Lecture 5

THE POSTULATES OF QM

MOTIVATION

OVERVIEW

PATH INTEGRAL POSTULATES

EXAMPLES FOR POSTULATES 1-3

Preface to the First Edition

Publish and perish—*Giordano Bruno*

Given the number of books that already exist on the subject of quantum mechanics, one would think that the public needs one more as much as it does, say, the latest version of the Table of Integers. But this does not deter me (as it didn't my predecessors) from trying to circulate my own version of how it ought to be taught. The approach to be presented here (to be described in a moment) was first tried on a group of Harvard undergraduates in the summer of '76, once again in the summer of '77, and more recently at Yale on undergraduates ('77–'78) and graduates ('78–'79) taking a year-long course on the subject. In all cases the results were very satisfactory in the sense that the students seemed to have learned the subject well and to have enjoyed the presentation. It is, in fact, their enthusiastic response and encouragement that convinced me of the soundness of my approach and impelled me to write this book.

The basic idea is to develop the subject from its postulates, after addressing some indispensable preliminaries. Now, most people would agree that the best way to teach any subject that has reached the point of development where it can be reduced to a few postulates is to start with the latter, for it is this approach that gives students the fullest understanding of the foundations of the theory and how it is to be used. But they would also argue that whereas this is all right in the case of special relativity or mechanics, a typical student about to learn quantum mechanics seldom has any familiarity with the mathematical language in which the postulates are stated. I agree with these people that this problem is real, but I differ in my belief that it should and can be overcome. This book is an attempt at doing just this.

It begins with a rather lengthy chapter in which the relevant mathematics of vector spaces developed from simple ideas on vectors and matrices the student is assumed to know. The level of rigor is what I think is needed to make a practicing quantum mechanic out of the student. This chapter, which typically takes six to eight lecture hours, is filled with examples from physics to keep students from getting too fidgety while they wait for the "real physics." Since the math introduced has to be taught sooner or later, I prefer sooner to later, for this way the students, when they get to it, can give quantum theory their fullest attention without having to battle with the mathematical theorems at the same time. Also, by segregating the mathematical theorems from the physical postulates, any possible confusion as to which is which is nipped in the bud.

This chapter is followed by one on classical mechanics, where the Lagrangian and Hamiltonian formalisms are developed in some depth. It is for the instructor to decide how much of this to cover; the more students know of these matters, the better they will understand the connection between classical and quantum mechanics. Chapter 3 is devoted to a brief study of idealized experiments that betray the inadequacy of classical mechanics and give a glimpse of quantum mechanics.

Having trained and motivated the students I now give them the postulates of quantum mechanics of a single particle in one dimension. I use the word "postulate" here to mean "that which cannot be deduced from pure mathematical or logical reasoning, and given which one can formulate and solve quantum mechanical problems and interpret the results." This is not the sense in which the true axiomatist would use the word. For instance, where the true axiomatist would just postulate that the dynamical variables are given by Hilbert space operators, I would add the operator identifications, i.e., specify the operators that represent coordinate and momentum (from which others can be built). Likewise, I would not stop with the statement that there is a Hamiltonian operator that governs the time evolution through the equation $i\hbar\partial|\psi\rangle/\partial t = H|\psi\rangle$; I would say the H is obtained from the classical Hamiltonian by substituting for x and p the corresponding operators. While the more general axioms have the virtue of surviving as we progress to systems of more degrees of freedom, with or without classical counterparts, students given just these will not know how to calculate anything such as the spectrum of the oscillator. Now one can, of course, try to "derive" these operator assignments, but to do so one would have to appeal to ideas of a postulatory nature themselves. (The same goes for "deriving" the Schrödinger equation.) As we go along, these postulates are generalized to more degrees of freedom and it is for pedagogical reasons that these generalizations are postponed. Perhaps when students are finished with this book, they can free themselves from the specific operator assignments and think of quantum mechanics as a general mathematical formalism obeying certain postulates (in the strict sense of the term).

The postulates in Chapter 4 are followed by a lengthy discussion of the same, with many examples from fictitious Hilbert spaces of three dimensions. Nonetheless, students will find it hard. It is only as they go along and see these postulates used over and over again in the rest of the book, in the setting up of problems and the interpretation of the results, that they will catch on to how the game is played. It is hoped they will be able to do it on their own when they graduate. I think that any attempt to soften this initial blow will be counterproductive in the long run.

Chapter 5 deals with standard problems in one dimension. It is worth mentioning that the scattering off a step potential is treated using a wave packet approach. If the subject seems too hard at this stage, the instructor may decide to return to it after Chapter 7 (oscillator), when students have gained more experience. But I think that sooner or later students must get acquainted with this treatment of scattering.

The classical limit is the subject of the next chapter. The harmonic oscillator is discussed in detail in the next. It is the first realistic problem and the instructor may be eager to get to it as soon as possible. If the instructor wants, he or she can discuss the classical limit after discussing the oscillator.

Postulate 1

Classical Mechanics

I. The state of a particle at any given time is specified by the two variables $x(t)$ and $p(t)$, i.e., as a point in a two-dimensional phase space.

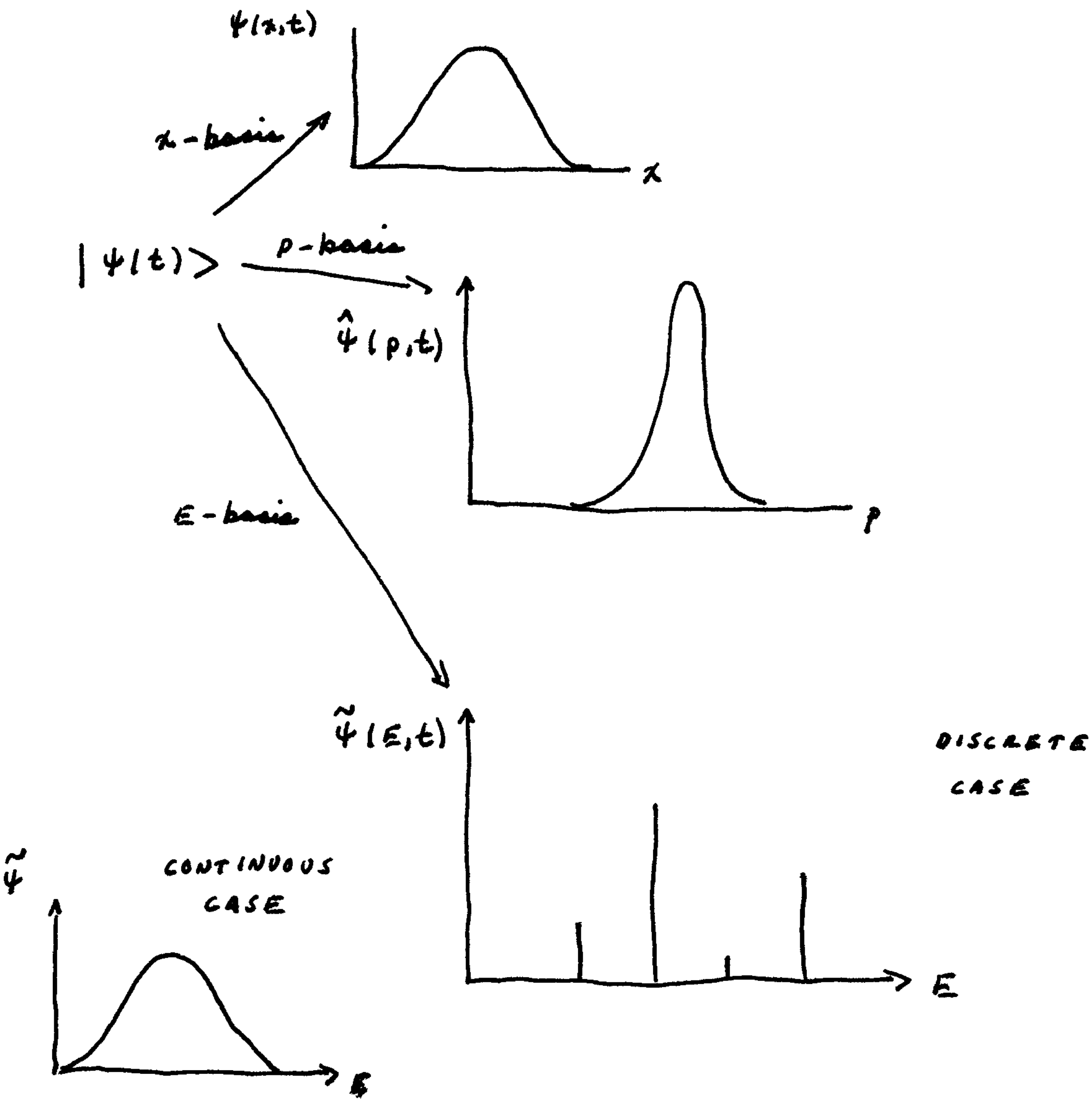
Quantum Mechanics

I. The state of the particle is represented by a vector $|\psi(t)\rangle$ in a Hilbert space.

CLASSICAL MECHANICS

CONFIGURATION SPACE	PHASE SPACE
1 PARTICLE IN 1d	2d $(x(t), p(t))$
1 PARTICLE IN 3d	6d $(\vec{x}(t), \vec{p}(t))$
N PARTICLES IN 3d	6N

	STATE SPACE
1 PARTICLE IN 1d	∞ $\quad |\psi(t)\rangle$



Postulate 2

II. Every dynamical variable ω is a function of x and p: $\omega = \omega(x, p)$.

II. The independent variables x and p of classical mechanics are represented by Hermitian operators X and P with the following matrix elements in the eigenbasis of X‡:

$$\langle x | X | x' \rangle = x\delta(x - x')$$
$$\langle x | P | x' \rangle = -i\hbar\delta'(x - x')$$

The operators corresponding to dependent variables $\omega(x, p)$ are given Hermitian operators

$$\Omega(X, P) = \omega(x \rightarrow X, p \rightarrow P)^{\S}$$

CM: $\omega(x, p)$

$$KE(x, p) = \frac{p^2}{2m}$$

$$PE(x, p) = \frac{1}{2} m \omega X^2$$

QM: $\Omega_{op}(X_{op}, P_{op}) = \omega(x \rightarrow X_{op},\ p \rightarrow P_{op})$

$$KE = \frac{P_{op}^2}{2m}$$

$$PE = \frac{1}{2} m \omega X_{op}^2$$

$$X_{op}\, f(x) = x\, f(x)$$

$$P_{op}\, f(x) = -i\hbar \frac{df(x)}{dx}$$

$$X_{op}\, g(p) = i\hbar \frac{dg(p)}{dp}$$

$$P_{op}\, g(p) = p\, g(p)$$

Postulate 3

III. If the particle is in a state given by x and p, the measurement[‖] of the variable ω will yield a value $\omega(x, p)$. The state will remain unaffected.

III. If the particle is in a state $|\psi\rangle$, measurement[‖] of the variable (corresponding to) Ω will yield one of the eigenvalues ω with probability $P(\omega) \propto |\langle\omega|\psi\rangle|^2$. The state of the system will change from $|\psi\rangle$ to $|\omega\rangle$ as a result of the measurement.

CM : Can measure without disturbing the system

QM : Measurement of Ω leaves the system as an $e\vec{v}$ of Ω

SUPPOSE WE MEASURE SOMETHING

PRESCRIPTION FOR MEASUREMENT

STEP 1: CONSTRUCT THE OPERATOR Ω

$$x \rightarrow x_{op}$$

$$p \rightarrow p_{op}$$

EXAMPLE $\quad KE = \dfrac{p^2}{2m}$

$$\Omega = \frac{p_{op}^2}{2m}$$

STEP 2: FIND THE ev's AND e$\vec{v}$'s OF Ω

"DIAGONALIZE Ω!"

STEP 3: EXPAND THE STATE VECTOR IN THE $|\omega_i\rangle$ BASIS

$$|\psi(t)\rangle = I\,|\psi(t)\rangle$$

$$= \sum |\omega_i\rangle\langle\omega_i|\psi(t)\rangle$$

STEP 4: IF WE MEASURE Ω, WE WILL OBTAIN ω_i's

WITH PROBABILITY

$$P(\omega_i) = \frac{|\langle \omega_i | \psi(t) \rangle|^2}{\langle \psi(t) | \psi(t) \rangle}$$

in terms of the projection operator

$$\mathbb{P}_i = |\omega_i\rangle\langle\omega_i|$$

$$P(\omega_i) = \frac{\langle \psi | \mathbb{P}_i | \psi \rangle}{\langle \psi | \psi \rangle} = \frac{\langle \psi | \omega_i \rangle \langle \omega_i | \psi \rangle}{\langle \psi | \psi \rangle}$$

$$= \frac{|\langle \omega | \psi \rangle|^2}{\langle \psi | \psi \rangle}$$

for normalized vectors $P(\omega_i) = |\langle \omega_i | \psi \rangle|^2$

ALL VECTORS $|\psi\rangle \rightarrow \alpha |\psi\rangle$ HAVE

THE SAME ~~PROBABILITY~~ PROBABILITY

COMPARE

$$P(\omega_i) = \frac{|\langle \omega_i | \psi \rangle|^2}{\langle \psi | \psi \rangle}$$

$$P'(\omega_i) = \frac{|\langle \omega_i | \alpha \psi \rangle|^2}{\langle \alpha \psi | \alpha \psi \rangle} = \frac{|\alpha|^2 \; |\langle \omega_i | \psi \rangle|^2}{|\alpha|^2 \langle \psi | \psi \rangle}$$

$$\Rightarrow \quad P'(\omega_i) = P(\omega_i)$$

N.B., THE ABSOLUTE PHASE IS NOT SET....

CAN ALWAYS MULTIPLY BY $e^{i\varphi}$

EIGENSTATES OF Ω

SPECIAL CASE WHEN $|\psi\rangle = |\omega_i\rangle$

$$P(\omega_i) = |\langle \omega_i | \psi \rangle|^2$$

$$= |\langle \omega_i | \omega_i \rangle|^2 = 1$$

Postulate 4

IV. The state variables change with time according to Hamilton's equations:

$$\dot{x} = \frac{\partial \mathscr{H}}{\partial p}$$

$$\dot{p} = -\frac{\partial \mathscr{H}}{\partial x}$$

IV. The state vector $|\psi(t)\rangle$ obeys the *Schrödinger equation*

$$i\hbar \frac{d}{dt} |\psi(t)\rangle = H |\psi(t)\rangle$$

where $H(X, P) = \mathscr{H}(x \to X, p \to P)$ is the quantum Hamiltonian operator and $\mathscr{H}$ is the Hamiltonian for the corresponding classical problem.

PATH INTEGRAL POSTULATES

(1) The probability $P(b,a)$ for a particle to move to point b from point a is the square of a complex number $U(b,a)$

$$P(b,a) = |U(b,a)|^2$$

(2) The propagator $U(b,a)$ is given by the sum of the phase factor $e^{iS/\hbar}$, where S is the classical action, taken over all possible paths from a to b

$$U(b,a) = \sum_{\substack{\text{ALL}\\ \text{PATHS}}} A\, e^{iS/\hbar}$$

where the constant A can be determined using

$$U(c,a) = \sum_{\substack{\text{ALL PATHS}\\ \text{ALL } b \text{ POINTS}}} U(c,b)\, U(b,a)$$

EXAMPLE

$$|\psi(t)\rangle = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

$$H = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

$$P = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 1 \\ 0 & 1 & 2 \end{bmatrix}$$

Q: IF WE MEASURE energy (momentum) AT TIME t, WHAT RESULTS WILL WE FIND AND WITH WHAT PROBABILITES WILL WE FIND THEM?

STEP 1: CONSTRUCT OPERATORS ✓

STEP 2: FIND ev's AND $e\vec{v}$'s OF OPERATORS

H   ev = 3   4   5

$$e\vec{v} = \begin{pmatrix}1\\0\\0\end{pmatrix} \quad \begin{pmatrix}0\\1\\0\end{pmatrix} \quad \begin{pmatrix}0\\0\\1\end{pmatrix}$$

p ev = 2   3   1

$$e\vec{v} = \begin{pmatrix}1\\0\\0\end{pmatrix} \quad \frac{1}{\sqrt{2}}\begin{pmatrix}0\\1\\1\end{pmatrix} \quad \frac{1}{\sqrt{2}}\begin{pmatrix}0\\1\\-1\end{pmatrix}$$

WHAT ENERGIES WILL WE FIND?   3, 4, 5

WHAT MOMENTA WILL WE FIND?   2, 3, 1

WITH WHAT PROBABILITIES WILL WE FIND THEM?

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SO, IF WE MEASURE THE ENERGY:

$$P(E=3) = \frac{|\langle \overset{E=3}{\hat{E}_3} \mid \psi(t)\rangle|^2}{\langle \psi(t) \mid \psi(t)\rangle}$$

$$= \frac{\left| \begin{pmatrix} 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \right|^2}{\begin{pmatrix} 1 & 2 & 3 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}} = \frac{1}{14}$$

$$P(E=4) = \frac{\left| \begin{pmatrix} 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \right|^2}{14} = \frac{4}{14}$$

$$P(E=5) = \frac{\left| \begin{pmatrix} 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \right|^2}{14} = \frac{9}{14}$$

CHECK $\frac{1}{14} + \frac{4}{14} + \frac{9}{14} = 1$ ✓

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NOW IF WE MEASURE THE MOMENTUM

$$P(p=2) = \frac{\left|(1\ 0\ 0)\begin{pmatrix}1\\2\\3\end{pmatrix}\right|^2}{14} = \frac{1}{14} = \frac{2}{28}$$

$$P(p=3) = \frac{\left|\frac{1}{\sqrt{2}}(0\ 1\ 1)\begin{pmatrix}1\\2\\3\end{pmatrix}\right|^2}{14} = \frac{25}{28}$$

$$P(p=1) = \frac{\left|\frac{1}{\sqrt{2}}(0\ 1\ -1)\begin{pmatrix}1\\2\\3\end{pmatrix}\right|^2}{14} = \frac{1}{28}$$

CHECK $\frac{2}{28} + \frac{25}{28} + \frac{1}{28} = 1$ ✓

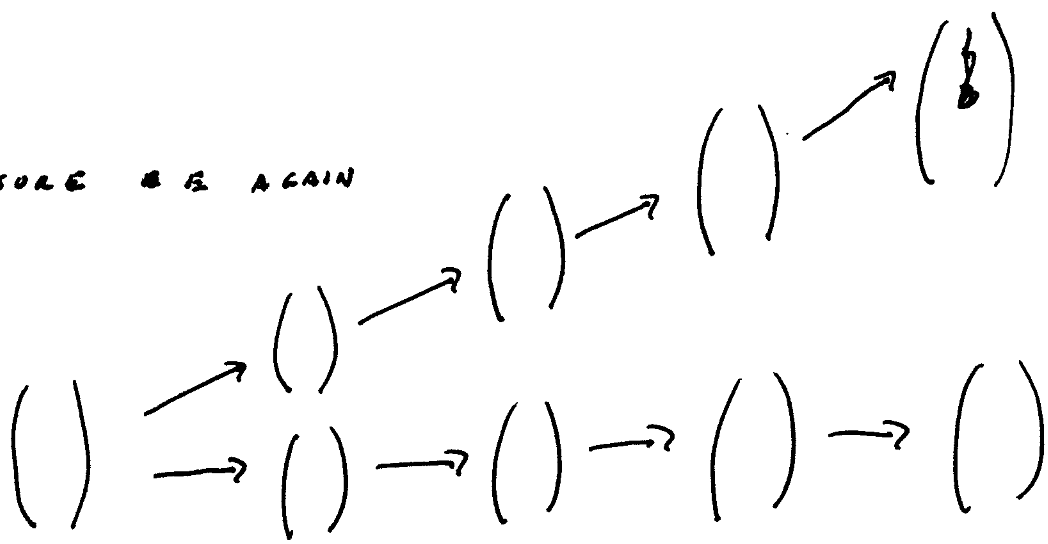
H TREE

$$\frac{1}{\sqrt{14}}\begin{pmatrix}1\\2\\3\end{pmatrix} \xrightarrow[\frac{1}{14}]{E=3} \begin{pmatrix}1\\0\\0\end{pmatrix}$$

$$\frac{1}{\sqrt{14}}\begin{pmatrix}1\\2\\3\end{pmatrix} \xrightarrow[4/14]{E=4} \begin{pmatrix}0\\1\\0\end{pmatrix}$$

$$\frac{1}{\sqrt{14}}\begin{pmatrix}1\\2\\3\end{pmatrix} \xrightarrow[\frac{9}{14}]{E=6} \begin{pmatrix}0\\0\\1\end{pmatrix}$$

MEASURE ~~E~~ E AGAIN



ALWAYS GET SAME ENERGY e_v

STAY IN SAME ENERGY $e\vec{v}$

P TREE

$$\frac{1}{\sqrt{14}}\begin{pmatrix}1\\2\\3\end{pmatrix} \xrightarrow[2/28]{p=2} \begin{pmatrix}1\\0\\0\end{pmatrix} \xrightarrow[100\%]{p=2} \begin{pmatrix} \\ \\ \end{pmatrix} \longrightarrow$$

$$\frac{1}{\sqrt{14}}\begin{pmatrix}1\\2\\3\end{pmatrix} \xrightarrow[25/28]{p=3} \begin{pmatrix}0\\1\\1\end{pmatrix} \xrightarrow[100\%]{p=3} \begin{pmatrix}0\\1\\1\end{pmatrix}$$

MEASURE IMMEDIATELY

$$\begin{pmatrix}0\\1\\1\end{pmatrix} \xrightarrow[\text{THEN MEASURE}]{\text{WAIT TIME } t} \begin{cases} p=3 & \begin{pmatrix}0\\1\\1\end{pmatrix} \\ p=1 & \begin{pmatrix}0\\1\\-1\end{pmatrix} \end{cases}$$

REPEATED MEASURMENTS WITH ZERO TIME DELAY

$$\begin{pmatrix}1\\2\\3\end{pmatrix} \xrightarrow[\frac{1}{14}]{E=3} \begin{pmatrix}1\\0\\0\end{pmatrix} \xrightarrow[100\%]{E=3} \begin{pmatrix}1\\0\\0\end{pmatrix}$$

$|\psi(0^-)\rangle$ $\quad$ $|\psi(0^+)\rangle$ $\quad$ $|\psi(0^{++})\rangle$

$$\begin{pmatrix}1\\2\\3\end{pmatrix} \xrightarrow[\frac{4}{14}]{E=4} \begin{pmatrix}0\\1\\0\end{pmatrix} \xrightarrow[100\%]{E=4} \begin{pmatrix}0\\1\\0\end{pmatrix}$$

IF IN AN $e\vec{v}$ A OR H, THEN MEASURE ASSOCIATED

ev 100% OF THE TIME

OF H

IF IN AN $e\vec{v}$ OF P AND YOU MEASURE P_{op} AT LATER TIMES

THEN IN GENERAL YOU WILL NOT STAY

IN $e\vec{v}$ OF P_{op}

FOR ZERO TIME DELAY

SAME THING FOR P

$$\begin{pmatrix}1\\2\\3\end{pmatrix} \xrightarrow[\frac{25}{28}]{P=3} \frac{1}{\sqrt{2}}\begin{pmatrix}0\\1\\1\end{pmatrix} \xrightarrow[100\%]{P=3} \frac{1}{\sqrt{2}}\begin{pmatrix}0\\1\\1\end{pmatrix}$$

$|\psi(0^-)\rangle$ $\qquad\qquad$ $|\psi(0^+)\rangle$

FOR ZERO TIME DELAY

WHAT IF YOU MEASURE E, P, E, P, ...

$$\begin{pmatrix}1\\2\\3\end{pmatrix} \xrightarrow[\frac{1}{14}]{E=3} \begin{pmatrix}1\\0\\0\end{pmatrix} \xrightarrow[100\%]{P=2} \begin{pmatrix}1\\0\\0\end{pmatrix} \xrightarrow[100\%]{E=3} \begin{pmatrix}1\\0\\0\end{pmatrix}$$

H AND P SHARE one EV $= \begin{pmatrix}1\\0\\0\end{pmatrix}$

$$\begin{pmatrix}1\\2\\3\end{pmatrix} \xrightarrow[\frac{4}{14}]{E=4} \begin{pmatrix}0\\1\\0\end{pmatrix}$$

$$\begin{pmatrix}0\\1\\0\end{pmatrix} \xrightarrow[50\%]{P=3} \begin{pmatrix}0\\1\\1\end{pmatrix}, \quad \begin{pmatrix}0\\1\\1\end{pmatrix} \xrightarrow[50\%]{E=4} \begin{pmatrix}0\\1\\0\end{pmatrix}, \quad \begin{pmatrix}0\\1\\1\end{pmatrix} \xrightarrow[E=5]{50\%} \begin{pmatrix}0\\0\\1\end{pmatrix}$$

$$\begin{pmatrix}0\\1\\0\end{pmatrix} \xrightarrow[50\%]{P=1} \begin{pmatrix}0\\1\\-1\end{pmatrix}, \quad \begin{pmatrix}0\\1\\-1\end{pmatrix} \xrightarrow[50\%]{E=4} \begin{pmatrix}0\\1\\0\end{pmatrix}, \quad \begin{pmatrix}0\\1\\-1\end{pmatrix} \xrightarrow[50\%]{E=5} \begin{pmatrix}0\\0\\1\end{pmatrix}$$

H AND P DO NOT SHARE E=4 EV

$$\begin{pmatrix}1\\2\\3\end{pmatrix} \xrightarrow[\frac{9}{14}]{E=5} \begin{pmatrix}0\\0\\1\end{pmatrix}$$

$$\begin{pmatrix}0\\0\\1\end{pmatrix} \xrightarrow[50\%]{p=3} \begin{pmatrix}0\\1\\1\end{pmatrix} \qquad \begin{pmatrix}0\\1\\1\end{pmatrix} \xrightarrow{E=4} \qquad \begin{pmatrix}0\\1\\1\end{pmatrix} \xrightarrow{E=5}$$

$$\begin{pmatrix}0\\0\\1\end{pmatrix} \xrightarrow[50\%]{p=1} \begin{pmatrix}0\\1\\-1\end{pmatrix}$$

(ZERO TIME DELAY)
IF MEASURE $p, E, p, E, \ldots$

SAME TREES

WHAT HAPPENS IF WE WAIT t BEFORE WE MAKE THE SECOND MEASUREMENT?

$$\begin{pmatrix}1\\2\\3\end{pmatrix} \xrightarrow{E=3} \begin{pmatrix}1\\2\\3\end{pmatrix} \xrightarrow[\text{MEASURE } E]{\text{WAIT } t} \begin{pmatrix}?\\?\\?\end{pmatrix}$$

$$|\psi(0^-)\rangle \qquad |\psi(0^+)\rangle \qquad |\psi(t)\rangle$$

$$\begin{pmatrix}1 & 0\\2 & 1\\3 & 1\end{pmatrix} \xrightarrow{p=3} \begin{pmatrix}0\\1\\1\end{pmatrix} \xrightarrow[\text{MEASURE } p]{\text{WAIT } t} \begin{pmatrix}?\\?\\?\end{pmatrix}$$

ev = ?

ev = ?

TIME EVOLUTION
POSTULATE 4

(1) EXPAND $|\psi(0)\rangle$ IN ENERGY EIGENBASIS:

$$\frac{1}{\sqrt{14}}\begin{pmatrix}1\\2\\3\end{pmatrix} = \frac{1}{\sqrt{14}}\begin{pmatrix}1\\0\\0\end{pmatrix} + \frac{2}{\sqrt{14}}\begin{pmatrix}0\\1\\0\end{pmatrix} + \frac{3}{\sqrt{14}}\begin{pmatrix}0\\0\\1\end{pmatrix}$$

$\vec{ev}$ OF H OBEY TOSE

$$H\,|E_i\rangle = E_i\,|E_i\rangle$$

$$H\,|E_i\rangle = i\hbar\frac{d}{dt}\,|E_i\rangle = E_i\,|E_i\rangle$$

$$|E_i(t)\rangle = e^{-iEt/\hbar}\,|E_i(0)\rangle$$

TIME EVOLUTION OF $\vec{ev}$'s OF H IS
VERY SIMPLE: ONLY THEIR PHASE CHANGES

(2) SIMPLY WRITE DOWN THE TIME DEPENDENCE:

$$|\psi(t)\rangle = \sum e^{-iE_it/\hbar}\,a_i\,|E_i\rangle$$

$$= \frac{1}{\sqrt{14}}e^{-i3t/\hbar}\begin{pmatrix}1\\0\\0\end{pmatrix} + \frac{2}{\sqrt{14}}e^{-i4t/\hbar}\begin{pmatrix}0\\1\\0\end{pmatrix}$$

$$+\frac{3}{\sqrt{14}}e^{-i5t/\hbar}\begin{pmatrix}0\\0\\1\end{pmatrix}$$

ef's
$e\vec{v}$'s OF H ARE CALLED STATIONARY STATES
es's

ONLY THEIR PHASE VARIES

$$\xrightarrow{E=3} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} e^{-i3t/\hbar} \xrightarrow[\text{then}]{\substack{\text{MEASURE} \\ \text{IMMEDIATELY} \\ E=3}} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} e^{-i3t/\hbar}$$

$$\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} e^{-i3t/\hbar} \xrightarrow[\text{THEN MEASURE}]{\text{WAIT } t'} \left(\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} e^{-i3t/\hbar} \right) e^{-i3t'/\hbar}$$

ALWAYS GET $E=3$

ALWAYS IN STATE $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} e^{i\theta_0}$

TIME EVOLUTION OF $e\vec{v}$ OF H:

$$e^{i\theta_0} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \xrightarrow{\text{WAIT } t} \left[\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} e^{i\theta_0} \right] e^{-i3t/\hbar}$$

TIME EVOLUTION OF e$\vec{v}$'S OF P

$$p=2 \text{ e}\vec{v} \text{ OF } P_{op} = \begin{pmatrix}1\\0\\0\end{pmatrix} = E=3 \text{ e}\vec{v} \text{ OF } H_{op}$$

P=2 IS STATIONARY
BECAUSE $|P=2\rangle \equiv |E=3\rangle$
AND $|E=3\rangle$ IS STATIONARY

SO ITS TIME EVOLUTION IS EXACTLY THE SAME

AS THE E=3 e$\vec{v}$ OF H

$$p=3 \text{ e}\vec{v} \text{ OF } P = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\1\\1\end{pmatrix}$$

NO EQUAL TO AN e$\vec{v}$ OF H
SO $|p=3\rangle$ IS TIME DEPENDENT

TO FIND
ITS TIME EVOLUTION

$$\frac{1}{\sqrt{2}}\begin{pmatrix}0\\1\\1\end{pmatrix} = \frac{1}{\sqrt{2}}\left[0\begin{pmatrix}1\\0\\0\end{pmatrix} + 1\begin{pmatrix}0\\1\\0\end{pmatrix} + 1\begin{pmatrix}0\\0\\1\end{pmatrix}\right]$$

EXPAND IN e$\vec{v}$'S OF H
BECAUSE WE KNOW THEIR
TIME EVOLUTION

PUT IN THEIR
TIME
DEPENDENCE

$$= \frac{1}{\sqrt{2}}\left[0\begin{pmatrix}1\\0\\0\end{pmatrix}e^{-i3t/\hbar} + 1\begin{pmatrix}0\\1\\0\end{pmatrix}e^{-i4t/\hbar} + 1\begin{pmatrix}0\\0\\1\end{pmatrix}e^{-i\cancel{4}^{5}t/\hbar}\right]$$

IF $|\Psi(0)\rangle = |p=3\rangle$ THEN $|\Psi(t)\rangle \neq |p=3\rangle$

WHAT DOES THIS LOOK LIKE:

$$\frac{1}{\sqrt{2}}\left[e^{-i4t/\hbar}\begin{pmatrix}0\\1\\0\end{pmatrix} + e^{-i5t/\hbar}\begin{pmatrix}0\\0\\1\end{pmatrix}\right]$$

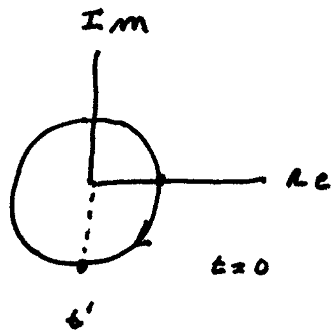
$$t=0 \quad \frac{1}{\sqrt{2}}\left[\begin{pmatrix}0\\1\\0\end{pmatrix} + \begin{pmatrix}0\\0\\1\end{pmatrix}\right] = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\1\\1\end{pmatrix}$$

TIME-DEPENDENT PHASE FACTORS

$e^{-i4t/\hbar}$

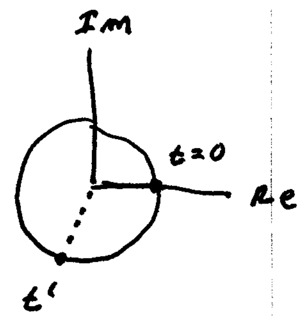
$e^{-i\omega t}$

$\omega = \frac{E}{\hbar}$



$e^{-i5t/\hbar}$

$e^{-i\omega' t}$



SO AT SOME TIME $e^{-i5t/\hbar}$ WILL BE 180^0 AHEAD OF $e^{-i4t/\hbar}$

$$\frac{1}{\sqrt{2}}\left[e^{-i4t/\hbar}\begin{pmatrix}0\\1\\0\end{pmatrix} + e^{-i5t/\hbar}\begin{pmatrix}0\\0\\1\end{pmatrix}\right]$$

$$\frac{1}{\sqrt{2}}e^{-i4t/\hbar}\left[\begin{pmatrix}0\\1\\0\end{pmatrix} + e^{-i\omega t/\hbar}\begin{pmatrix}0\\0\\1\end{pmatrix}\right] \begin{array}{l}\nearrow \begin{pmatrix}0\\1\\1\end{pmatrix}e^{i\theta} \\ \searrow \begin{pmatrix}0\\1\\-1\end{pmatrix}e^{i\theta'}\end{array}$$

START IN $P=3$ eV OF P_{op}

$\longleftarrow$ IN BETWEEN 50:50

LATER IN $P=1$ eV OF P_{op}

$\longleftarrow$ AGAIN IN BETWEEN

BACK IN $P=3$ eV OF P_{op}
JUST LIKE COUPLED PENDULA

IN BETWEEN, YOU ARE
IN A LINEAR COMBINATION
OF $|E=4\rangle$ AND $|E=5\rangle$