For the following four topics include a sketch the potential and the wavefunctions with your description in words
The transmission and reflection coefficients for a potential step up
The transmission and reflection coefficients for a potential step down
The transmission and reflection coefficients for a potential well (down)
The transmission and reflection coefficients for a potential barrier (up)
Why study the square well?
Sketch the first four energy eigenfunctions for the square well in x-space
Sketch the corresponding first four probability densities for the square well in x-space
Sketch the first four energy eigenfunctions for the square well in p-space
Sketch the first four energy eigenfunctions for the square well in E-space
Obtaining the eigenfunctions for the square well
Obtaining the eigenenergies for the square well
The past, present, and future time evolution of a 1d free-particle Gaussian wave packet
The spreading of a Gaussian wave packet and the dispersion of empty space for matter waves
The eigenstates of momentum for the one-dimensional free particle
The phase velocity and the group velocity for the one-dimensional free particle
The minimum uncertainty state for the free particle
Why study the harmonic oscillator?
Sketch the first four energy eigenfunctions for the harmonic oscillator in x-space
Sketch the corresponding first four probability densities for the harmonic oscillator in x-space
Sketch the first four energy eigenfunctions for the harmonic oscillator in p-space
Sketch the first four energy eigenfunctions for the harmonic oscillator in E-space
Obtaining the eigenenergies for the harmonic oscillator via the separation of variables
Obtaining the eigenfunctions for the harmonic oscillator via the separation of variables
Charles Hermite, the Hermite equation, and the Hermite polynomials
Factoring the Hamiltonian for the harmonic oscillator
The ladder operators for the harmonic oscillator in Hilbert space
The ladder operators for the harmonic oscillator in position space
Obtaining the eigenenergies for the simple harmonic oscillator using the ladder operators
Obtaining the eigenfunctions for the simple harmonic oscillator using the ladder operators
The minimum uncertainty state for the harmonic oscillator
The zero-point energy and the zero-point motion of the harmonic oscillator

Transmission and Reflection

Consider the transmission and reflection coefficients for a potential with two steps down as shown in the attached figure.

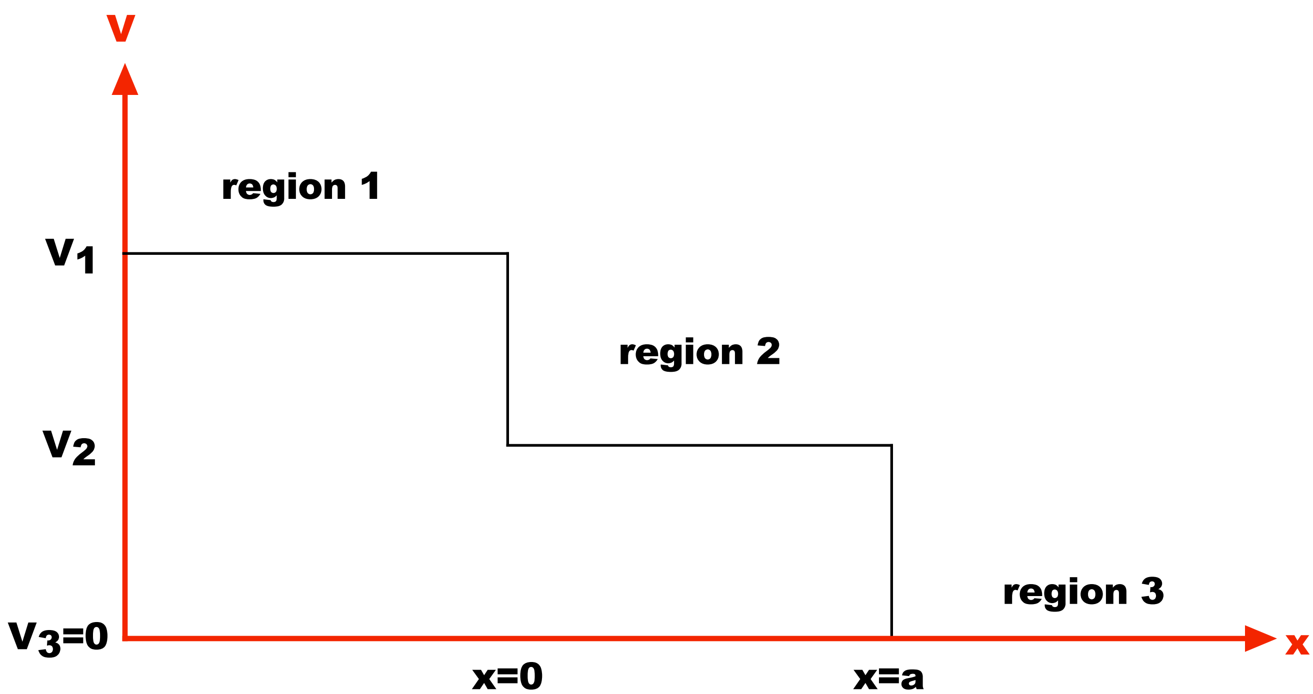
(a) Sketch the components of the wavefunctions in each region.

(b) Write down the functional form of the wavefunction in the three regions.

(c) Match the boundary conditions on the wavefunctions at each step to obtain the corresponding set of two equations relating the amplitudes.

(d) Match the boundary conditions on the derivatives of the wavefunctions at each step to obtain the corresponding set of two equations relating the amplitudes.

(e) Naively, there are six unknowns, but only four equations. Explain how to use these four equations to solve the problem. You do not need to solve the problem, you only need to explain.


v
region 1
v1
region 2
v2
region 3
v3=0
x=0
x=a
x

SHO Eigenstates and Superposition States

First consider the simple harmonic oscillator in one of its energy eigenstates $|n>$

(a) Calculate $< x(t) >$ using ladder operators.

(b) Calculate $< x^2(t) >$ using ladder operators.

(c) Calculate $< p(t) >$ using ladder operators.

(d) Calculate $< p^2(t) >$ using ladder operators.

(e) Explain why your results do not depend on time.

(f) Use your results above to calculate $\Delta x \Delta p$.

(g) Explain why the ground state is a minimum uncertainty state.

(h) Explain why the excited states are not minimum uncertainty states.

Now consider a simple harmonic oscillator in a superposition of its ground state $| 0 >$ and its first excited state $| 1 >$ at $t = 0$

$$|\psi(0) >= \sqrt{\frac{2}{3}} \mid 0 > + \sqrt{\frac{1}{3}} \mid 1 > .$$

(i) Express the normalized time-dependent state vector $|\psi(t) >$ in terms of the energy eigenkets.

(j) Calculate $< x(t) >$ using ladder operators.

(k) Calculate $< x^2(t) >$ using ladder operators.

(l) Calculate $\Delta x(t)$ using your results above.

(m) Write down—but do not calculate—the position-space integral that you would need to evaluate in order to obtain $< x(t) >$.

(n) Write down—but do not calculate—the position-space integral that you would need to evaluate in order to obtain $< x^2(t) >$.

(o) Compare and contrast solving this problem using Dirac notation—i.e., using ladder operators in the Hilbert space—with solving the problem by explicitly calculating the integrals.