Physics 441 Exam 1

Nominal Due Date February 15, 2012

Physics is about much more than equations---it is about ideas !!! Mathematics is the language that allows us to express those ideas in a compact, precise form. Some ideas require both the language of mathematics and the language of people to express them---quantum mechanics certainly does. Other ideas can be expressed using only words---because people intuitively understand the math. However, before some training, most people do not intuitively understand the mathematics used in quantum mechanics.

I want you to understand quantum mechanics, which to me means being able to express it in words, in pictures, and in equations. Rutherford said that if you really understand something you should be able to explain it to your grandmother. I am not asking you to explain quantum mechanics in a way that your grandmother would understand it, because your grandmother might not understand linear algebra and its infinite dimensional generalization called Hilbert space. Explain it so I know that you understand it.

For each of the topics on the next page, write clear, concise, physical descriptions that demonstrate you really understand the important qualitative aspects of quantum mechanics. You should be able to do this in a few sentences to a paragraph for each topic.

Explain the physics for each topic in your own words. You do not have to write a perfect essay on each topic, but do write enough to convince me that you really do understand the topic. Make sure to include any important pictures, graphs, and equations.

Try to write down, or draw, three or four important things for each topic.

	A
1	Linear vector spaces and Hilbert spaces
2	Dirac notation, bras, and kets
3	Changing basis using Dirac notation
4	Complex conjugation and the adjoint operation
5	Inner products, outer products, and projection operators
6	Hermitian operators and physical observables
7	A complete set of states (continuous and discrete)
8	Resolutions of the identity (continuous and discrete)
9	The canonical commutation relations
10	Degenerate Hermitian operators
11	Simultaneous diagonalization of Hermitian operators
12	A complete set of commuting observables
13	Unitary operators, time evolution, and changing bases
14	The propagator
15	Compatible, partially compatible, and completely incompatible operators
16	The position, momentum, and energy bases
17	Eigenvalues, eigenvectors, and eigenfunctions
18	The analogy between classical normal modes and quantum stationary states
19	Measurement possibilities and measurement probabilities
20	The time-independent Schrodinger equation
21	Solving the TISE by finding the stationary states
22	The time-dependent Schrodinger equation
23	Solving the TDSE by expanding the initial state in terms of the stationary states
24	Time-dependent and time-independent expectation values
25	Time-dependent and time-independent uncertainties
26	The first postulate of quantum mechanics
27	The second postulate of quantum mechanics
28	The third postulate of quantum mechanics
29	The fourth postulate of quantum mechanics
30	Measurement and the collapse of the wavefunction in the double slit experiment

Problem 1. Quantum Mechanics in a 3d State Space

Consider a system described by the Hamiltonian

$$H = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

(1) Find the associated eigenvalues of the Hamiltonian. Then find the normalized eigenvectors. Whether you solve this problem by inspection, or by an elaborate calculation, be sure to show that your eigenvalues and eigenvectors work!!!

At $t = 0$ this system is in the initial state

$$\mid \psi(t=0) > = N \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

(2) If you were to measure the energy at time $t = 0$, what results could you obtain, and with what probabilities would you obtain them?

(3) Calculate the expectation value of the energy of this system at $t = 0$, and show that your result agrees with the possibilities and probabilities that you obtained in part (2), *i.e.*, show that

$$< E > = < H > = < \psi(0)|H|\psi(0) > = \sum_i P(E_i)\ E_i$$

(4) Calculate the standard deviation of the energy of this system at $t = 0$, and show that your result agrees with the possibilities and probabilities that you obtained in part (2), *i.e.*, show that

$$\Delta E\ = \Delta H\ = \sqrt{\Big(< \psi(0)|(H- < H >)^2|\psi(0) > \Big)} = \sqrt{\sum_i P(E_i)\ (E_i- < H >)^2}$$

(5) Plot your calculated probabilities $P(E_i)$ versus E. Show your calculated expectation value and your calculated standard deviation on your plot. Discuss what this plot shows, *i.e.*, explain how the expectation value and the standard deviation are related to the outcome of a series of energy measurements.

(6) Now expand the initial state $\mid \psi(0) >$ in the energy eigenbasis, and write down the full time-dependent state vector $\mid \psi(t) >$. Express your result in both Dirac and vector notation.

(7) Calculate the time-dependent possibilities and probabilities of energy measurements at time t. Be sure to show that these possibilities and probabilities do not depend on time, and be sure to explain why they do not depend on time.

Consider a second observable operatorΩ

$$\Omega = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 3 \end{pmatrix}$$

(8) Find the eigenvalues and the eigenvectors of Ω.

(9) Write down the possibilities for Ω measurements, and calculate the time-dependent probabilities for Ω measurements.

(10) Explain why the results of Ω measurements are time-dependent and why energy measurements are not. Explain this in terms of the eigenvectors you calculated. Specifically, are H andΩ completely compatible, partially compatible, or completely incompatible? Then calculate the commutator $[\, H,\ \Omega\,]$ and explain how it is related to the time-dependence of Ω measurements.

Problem 2. Quantum Mechanics of the Spin of the Electron

Consider a system described (in the S_z basis) by the Hamiltonian $\mathbf{H}$

$$\mathbf{H} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Work this problem in the S_z basis.

At $t = 0$, the system is initially in the state $\mid \psi(0) >$ represented by $N \begin{pmatrix} (1-i) \\ (1+i) \end{pmatrix}$

(a) Calculate the normalization constant N.

(b) Find the eigenvalues and the eigenvectors of the Hamiltonian operator. Whether you find them by inspection, or by full calculation, show that they work!

(c) If you were to measure the energy H at time $t = 0$, what results could you obtain, and with what probabilities would you obtain them? What would the state vector be right after each energy measurement? Express the possible state vectors in both Dirac and vector notation.

(d) If you were to measure S_x at time $t = 0$, what results could you obtain, and with what probabilities would you obtain them? What would the state vector be right after each measurement? Express the possible state vectors in both Dirac and vector notation.

(e) If you were to measure S_y at time $t = 0$, what results could you obtain, and with what probabilities would you obtain them? What would the state vector be right after each measurement? Express the possible state vectors in both Dirac and vector notation.

(f) If you were to measure S_z at time $t = 0$, what results could you obtain, and with what probabilities would you obtain them? What would the state vector be right after each measurement? Express the possible state vectors in both Dirac and vector notation.

Consider the time-dependence of this system.

(g) Write down the zero-time state vector $\mid \psi(0) >$ in both Dirac and vector notation. Then write down the time-dependent state vector $\mid \psi(0) >$ again in both Dirac and vector notation.

(h) If you were to measure the energy H at time t, what results could you obtain, and with what probabilities would you obtain them? What would the state vector be right after each energy measurement? Express the possible state vectors in both Dirac and vector notation.

(i) If you were to measure S_x at time t, what results could you obtain, and with what probabilities would you obtain them? What would the state vector be right after each measurement? Express the possible state vectors in both Dirac and vector notation.

(j) If you were to measure S_y at time t, what results could you obtain, and with what probabilities would you obtain them? What would the state vector be right after each measurement? Express the possible state vectors in both Dirac and vector notation.

(k) If you were to measure S_z at time t, what results could you obtain, and with what probabilities would you obtain them? What would the state vector be right after each measurement? Express the possible state vectors in both Dirac and vector notation.