 1, 2, 3$. For each n, show all allowed values of l and m.

Problem 2. Hydrogen Superpositions

Putting n and l and m all together

Consider a hydrogen atom that is in the following superposition of its energy eigenkets $| \, n, l, m >$ at $t = 0$

$$|\psi(t=0)> = \; N \left[\; \sqrt{3} \, | \, 100 > + \, | \, 211 > + \, \sqrt{2} \, | \, 310 > + \, \sqrt{3} \, | \, 321 > \; \right].$$

(a) Calculate the normalization constant N.

(b) Write down the normalized time-dependent state vector $| \, \psi(t) >$ in the Hilbert space, *i.e.,* in Dirac notation.

(c) Write down the normalized time-dependent wavefunction $\psi(x, t)$ in position space, *i.e.,* in terms of the stationary states $\psi_{nlm}(x)$ of hydrogen.

(d) If you measure the energy at time t, what are the possibilities and what are the probabilities? Plot the probability distribution $P(E)$.

(e) If you measure L^2 at time t, what are the possibilities and what are the probabilities? Plot the probability distribution $P(L^2)$.

(f) If you measure L_z at time t, what are the possibilities and what are the probabilities? Plot the probability distribution $P(L_z)$.

(g) Are the probability distributions time-dependent? Explain.