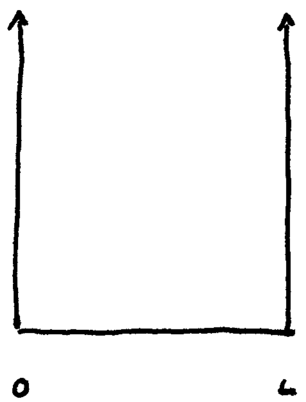
# Lecture 9

INFINITE SQUARE WELL

"PARTICLE IN A BOX"




SOLVE TISE $\Rightarrow$ STATIONARY STATES (TI)

$\Downarrow$

TIME-DEPENDENT STATES

$$-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} + V(x)\,\psi(x) = E\,\psi(x)$$

$$\frac{d^2\psi}{dx^2} + \frac{2mE}{\hbar^2}\psi = 0$$

$$\frac{d^2\psi}{dx^2} + K^2\psi = 0$$

$$\Rightarrow \quad A e^{iKx} + B e^{-iKx} \qquad \sqrt{2mE/\hbar^2} = K$$

GENERAL SOLUTION

GENERAL SOLUTION
+
BOUNDARY CONDITIONS $\Rightarrow$ SPECIFIC SOLN'S

$$\psi(0) = 0 \Rightarrow A \sin(kx)$$

$$\psi(L) = 0 \Rightarrow A \sin\left(\frac{n\pi}{L} x\right)$$

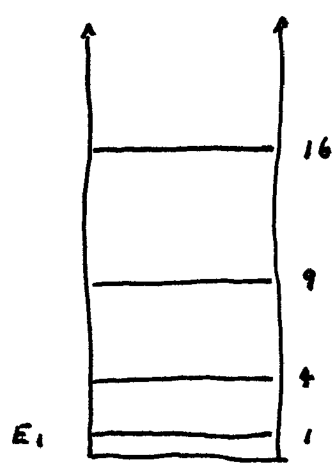
$$A \sin(k_n x)$$

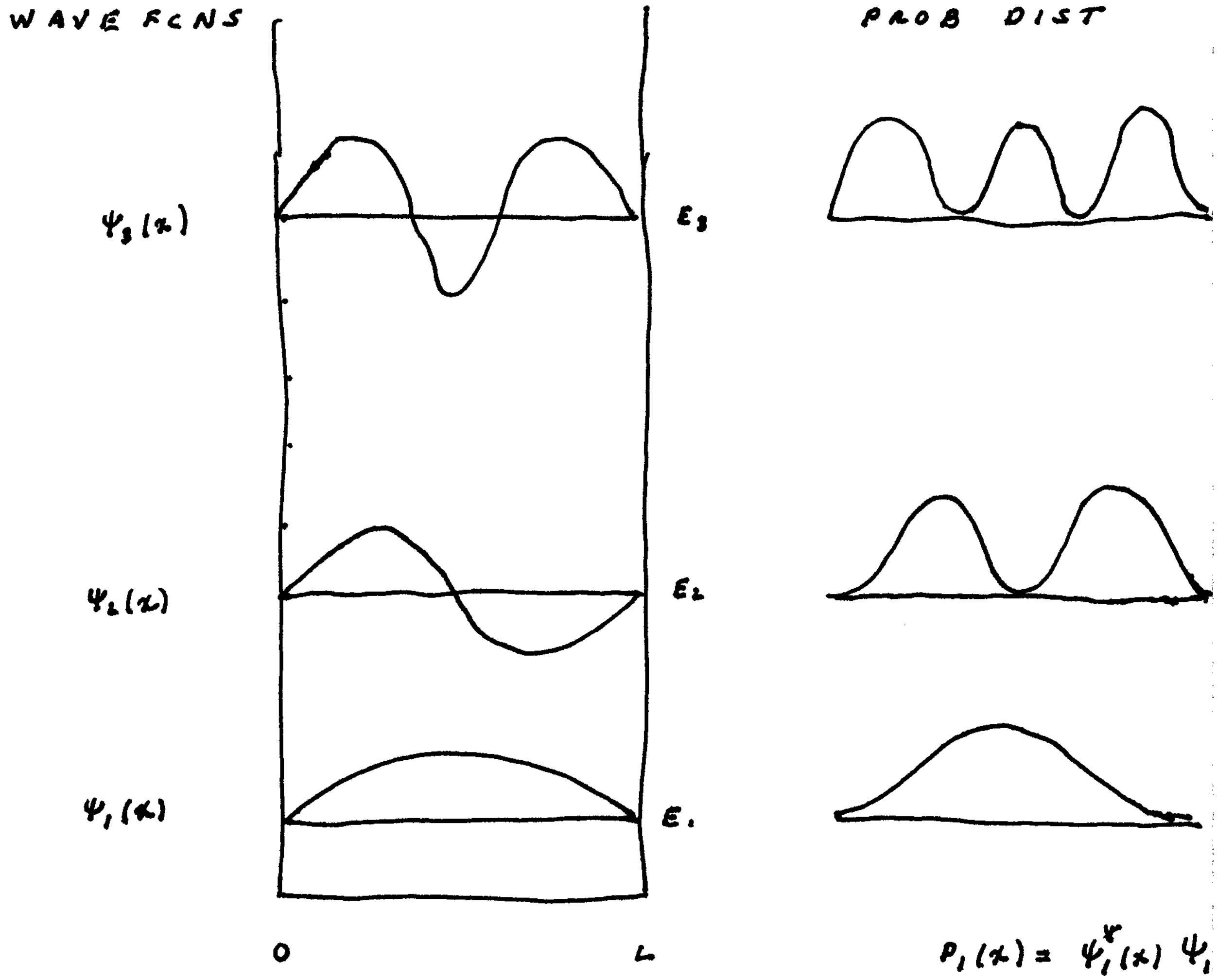
$$k_n = \frac{n\pi}{L}$$

NORMALIZED STATIONARY STATES

$$\psi_n(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi}{L} x\right)$$

$$k_n = \frac{n\pi}{L} \Rightarrow E_n = \frac{\hbar^2 k^2}{2m} = \left(\frac{\pi^2 \hbar^2}{2mL^2}\right) n^2 = E_1 n^2$$






$$P_1(x) = \psi_1^*(x)\,\psi_1(x)$$
$$= |\psi_1(x)|^2$$

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$$E_n \sim n^2 / L^2$$

The larger the well, the lower the energy

The higher the quantum number n, the higher the energy

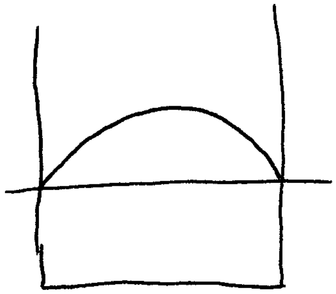
E = KE + PE

The potential energy for the square well is the same everywhere except at the boundaries where it is infinite. Consequently, the wavefunction must be zero at the boundaries.

The kinetic energy for the square well is proportional to the curvature of the wavefcn.

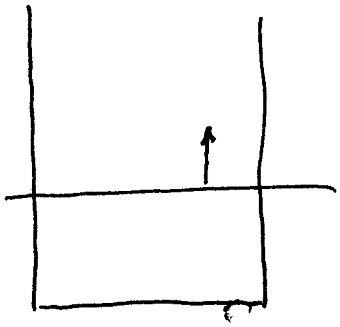
The ground state wavefunction has the minimum curvature---that is not zero everywhere and is zero at its ends.

IF YOU MEASURE POSITION



BEFORE

$\psi_1(x)$ eigenfunction of energy

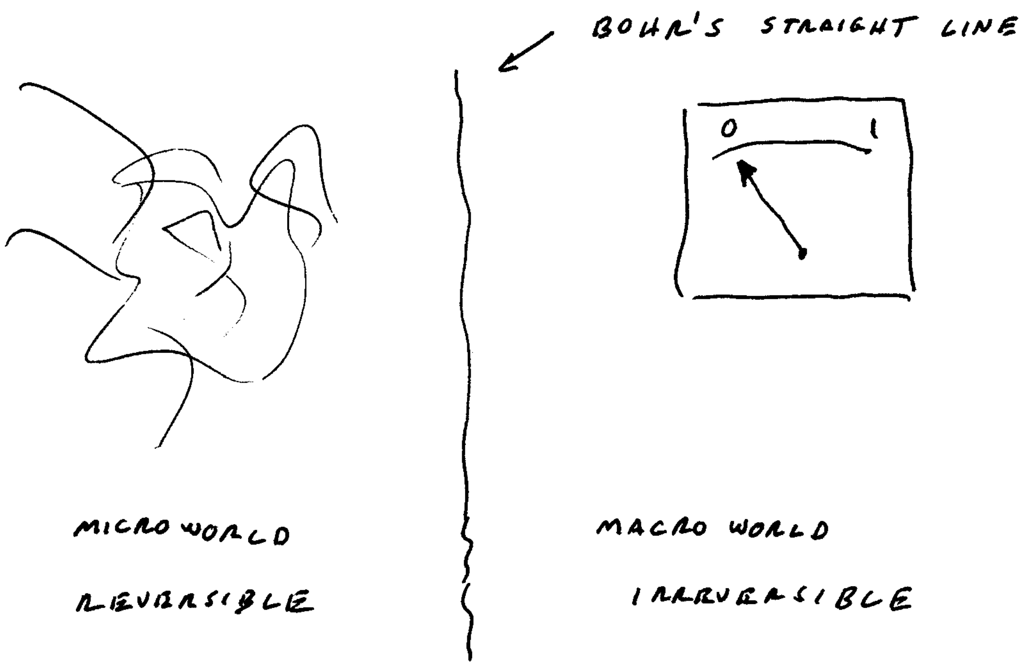


AFTER

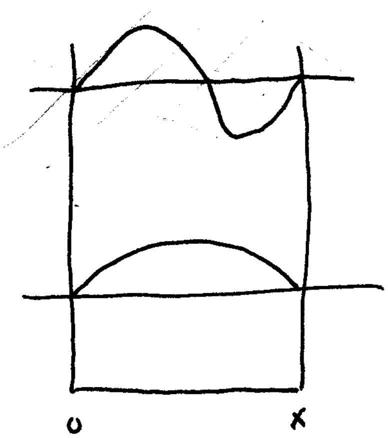
eigenfunction of position

NRQM: This is instantaneous

RQM: NOT INSTANTANEOUS



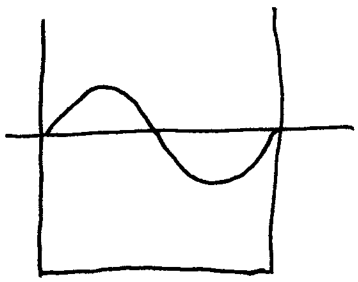
MEASURE POSITION ?




FIRST STATIONARY STATE $\psi_1(x)$

$PROB(x)\,dx = P(x)\,dx = |\psi_1(x)|^2\,dx$

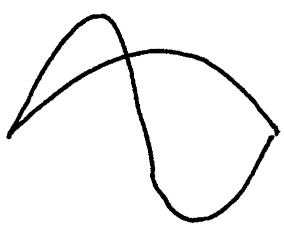
$P(x)$ is stationary



SECOND STATIONARY STATE $\psi_2(x)$

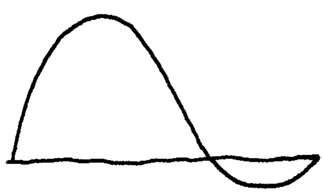
$P(x)\,dx = |\psi_2(x)|^2\,dx$

$P(x)$ is stationary



SUPERPOSITION STATE

$$\psi(x,0) = \left(\psi_1(x) + \psi_2(x)\right)\frac{1}{\sqrt{2}}$$



NOW $P(x)$ is not stationary!

$\psi(x,0)$

$P(x,0) = |\psi(x,0)|^2$

TIME-DEPENDENCE OF THE ENERGY EIGENKETS

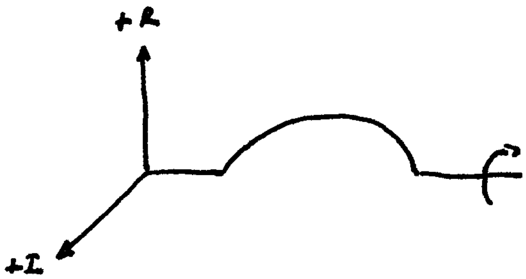
$$|E_n(t)\rangle = e^{-iE_nt/\hbar}\,|E_n(0)\rangle$$

TIME-DEPENDENCE OF THE STATIONARY STATES

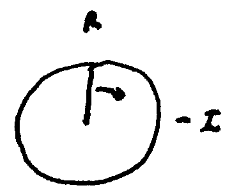
$$\psi_n(x,t) = e^{-iE_nt/\hbar}\,\psi_n(x)$$

IN POSITION SPACE

CLOCK HAND




$\omega_1 = E_1/\hbar$




$\omega_2 = E_2/\hbar = 4E_1/\hbar$

$\omega_2 = 4\omega_1$

$\omega_3 = E_3/\hbar = 9\omega_1$

# THE QUANTUM BABY RATTLE

$$| \psi(0) \rangle = \frac{1}{\sqrt{2}} | E_1 \rangle + \frac{1}{\sqrt{2}} | E_2 \rangle$$

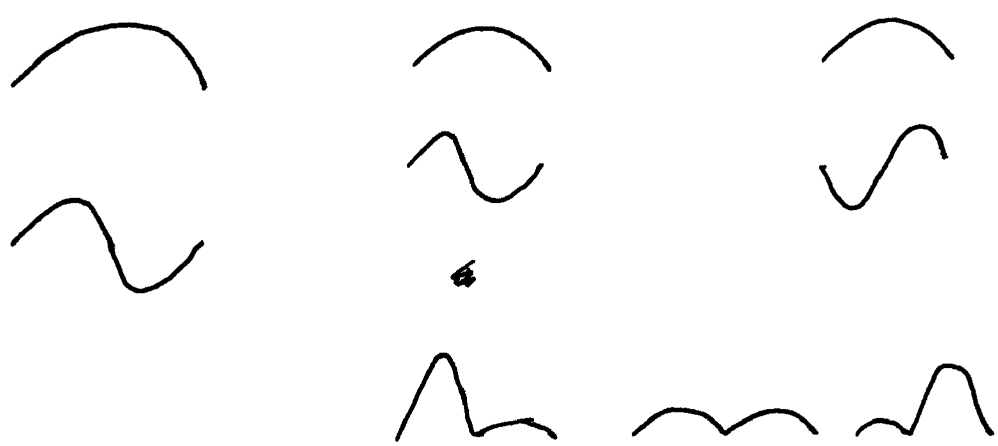
$$\psi(x,0) = \frac{1}{\sqrt{2}} \psi_1(x) + \frac{1}{\sqrt{2}} \psi_2(x)$$

TD

$$| \psi(t) \rangle = \frac{1}{\sqrt{2}} | E_1 \rangle e^{-iE_1 t/\hbar} + \frac{1}{\sqrt{2}} | E_2 \rangle e^{-iE_2 t/\hbar}$$

$$\psi(x,t) = \frac{1}{\sqrt{2}} \psi_1(x) e^{-iE_1 t/\hbar} + \frac{1}{\sqrt{2}} \psi_2(x) e^{-iE_2 t/\hbar}$$

CYCLIC BEHAVIOR : in phase, perp, 180° out, perp



50:50 SUPERPOSITION STATE

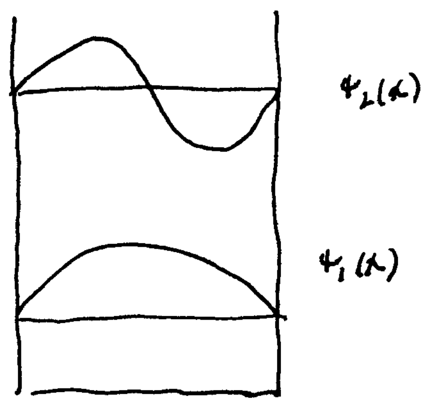
$$|\psi(0)\rangle = \frac{1}{\sqrt{2}}|E_1\rangle + \frac{1}{\sqrt{2}}|E_2\rangle$$

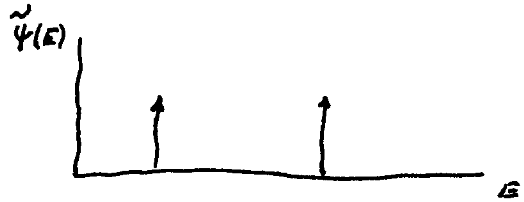
$$|\psi(t)\rangle = \frac{1}{\sqrt{2}}|E_1\rangle e^{-iE_1t/\hbar} + \frac{1}{\sqrt{2}}|E_2\rangle e^{-iE_2t/\hbar}$$

$\langle x|$ $$\psi(x,t) = \frac{1}{\sqrt{2}}\psi_1(x)\, e^{-iE_1t/\hbar} + \frac{1}{\sqrt{2}}\psi_2(x)\, e^{-iE_2t/\hbar}$$

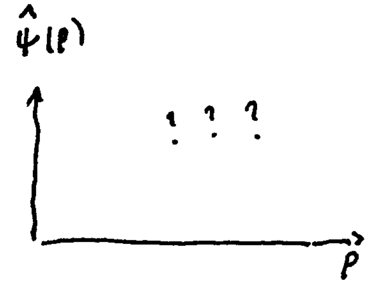
$\langle p|$ $$\hat{\psi}(p,t) = \frac{1}{\sqrt{2}}\hat{\psi}_1(p)\, e^{-iE_1t/\hbar} + \frac{1}{\sqrt{2}}\hat{\psi}_2(p)\, e^{-iE_2t/\hbar}$$

$\langle E|$ $$\tilde{\psi}(E,t) = \frac{1}{\sqrt{2}}\delta(E-E_1)\, e^{-iE_1t/\hbar} + \frac{1}{\sqrt{2}}\delta(E-E_2)\, e^{-iE_2t/\hbar}$$




TWO QUESTIONS:



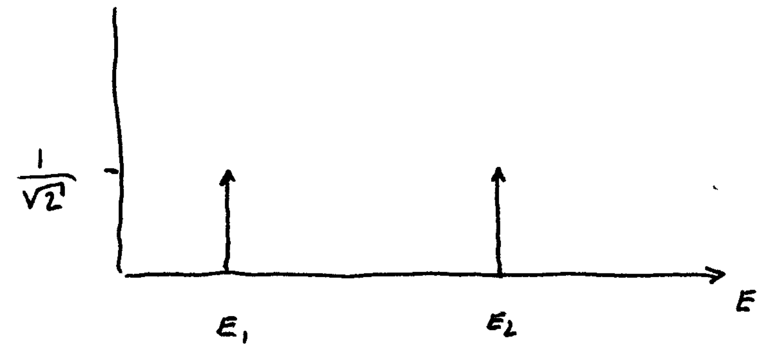

$\langle x(t)\rangle$

IF YOU MEASURE ....

ENERGY
POSSIBLE
RESULTS : EIGENVALUES OF THE HAMILTONIAN

EIGENENERGIES

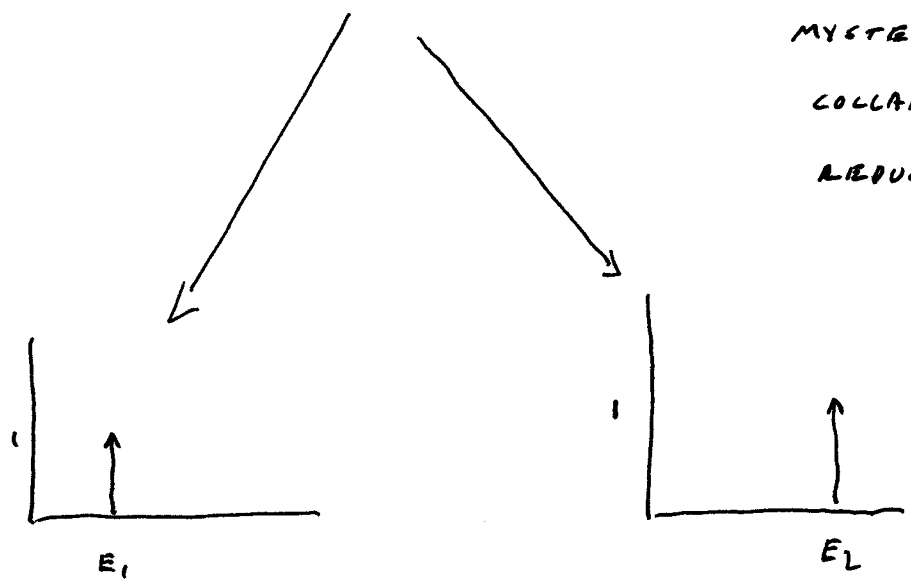
PROBABILITIES : HOW MUCH OF THAT EIGENSTATE IS IN $|\psi\rangle$




MYSTERIOUS QUANTUM JUMP

COLLAPSE OF THE WAVEFCN

REDUCTION OF THE STATE VECTOR




MYSTERIOUS BECAUSE :

MEASURE ENERGY $\Rightarrow$ WHAT ARE POSSIBLE RESULTS?

WHAT ARE THE ASSOCIATED PROBS?

COMMON SENSE

$$|\psi(0)\rangle = \frac{1}{\sqrt{2}}|E_1\rangle + \frac{1}{\sqrt{2}}|E_2\rangle$$

EQUAL MIXTURE OF $E_1$ AND $E_2$

EQUAL PROB OF $E_1$ AND $E_2$

TOTAL PROB = 1

$PROB(E_1) = \frac{1}{2}$

$PROB(E_2) = \frac{1}{2}$

$$\left(\frac{1}{\sqrt{2}}\right)^2 = \frac{1}{2}$$

$$PROB = |a_j|^2 \qquad \text{if } \sum_j |a_j|^2 = 1$$

FORMAL METHOD

MEASURE ENERGY

POSSIBILITIES : ev's OF $H$

$$E_1 , E_2 , E_3 , \ldots$$

PROB OF EACH ONE : 50% $E_1$ 50% $E_2$

$$\text{PROB}(E_1) = \left| \langle E_1 | \Psi(t) \rangle \right|^2$$

$$= \left| \left( \langle E_1 | \left( \frac{1}{\sqrt{2}} |E_1\rangle e^{-iE_1 t/\hbar} + \frac{1}{\sqrt{2}} |E_2\rangle e^{-iE_2 t/\hbar} \right) \right) \right|^2$$

$$= \left| \frac{1}{\sqrt{2}} e^{-iE_1 t/\hbar} \right|^2 = \frac{1}{2}$$

$$\text{PROB}(E_1) = \left| \int_0^a \psi_1^*(x) \left( \frac{1}{\sqrt{2}} \psi_1(x) e^{-iE_1 t/\hbar} + \frac{1}{\sqrt{2}} \psi_2(x) e^{-iE_2 t/\hbar} \right) dx \right|^2$$

$$= \left| \frac{1}{\sqrt{2}} \frac{2}{a} \int_0^a \sin\left(\frac{\pi}{a} x\right) \sin\left(\frac{\pi}{a} x\right) e^{-iE_1 t/\hbar} \, dx \right.$$

$$\left. + \frac{1}{\sqrt{2}} \frac{2}{a} \int_0^a \sin\left(\frac{\pi}{a} x\right) \sin\left(\frac{2\pi}{a} x\right) e^{-iE_2 t/\hbar} \, dx \right|^2$$

$$= \left| \frac{1}{\sqrt{2}} \right|^2 = \frac{1}{2} \implies 50\%$$

WORKS FOR ANY $f(x) = \Psi(x,0)$

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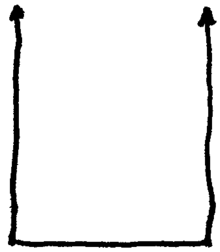
# THREE CONVENTIONS FOR THE SQUARE WELL

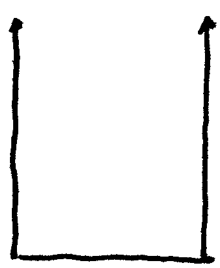


$-a \qquad +a$

$$\psi_n(x) = \sqrt{\frac{1}{a}}\,\cos\left(\frac{n\pi}{2a}x\right) \qquad n \text{ ODD}$$

$$= \sqrt{\frac{1}{a}}\,\sin\left(\frac{n\pi}{2a}x\right) \qquad n \text{ EVEN}$$

$$E_n = \left(\frac{\hbar^2\pi^2}{8ma^2}\right)n^2 \qquad L = 2a$$

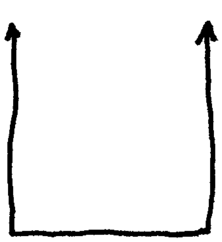


$-\frac{a}{2} \qquad +\frac{a}{2}$

$$\psi_n(x) = \sqrt{\frac{2}{a}}\,\cos\left(\frac{n\pi}{a}x\right) \qquad n \text{ odd}$$

$$= \sqrt{\frac{2}{a}}\,\sin\left(\frac{n\pi}{a}x\right) \qquad n \text{ even}$$

$$E_n = \left(\frac{\hbar^2\pi^2}{2ma^2}\right)n^2$$

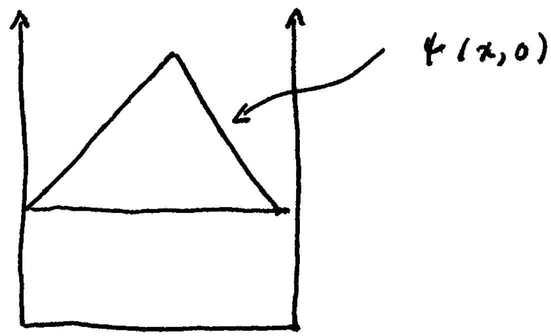


$0 \qquad a$

$$\psi_n(x) = \sqrt{\frac{2}{a}}\,\sin\left(\frac{n\pi}{a}x\right)$$

$$E_n = \left(\frac{\hbar^2\pi^2}{2ma^2}\right)n^2$$

# TIME-DEPENDENCE OF AN ARBITRARY STATE




FOUR-STEP PLAN

(1) DIAGONALIZE $H$

(2) EXPAND $|\psi(0)\rangle$

(3) WRITE DOWN $|\psi(t)\rangle$

(4) CALCULATE EVERYTHING $\quad \langle x\rangle \quad \Delta x$

$\langle p\rangle \quad \Delta p$

$\langle E\rangle \quad \Delta E$

$\vdots$

$$|\psi(0)\rangle = \sum_n |E_n\rangle\langle E_n|\psi(0)\rangle$$

$$|\psi(t)\rangle = \sum_n e^{-iE_nt/\hbar}\,|E_n\rangle\,\underbrace{\langle E_n|\psi(0)\rangle}_{a_n}$$

$$|\psi(t)\rangle = \sum_n a_n\,|E_n\rangle\, e^{-iE_nt/\hbar}$$

in x basis

$$\psi(x,t) = \sum_n a_n\,\psi_n(x)\, e^{-iE_nt/\hbar}$$

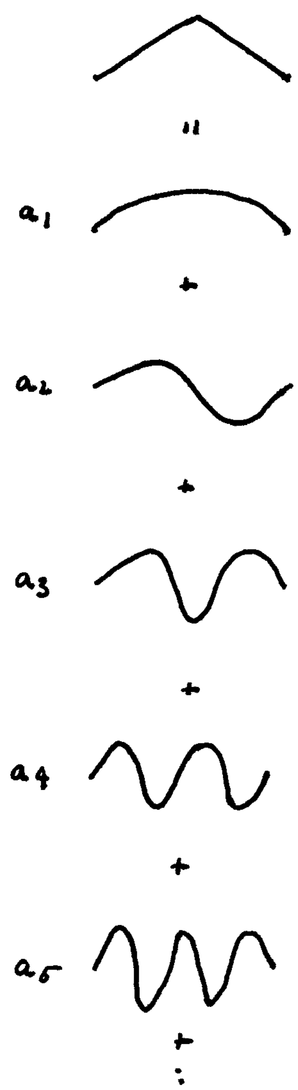
## EXPANSION COEFFICIENTS

$$a_m = \langle E_m | \psi(0) \rangle = \langle E_m | I | \psi(0) \rangle$$

$$= \int \underbrace{\langle E_m | x \rangle}_{\psi_m^*(x)} \underbrace{\langle x | \psi(0) \rangle}_{\psi(x,0)} \, dx$$

$$a_m = \int \psi_m^*(x) \; \psi(x,0) \; dx$$

## CARTOON VERSION




SYMMETRY => ONLY ODD TERMS

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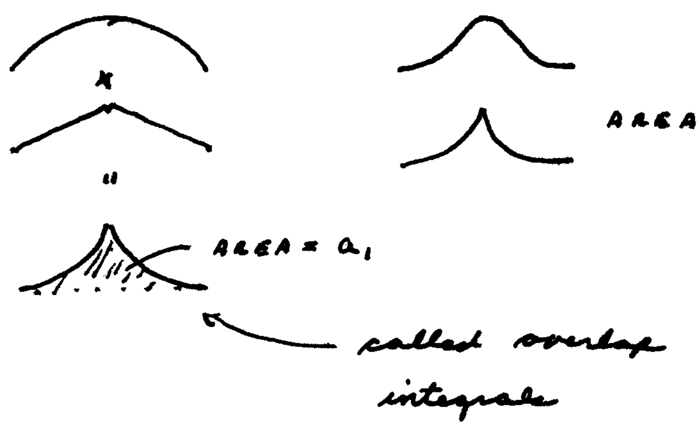
TO CALCULATE THE EXPANSION COEFFS

$$a_n = \int_0^L \psi_n^*(x)\ \psi(x,0)\ dx$$

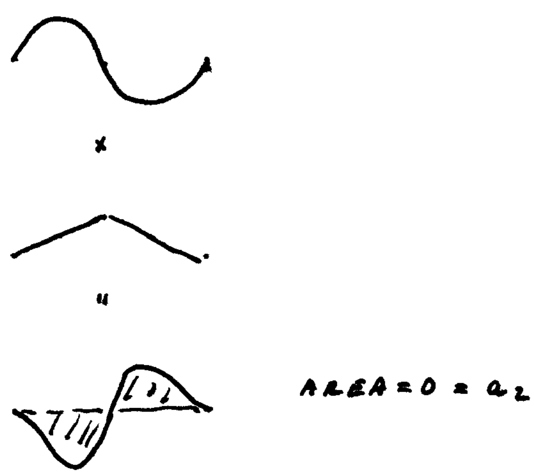
$$= \int_0^L \sqrt{\frac{2}{L}}\ \sin\left(\frac{n\pi}{L}x\right)\ \psi(x,0)\ dx$$

$\psi$ $\qquad$ $|\psi|^2$

$a_1$




$a_2$

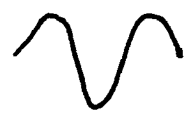


TIME - DEPENDENCE

$\psi(x,t) = \quad a_1 e^{-i\omega_1 t}$

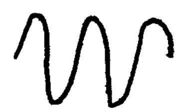
$T_1$

$+ a_3 e^{-i\omega_3 t}$



$T_3 = \frac{T_1}{9}$

$+ a_5 e^{-i\omega_5 t}$



$T_5 = \frac{T_1}{25}$

$\Rightarrow$ RECURRENCE TIME $= T_1$ !

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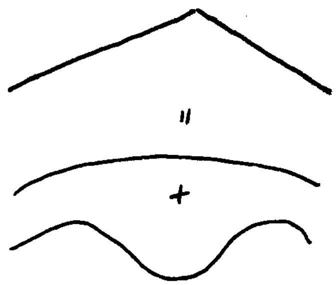
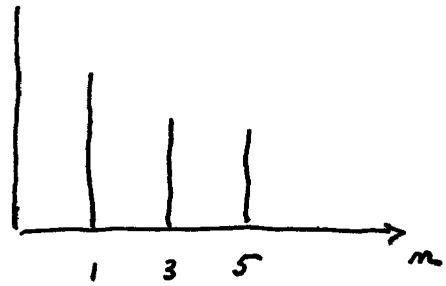
FOR THE SQUARE WELL



FOURIER SERIES




$a_n$'s are the FOURIER SERIES EXPANSION COEFFS! DISCRETE

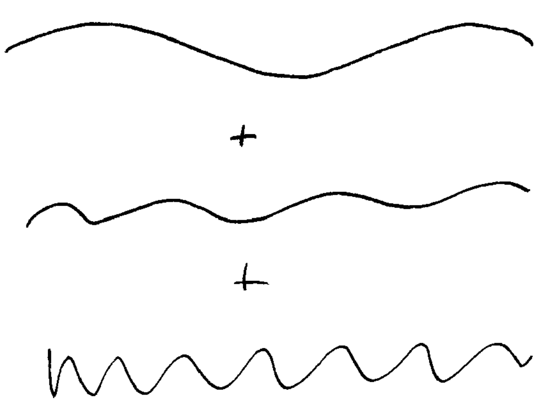
FOR THE FREE PARTICLE

Gaussian

FOURIER TRANSFORM

Gaussian

=



$a_n$'s are given by the Fourier transform coeffs

CONTINUOUS

SHOW MOVIES HERE!?

EXPECTATION VALUES

IF $[H, \Omega] = 0$ THEN $\langle \Omega \rangle$ and $\langle \Omega^2 \rangle$ are constant

SINCE $[H, H] = 0$ $\langle H \rangle = \langle E \rangle$ constant

$\langle H^2 \rangle = \langle E^2 \rangle$ constant

FOR THE QUANTUM RATTLE

$$\langle E(0) \rangle = \langle \psi(0) | H | \psi(0) \rangle$$

$$= \frac{1}{\sqrt{2}} \left( \langle E_2| + \langle E_1| \right) H \frac{1}{\sqrt{2}} \left( |E_1\rangle + |E_2\rangle \right)$$

$$= \frac{1}{2} (E_1 + E_2)$$

$$= \frac{5}{2} E_1 \qquad E_2 = 4 E_1$$

$$\langle E(t) \rangle = \langle \psi(t) | H | \psi(t) \rangle$$

$$= \frac{1}{\sqrt{2}} \left( \langle E_2| \, e^{+iE_2 t/\hbar} + \langle E_1| \, e^{+iE_1 t/\hbar} \right) H$$

$$\frac{1}{\sqrt{2}} \left( |E_1\rangle \, e^{-iE_1 t/\hbar} + |E_2\rangle \, e^{-iE_2 t/\hbar} \right)$$

$$= \frac{5}{2} E_1$$

$$\langle \psi | H^2 | \psi \rangle = \left( \langle E_2 | + \langle E_1 | \right) \frac{1}{\sqrt{2}} \quad H^2 \; \frac{1}{\sqrt{2}} \left( E_1 + E_2 \right)$$

$$= \frac{1}{2} \left( \langle E_2 + \langle E_1 | \right) \left( E_1^2 | E_1 \rangle + E_2^2 | E_2 \rangle \right)$$

$$= \frac{1}{2} E_1^2 + \frac{1}{2} E_2^2$$

since $E_2 = 4 E_1$

$$= \left( \frac{1}{2} + \frac{16}{2} \right) E_1^2$$

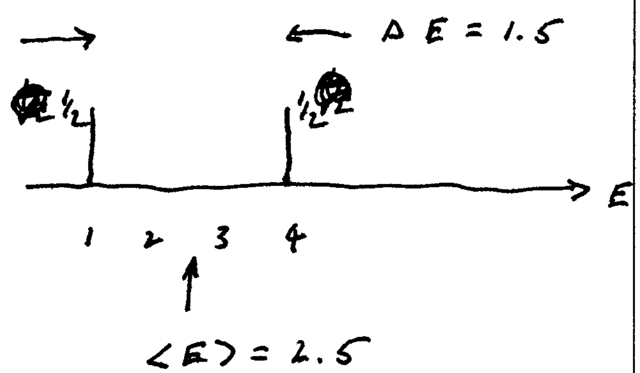
$$= \frac{17}{2} E_1^2 \qquad \text{TI}$$

$$\Delta H = \sqrt{\left( \langle H^2 \rangle - \langle H \rangle^2 \right)}$$

$$= \sqrt{\frac{17}{2} E_1^2 - \left( \frac{5}{2} E_1 \right)^2}$$

$$= \sqrt{\frac{34 - 25}{4} E_1^2} \qquad \text{TI}$$

$$= \frac{3}{2} E_1$$

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POSITION: $\langle x \rangle \quad \langle x^2 \rangle \quad \Delta x$

$$[x, H] = x\left(\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\right) - \left(\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\right)x$$

IN GENERAL

$$[x, H] \neq 0$$

$$[x^2, H] \neq 0$$

BUT FOR THE STATIONARY STATES

$$\langle E_n | (xH - Hx) | E_n \rangle = \langle E_n | (-Hx + xH) | E_n \rangle$$

$$= \langle E_n | (-E_n x + x E_n) E_n \rangle$$

$$= 0$$

$$\langle x(t) \rangle = \langle \psi(t) | x | \psi(t) \rangle$$

$$= \frac{1}{\sqrt{2}} \left( \langle E_1 | e^{+iE_1 t/\hbar} + \langle E_2 | e^{+iE_2 t/\hbar} \right) x$$

$$\frac{1}{\sqrt{2}} \left( |E_1\rangle e^{-iE_1 t/\hbar} + |E_2\rangle e^{-iE_2 t/\hbar} \right)$$

$$= \frac{1}{2} \int x \, dx \left( \psi_1^*(x) e^{i\omega_1 t} + \psi_2^*(x) e^{i\omega_2 t} \right)$$
$$\left( \psi_1(x) e^{-i\omega_1 t} + \psi_2(x) e^{-i\omega_2 t} \right)$$

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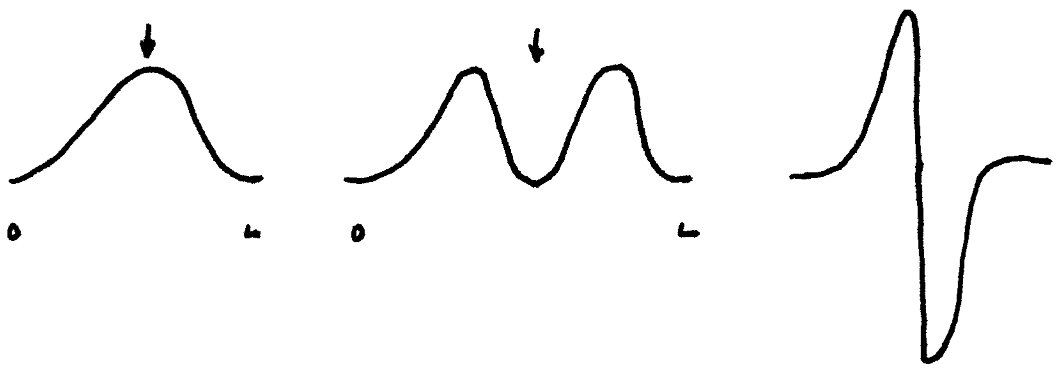
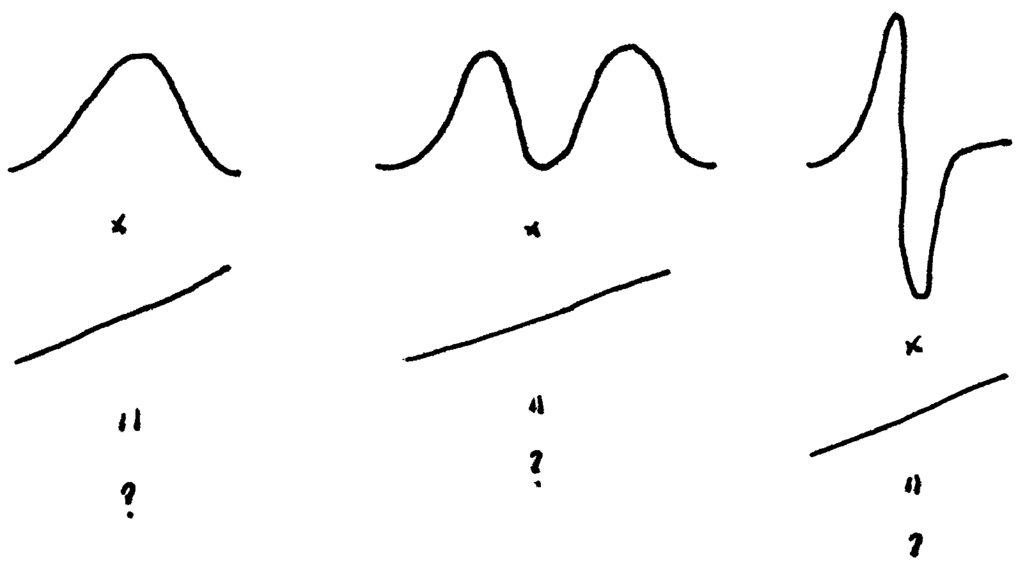
$$\langle x(t)\rangle = \int_0^L x\,dx\left[\frac{1}{2}\left|\psi_1(x)\right|^2 + \frac{1}{2}\left|\psi_2(x)\right|^2 + \psi_1(x)\,\psi_2(x)\,\cos(\Omega t)\right]$$

$$\Omega = \omega_2 - \omega_1 = (E_2 - E_1)/\hbar = 3E_1/\hbar = 3\omega_1$$



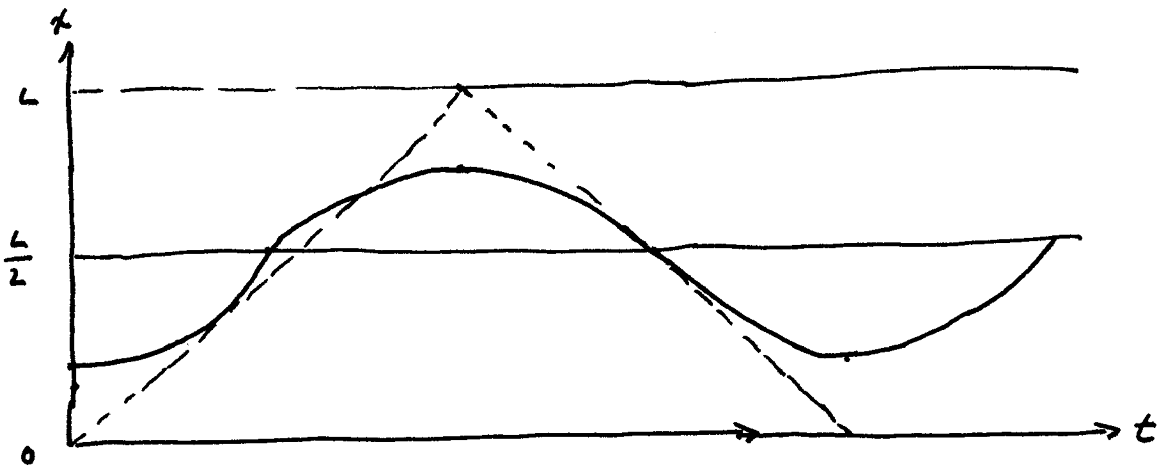

OSCILLATING

DOING THE INTEGRALS



STAEDTLER® No. 937 811E Engineer's Computation Pad

$$\langle \psi(t) | x | \psi(t) \rangle = \langle x(t) \rangle = \frac{L}{2} - \frac{16L}{9\pi^2} \cos(\Omega t)$$




quantum particle feels the walls sooner than the classical point particle

$$\frac{2}{L} \int_0^L \sin\left(\frac{\pi}{L} x\right) x \sin\left(\frac{2\pi}{L} x\right) dx = -\frac{16L}{9\pi^2}$$

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EXPECTATION VALUES IN MOMENTUM SPACE

in general $[p, H] \neq 0$

for stationary states $[p, H] = 0$

$$\langle E_m | pH - Hp | E_m \rangle$$

$$\langle E_m | -Hp + pH | E_m \rangle$$

$$\langle E_m | -E_m p + p E_m | E_m \rangle$$

$$0$$

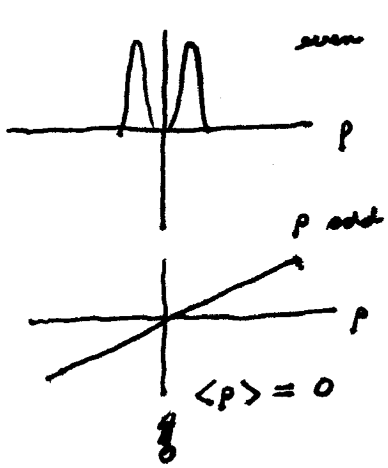
FOR THE ENERGY EIGENSTATES

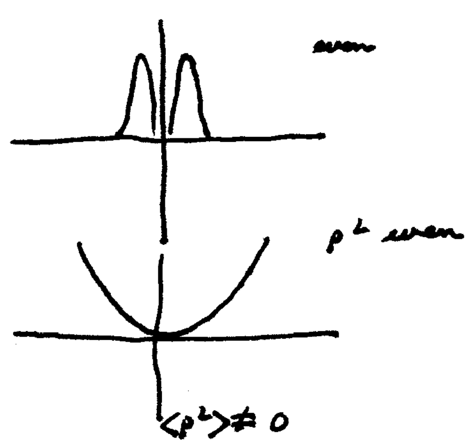
$$\langle p \rangle = 0$$

$$\langle p^2 \rangle = \left( \frac{\hbar^2 \pi^2}{a^2} \right) m^2$$

$$\Delta p = \left( \frac{\hbar \pi}{a} \right) m$$

FROM COMPUTER SIMULATION



QM BABY RATTLE

$$|\psi(0)\rangle = \frac{1}{\sqrt{2}}\left( |E_1\rangle + |E_2\rangle \right)$$

$$\langle p(0)\rangle = \langle \psi(0)|\ p\ |\psi(0)\rangle$$

$$= \frac{1}{\sqrt{2}}\left( \langle E_1| + \langle E_2| \right)\ p\ \frac{1}{\sqrt{2}}\left( |E_1\rangle + |E_2\rangle \right)$$

in $x$-space

$$= \frac{1}{2}\int \left[\psi_1^*(x) + \psi_2^*(x)\right] -i\hbar\frac{d}{dx}\left[\psi_1(x) + \psi_2(x)\right] dx$$

in $p$-space

$$= \frac{1}{2}\int \left[\hat{\psi}_1^*(p) + \hat{\psi}_2^*(p)\right]\ p\ \left[\hat{\psi}_1(p) + \hat{\psi}_2(p)\right] dp$$

$$\langle p(t)\rangle = \langle \psi(t)|\ p\ |\psi(t)\rangle$$

similar to $\langle x(t)\rangle$

$$\int p\, dp\ \frac{1}{2}\left(|\hat{\psi}_1(p)|^2 + |\hat{\psi}_2(p)|^2\right) + \left[\hat{\psi}_1^*(p)\,\hat{\psi}_2(p) + \hat{\psi}_1(p)\,\hat{\psi}_2^*(p)\right] \sin(\Omega t)$$

WAVEFUNCTIONS IN MOMENTUM SPACE

$$\psi_m(x) = \sqrt{\frac{2}{a}} \sin\left(\frac{m\pi}{a} x\right)$$

$$\hat{\psi}_m(p) = FT\Big(\psi_m(x)\Big)$$

$$= \sqrt{\frac{1}{2\pi\hbar}} \int_0^a \sqrt{\frac{2}{a}} \sin\left(\frac{m\pi}{a} x\right) e^{-ipx/\hbar}\, dx$$

$$\frac{1}{2i}\left[ e^{i(\ )} - e^{-i(\ )} \right]$$

$$= \frac{1}{2i} \sqrt{\frac{1}{2\pi\hbar}} \sqrt{\frac{2}{a}} \int_0^a \left[ e^{i\left(\frac{m\pi}{a} - \frac{p}{\hbar}\right)x} - e^{-i\left(\frac{m\pi}{a} + p/\hbar\right)x} \right] dx$$

$$\frac{\exp\left(\frac{m\pi}{a} - \frac{p}{\hbar}\right)x - 1}{i\left(\frac{m\pi}{a} - \frac{p}{\hbar}\right)} - \frac{\exp\left(\frac{m\pi}{a} + \frac{p}{\hbar}\right) - 1}{-i\left(\frac{m\pi}{a} + \frac{p}{\hbar}\right)}$$

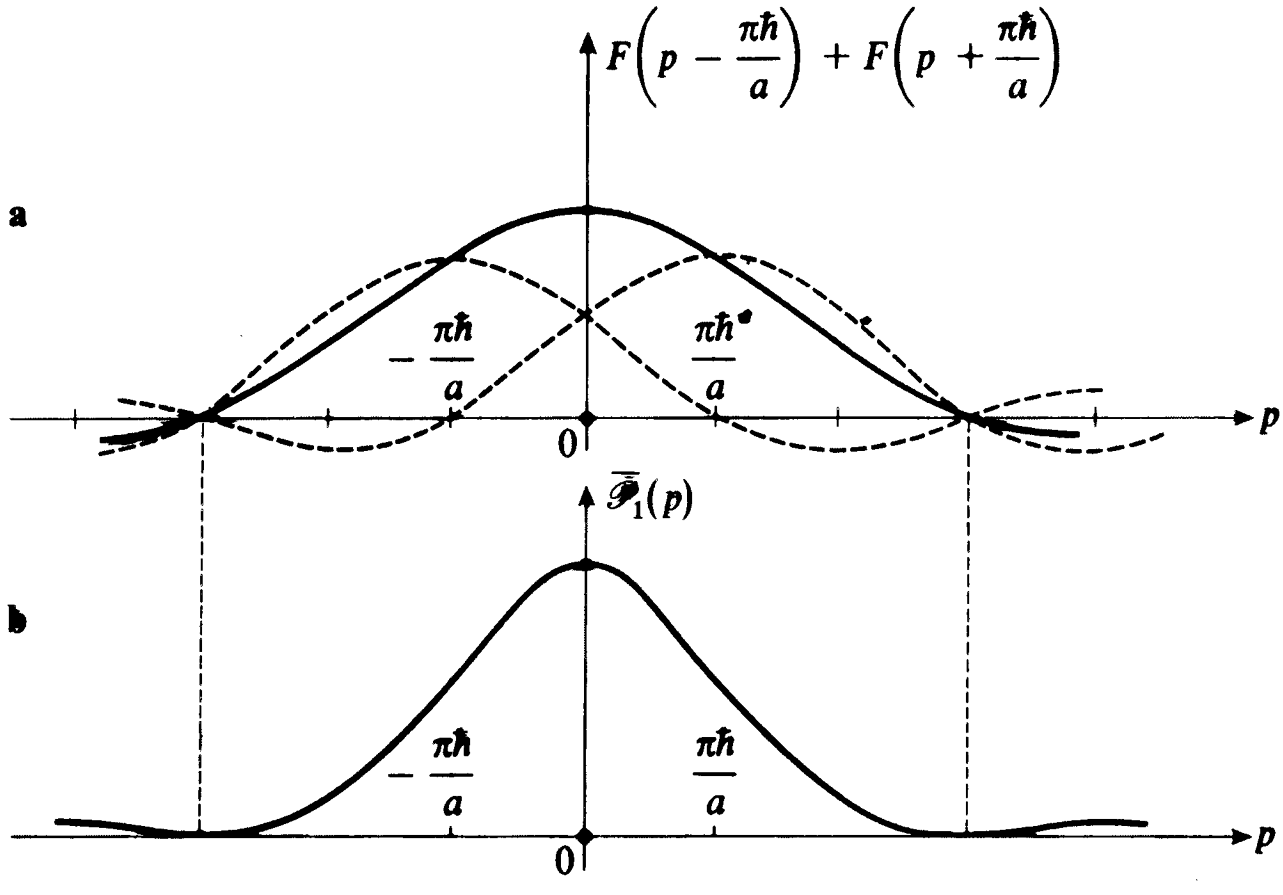
$$\hat{\psi}_n(p) = \frac{1}{2i}\sqrt{\frac{a}{\pi\hbar}}\; e^{i\left(\frac{n\pi}{2} - \frac{pa}{2\hbar}\right)}$$

$$\times \left[ F\left(p - \frac{n\pi\hbar}{a}\right) + (-1)^{n+1} F\left(p + \frac{n\pi\hbar}{a}\right) \right]$$

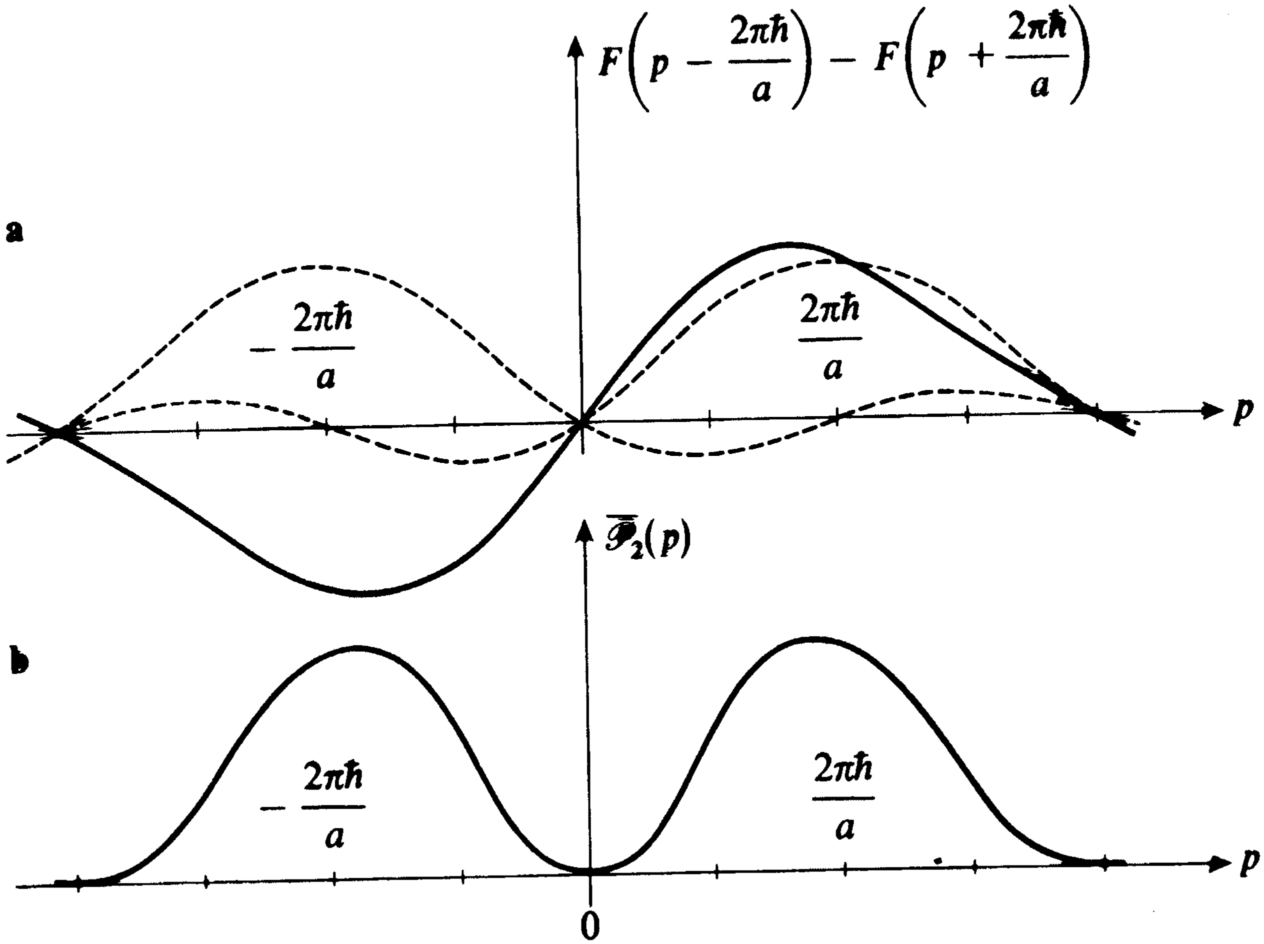
$$F(p) = \frac{\sin(pa/2\hbar)}{(pa/2\hbar)}$$

$$= \frac{\sin x}{x}$$

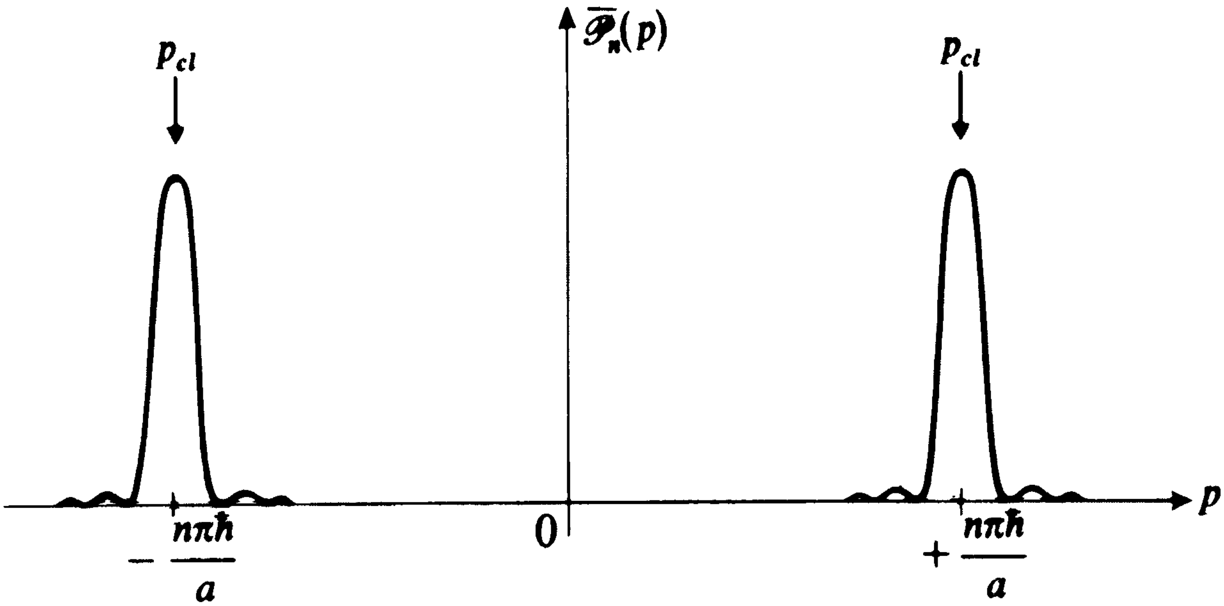
$$= \operatorname{sinc}(x)$$



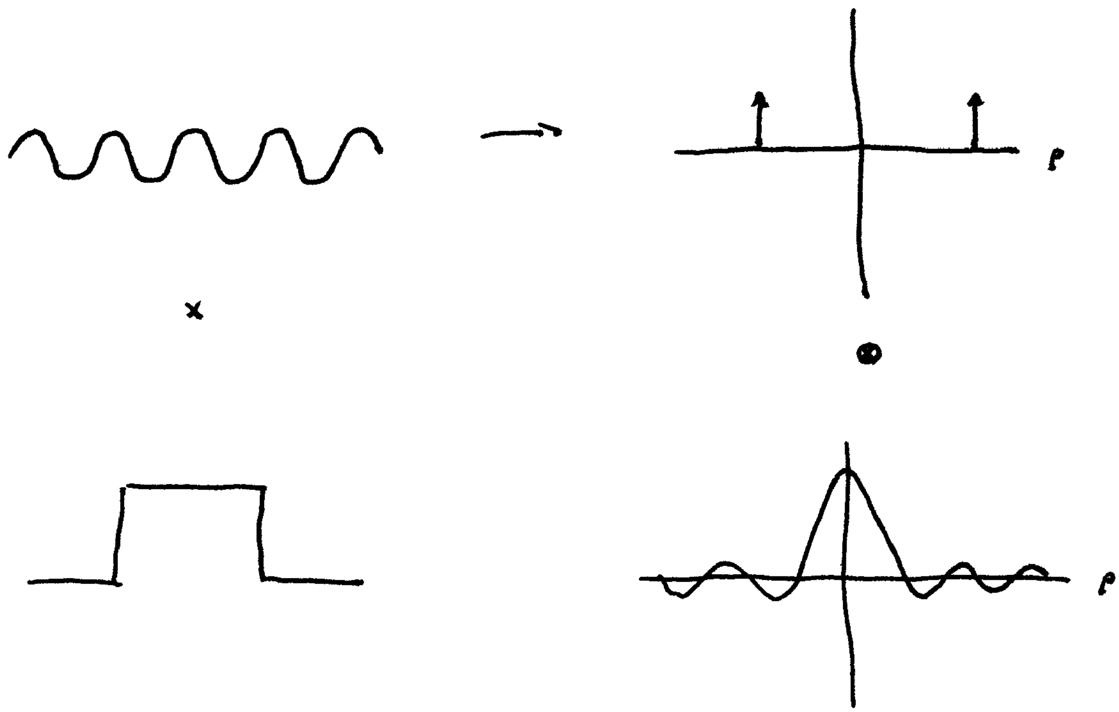
$F\left(p - \frac{\pi\hbar}{a}\right) + F\left(p + \frac{\pi\hbar}{a}\right)$
a
$-\frac{\pi\hbar}{a}$
$\frac{\pi\hbar}{a}$
0
p
$\bar{\bar{\mathscr{P}}}_1(p)$
b
$-\frac{\pi\hbar}{a}$
$\frac{\pi\hbar}{a}$
0
p



$F\left(p - \frac{2\pi\hbar}{a}\right) - F\left(p + \frac{2\pi\hbar}{a}\right)$
a
$-\frac{2\pi\hbar}{a}$
$\frac{2\pi\hbar}{a}$
$p$
$\overline{\mathscr{P}}_2(p)$
b
$-\frac{2\pi\hbar}{a}$
$\frac{2\pi\hbar}{a}$
$p$
0


$\overline{\mathscr{P}}_n(p)$
$p_{cl}$
$p_{cl}$
$p$
$0$
$-\frac{n\pi\hbar}{a}$
$+\frac{n\pi\hbar}{a}$

CARTOON VERSION

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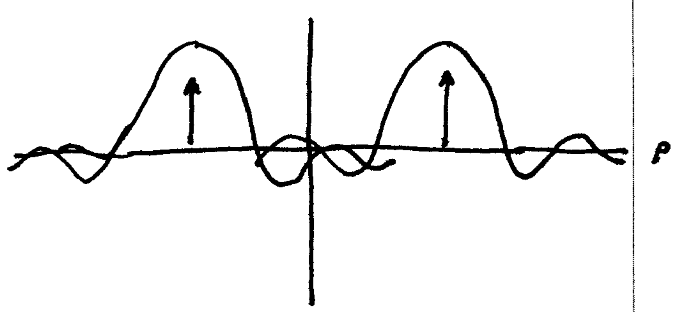
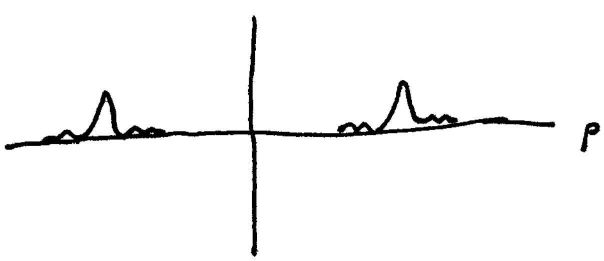


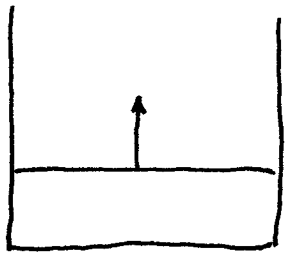

$$\boxed{A \cdot B = C \iff \hat{A} \otimes \hat{B} = \hat{C}}$$

CONVOLUTION THEOREM

as $n \uparrow$, more and more like a simple standing wave



MEASURE X FIND IN CENTER OF WELL



BY SYMMETRY => ONLY ODD $n$

yes

yes

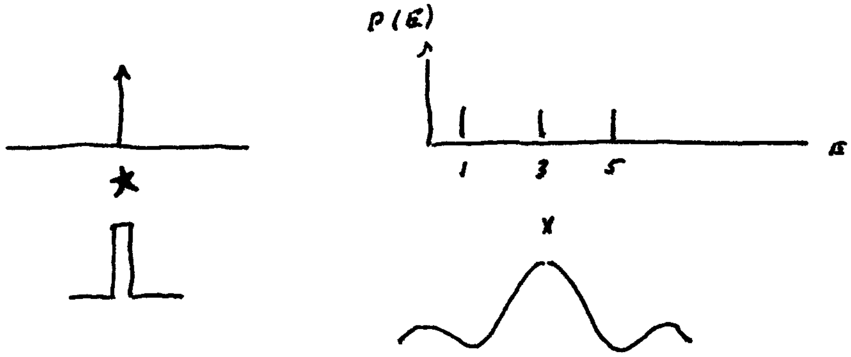
no

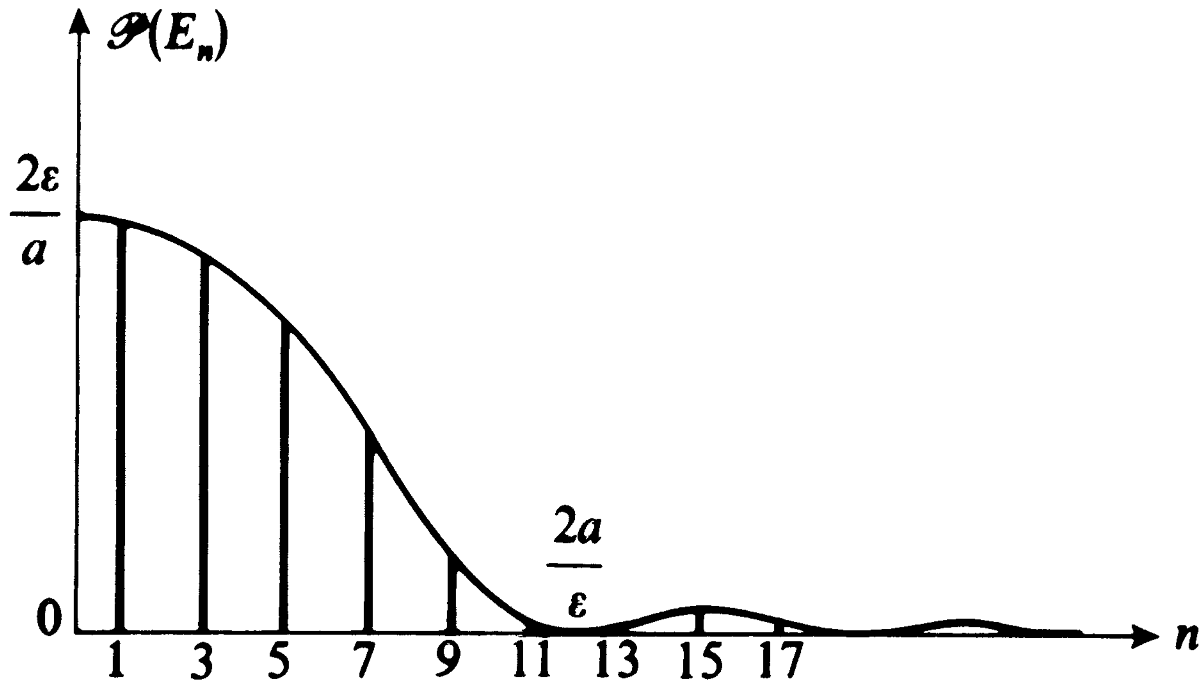
$$P(E_n) = \frac{2}{a} \quad n \text{ odd}$$

$$= 0 \quad n \text{ even}$$

=> $\infty$ ENERGY

FINITE X RESOLUTION





$\mathscr{P}(E_n)$
$\frac{2\varepsilon}{a}$
0
$\frac{2a}{\varepsilon}$
1
3
5
7
9
11
13
15
17
$n$