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The Postulates—a General Discussion

Having acquired the necessary mathematical training and physical motivation, you are now ready to get acquainted with the postulates of quantum mechanics. In this chapter the postulates will be stated and discussed in broad terms to bring out the essential features of quantum theory. The subsequent chapters will simply be applications of these postulates to the solution of a variety of physically interesting problems. Despite your preparation you may still find the postulates somewhat abstract and mystifying on this first encounter. These feelings will, however, disappear after you have worked with the subject for some time.

4.1. The Postulates‡

The following are the postulates of nonrelativistic quantum mechanics. We consider first a system with one degree of freedom, namely, a single particle in one space dimension. The straightforward generalization to more particles and higher dimensions will be discussed towards the end of the chapter. In what follows, the quantum postulates are accompanied by their classical counterparts (in the Hamiltonian formalism) to provide some perspective.

Classical Mechanics	Quantum Mechanics	
I. The state of a particle at any given time is specified by the two variables $x(t)$ and $p(t)$, i.e., as a point in a two-dimensional phase space.	I. The state of the particle is represented by a vector $\lvert\psi(t)\rangle$ in a Hilbert space.	1
II. Every dynamical variable ω is a function of x and p: $\omega=\omega(x, p)$.	II. The independent variables x and p of classical mechanics are represented	2

‡ Recall the discussion in the Preface regarding the sense in which the word is used here.

by Hermitian operators X and P with the following matrix elements in the eigenbasis of X‡

$$\langle x|X|x'\rangle = x\delta(x-x')$$ 2
$$\langle x|P|x'\rangle = -i\hbar\delta'(x-x')$$

The operators corresponding to dependent variables $\omega(x, p)$ are given Hermitian operators

$$\Omega(X, P) = \omega(x \to X, p \to P)§$$

III. If the particle is in a state given by x and p, the measurement‖ of the variable ω will yield a value $\omega(x, p)$. The state will remain unaffected.

III. If the particle is in a state $|\psi\rangle$, measurement‖ of the variable (corresponding to) Ω will yield one of the eigenvalues ω with probability $P(\omega) \propto |\langle\omega|\psi\rangle|^2$. The state of the system will change from $|\psi\rangle$ to $|\omega\rangle$ as a result of the measurement. 3

IV. The state variables change with time according to Hamilton's equations:

$$\dot{x} = \frac{\partial \mathscr{H}}{\partial p}$$

$$\dot{p} = -\frac{\partial \mathscr{H}}{\partial x}$$

IV. The state vector $|\psi(t)\rangle$ obeys the *Schrödinger equation*

$$i\hbar \frac{d}{dt}|\psi(t)\rangle = H|\psi(t)\rangle$$ 4

where $H(X, P) = \mathscr{H}(x \to X, p \to P)$ is the quantum Hamiltonian operator and $\mathscr{H}$ is the Hamiltonian for the corresponding classical problem.

4.2. Discussion of Postulates I–III

The postulates (of classical and quantum mechanics) fall naturally into two sets: the first three, which tell us how the system is depicted at a given time, and the last, which specifies how this picture changes with time. We will confine our attention to the first three postulates in this section, leaving the fourth for the next.

The first postulate states that a particle is described by a ket $|\psi\rangle$ in a Hilbert space which, you will recall, contains *proper vectors* normalizable to unity as well as

‡ Note that the X operator is the same one discussed at length in Section 1.10. Likewise $P = \hbar K$, where K was also discussed therein. You may wish to go over that section now to refresh your memory.
§ By this we mean that Ω is the same function of X and P as ω is of x and P.
‖ That is, in an ideal experiment consistent with the theory. It is assumed you are familiar with the ideal classical measurement which can determine the state of the system without disturbing it in any way. A discussion of ideal quantum measurements follows.