Consider a particle moving in the infinitely deep square well (remember that our square wells go from $-L$ to $+L$), which initially has the state vector

$$|\,\psi(0)> = A\,(\,|\,1> + |\,2>\,).$$

(a) First, calculate the normalization constant A. Then, express the normalized time-dependent state vector $|\,\psi(t)>$ in terms of the energy eigenkets. Finally, express the position-space time-dependent wavefunction $\psi(x,t) \;\; = < x \mid \psi(t) >$ in terms of the energy eigenfunctions in position space.

(b) Calculate the time-dependent expectation value of the position of the particle in the well $<x(t)>$.

Hint: Remember how to integrate the products of even and odd functions over symmetric limits! Also, you might find the following wavefunctions and definite integral helpful:

$$\psi_n(x) = \sqrt{\frac{1}{L}}\,\cos\left(\frac{n\pi x}{2L}\right) \qquad \text{when n is odd}$$

$$\psi_n(x) = \sqrt{\frac{1}{L}}\,\sin\left(\frac{n\pi x}{2L}\right) \qquad \text{when n is even}$$

$$\frac{1}{L}\int_{-L}^{+L} \cos\left(\frac{\pi x}{2L}\right)\,x\,\sin\left(\frac{2\pi x}{2L}\right)\,dx = \frac{16L}{9\pi^2}$$

(c) Sketch the ground state wavefunction $\psi_0(x)$ and the first excited state wavefunction $\psi_1(x)$ inside the well. Then explain how the sum of these two functions (times their respective time-dependent phase factors) produce the time-dependent motion of the particle in the well. Make a qualitative sketch of the shape of the probability distribution at three times: when the particle has its maximum and minimum expectation values $<x> = \pm\, 32L/9\pi^2$, and when it has an expectation value of zero, $<x> = 0$. Explain how your sketches agree qualitatively with your answer to part b.

(d) If the position is measured at time t, what results can be found, and with what probabilities will these results be found?

(e) If the energy is measured at time t, what results can be found, and with what probabilities will these results be found?

(f) Calculate the expectation value of the energy $<E>$ and the standard deviation of the energy ΔE. Then sketch $P(E)$ *versus* E and show your values of $<E>$ and ΔE on your sketch.