February 6, 2012

# Three Topics Left in this Module

(0) Square Well

(1) Free Particle

(2) Piecewise Constant Potentials

(3) Simple Harmonic Oscillator

# Then boldly go to three-dimensions

(1) Angular Momentum

(2) Hydrogen

# Transmission and Reflection Applets

http://www.youtube.com/watch?v=4-PO-RHQsFA

http://www.youtube.com/watch?v=_3wFXHwRP4s

http://www.rkm3d.com/How-3D-Works/3D-circularly-polarized-light-animation.html

http://www.youtube.com/watch?v=FfhAWZeEX78

http://phet.colorado.edu/en/simulation/quantum-tunneling

http://www.fen.bilkent.edu.tr/~yalabik/applets/t_d_quant.html

http://www.st-andrews.ac.uk/~qmanim/

http://www.st-andrews.ac.uk/~qmanim/animations_2/Scattering_Finite_Well_V6.swf

http://www.st-andrews.ac.uk/~qmanim/animations_2/Quantum_tunneling_V3.swf

http://www.st-andrews.ac.uk/~qmanim/animations_2/Wave_Packet_Potential_Step_V5.swf

http://www.st-andrews.ac.uk/~qmanim/animations_2/Expansion_Eigenstates_V3.swf

http://www.colorado.edu/physics/2000/isotopes/radioactive_decay3.html

http://demonstrations.wolfram.com/WavepacketForAFreeParticle/

http://demonstrations.wolfram.com/TransmissionAndReflectionCoefficientsOfQuantumParticles/

http://demonstrations.wolfram.com/TimeEvolutionOfAQuantumFreeParticleIn1D/

http://demonstrations.wolfram.com/EvolutionOfAGaussianWavePacket/

http://demonstrations.wolfram.com/DegenerateEigenstates/

http://demonstrations.wolfram.com/FundamentalCommutationRelationsInQuantumMechanics/

http://demonstrations.wolfram.com/SchroedingersCatOnCatnip/

http://demonstrations.wolfram.com/ScatteringByASquareWellPotential/

## The Double Slit Experiment

http://www.fen.bilkent.edu.tr/~yalabik/applets/collapse.html wavefunction collapse
http://ophelia.princeton.edu/~page/single_photon.html photons
http://www.hqrd.hitachi.co.jp/rd/moviee/doubleslite-n.mpeg electrons
http://www.quantum-physics.polytechnique.fr/index.html french

Cemal Yalabik has a sense of humor
http://www.fen.bilkent.edu.tr/~yalabik/applets/galaxy.html galaxy

## Barrier Transmission and Reflection

http://phet.colorado.edu/simulations/
quantum-tunneling_en.jar
quantum-wave-interference_en.jar
bound-states_en.jar
http://www.fen.bilkent.edu.tr/~yalabik/applets/t_d_quant.html

## More Applets

http://webphysics.davidson.edu/applets/QTime/QTime_Examples.html
http://www.ks.uiuc.edu/Services/Class/PHYS386/images/movies/Eigenstates.html
http://www.physics.nus.edu.sg/einstein/lect14/applets/propa.html
http://www.brainflux.org/java/classes/Schrodinger1D.html
http://www.folksemantic.com/visits/70038

## One-dimensional bound states

http://www.falstad.com/qm1d/ Paul Falstad
http://www.fen.bilkent.edu.tr/~yalabik/applets/1d.html Cemal Yalbik

# Conclusion

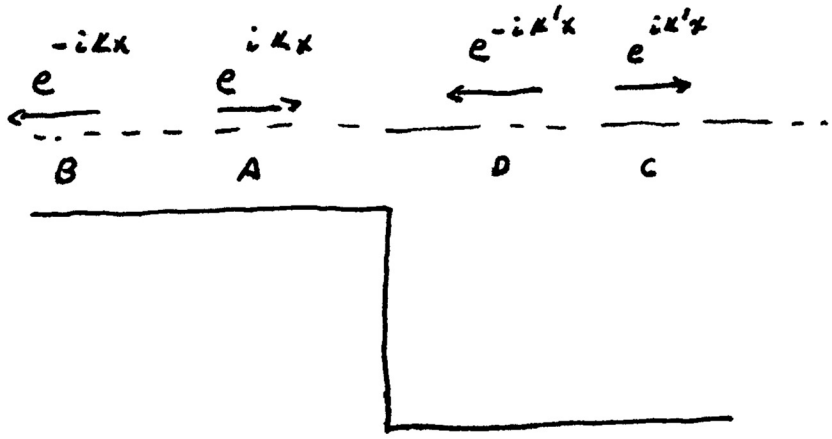
- Quantum tunneling gets subatomic particles across barriers they normally can not cross.
- The number that gets across drops exponentially with the barrier thickness.
- The number that gets across increases with the energy of the particles.

# Conclusion

- Quantum tunneling is a purely quantum effect.
- It has no counter part in Classical mechanics.
- It could theoretically occur on the macroscopic scale but the odds are too small to ever see.

1d SCATTERING

1d TRANSMISSION AND REFLECTION




TWO KNOWNS : $k$ AND $k'$

FOUR UNKNOWNS : A, B, C, D

TWO EQUATIONS : $\psi$ CONTINUOUS

$\psi'$ CONTINUOUS

$D = 0$ UNLESS WE SEND IN A WAVE

THREE UNKNOWNS: A, B, C

MEASURE RELATIVE TO A: $\frac{B}{A}$ , $\frac{C}{A}$

$$\boxed{\begin{array}{l}\text{2 EQN'S} \\ \text{2 UNKNOWNS}\end{array}}$$

TISE $\qquad H|\psi\rangle = E|\psi\rangle$

$$\left[ \frac{-\hbar^2}{2m} \frac{d^2}{dx^2} + V(x) \right] \psi = E\psi$$

$$\frac{d^2\psi}{dx^2} = \left[ \frac{2m}{\hbar^2} (V - E) \right] \psi$$

$$\frac{d^2\psi}{dx^2} = \Omega \psi$$

SOLN'S $\qquad \psi(x) = \exp\left( \pm \sqrt{\Omega}\, x \right)$

THREE CASES:

(1) $\quad E > V \Rightarrow \Omega < 0 \Rightarrow \pm\sqrt{\Omega} = \pm i k$

$\psi(x) = A e^{ikx} + B e^{-ikx}$

(2) $\quad E < V \Rightarrow \Omega > 0 \Rightarrow \pm\sqrt{\Omega} = \pm \kappa$

$\psi(x) = C e^{+\kappa x} + D e^{-\kappa x}$

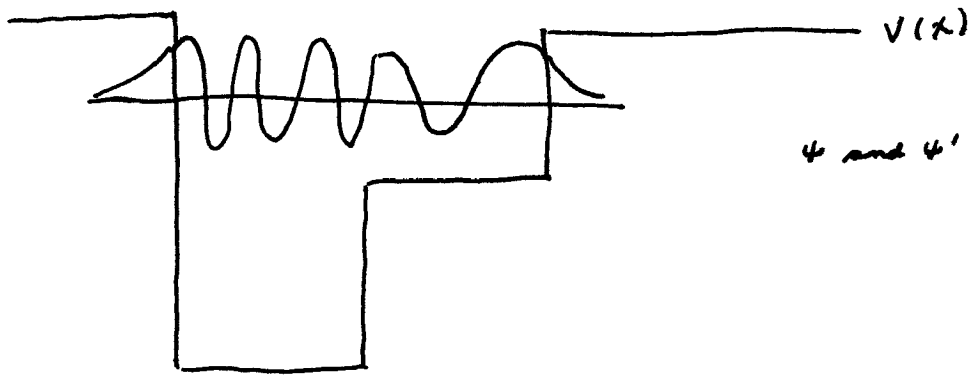
(3) $\quad E = V \Rightarrow \Omega = 0$

$\psi(x) = E x + F$

SKETCHING BOUND STATE WAVEFCN'S

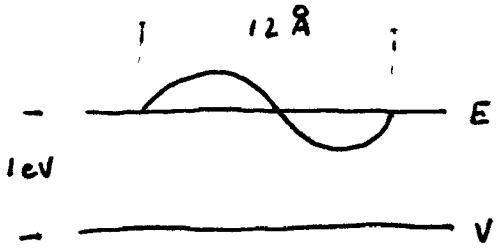
$E > V$ OSCILLATING

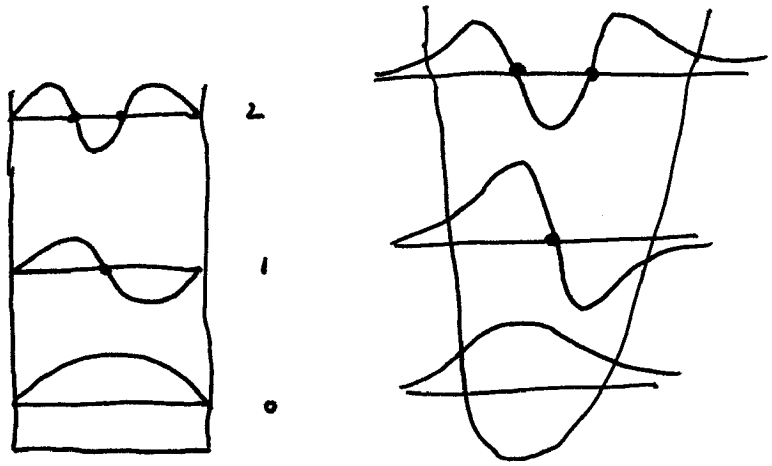
$E < V$ DECAYING

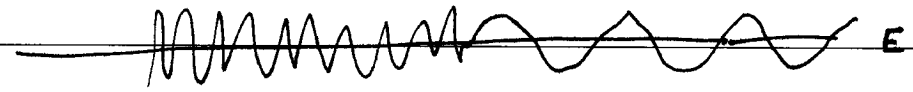



$\psi$ and $\psi'$ CONTINUOUS

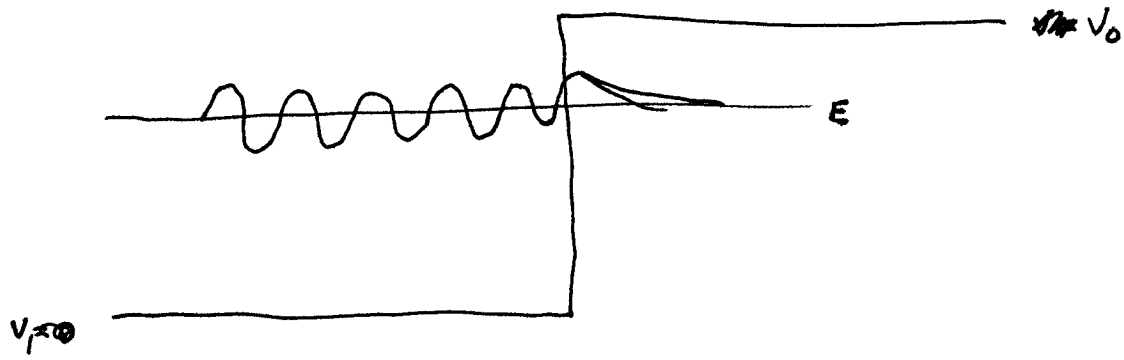
FOR ELECTRONS: $\lambda = \dfrac{12}{\sqrt{E - V}}$ Å, eV






SCATTERING




CLASSICALLY: $E < V_0$ ALL REFLECTED

BALL: all T

$E > V_0$ wave $\begin{cases} \text{SOME REFLECTED} \\ \text{SOME TRANSMITTED} \end{cases}$

LEFT REGION $E > V$

$$\psi_1(x) = \underbrace{A e^{ik_1 x}}_{\text{incident}} + \underbrace{B e^{-ik_1 x}}_{\text{reflected}}$$

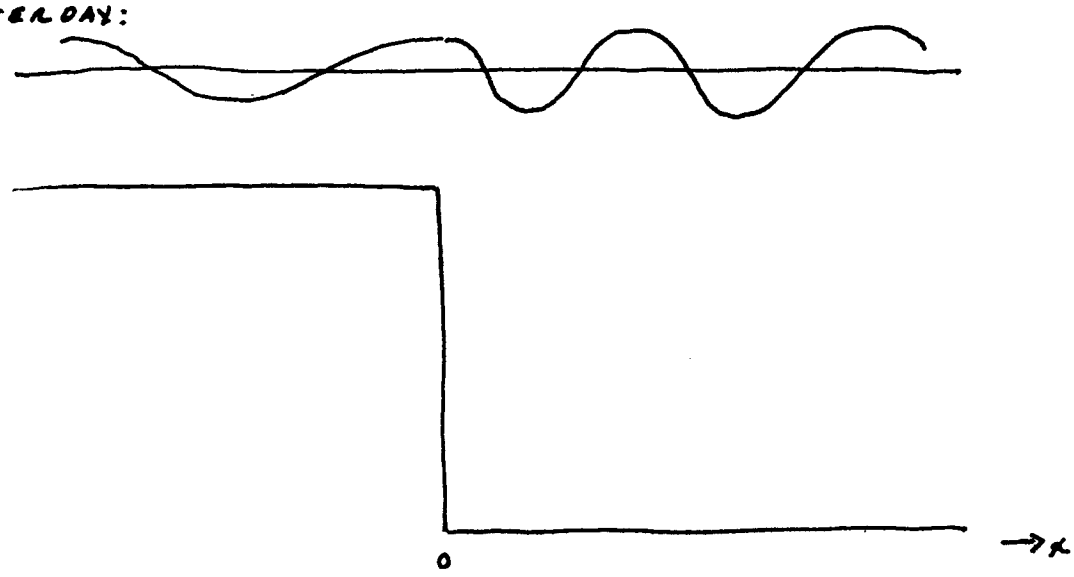
$$k_1 = \sqrt{2m(E - V_1)/\hbar^2} = p_1/\hbar$$

RIGHT REGION

BC at $+\infty$ no incoming wave

$$\psi_2(x) = \underbrace{C e^{ik_2 x}}_{\text{TRANSMITTED}} + \cancel{D e^{-ik_2 x}}$$

YESTERDAY:




$$\psi_1(x) = A e^{ik_1 x} + B e^{-ik_1 x} \qquad \psi_2(x) = C e^{ik_2 x}$$

$$k_1 = \sqrt{2m(E - V_0)/\hbar^2} \qquad k_2 = \sqrt{2mE/\hbar^2}$$

LOWER SPATIAL FREQUENCY

HIGH SPATIAL FREQUENCY

"

SHORT SPATIAL WAVE LENGTH

B.C.'S

$\psi$ CONTINUOUS $\qquad \psi_1(x=0) = \psi_2(x=0)$

$\frac{d\psi}{dx}$ CONTINUOUS $\qquad \left.\frac{d\psi_1}{dx}\right|_{x=0} = \left.\frac{d\psi_2}{dx}\right|_{x=0}$

WAVEFCN CONTINUOUS

$$\psi_1(0) = \psi_2(0)$$

$$A e^{ik_1(0)} + B e^{-ik_1(0)} = C e^{ik_2(0)}$$

$$\boxed{A + B = C}$$

DERIVATIVE CONTINUOUS
SLOPE CONTINUOUS

$$\left.\frac{d\psi_1}{dx}\right|_{x=0} = \left.\frac{d\psi_2}{dx}\right|_{x=0}$$

$$\frac{d}{dx}\left( A e^{ik_1 x} + B e^{-ik_1 x} = C e^{ik_2 x} \right)$$

$$ik_1 A e^{ik_1 x} - ik_1 B e^{-ik_1 x} = \cancel{e^{ik_2}}$$

$$ik_2 C e^{ik_2 x}$$

$$e^{ik(0)} = 1$$

$$\boxed{k_1(A - B) = k_2 C}$$

FINITE POTENTIAL => $\psi$ and $\psi'$ CONTINUOUS

$$\psi_1(0) = \psi_2(0)$$

$$A e^{ik_1(0)} + B e^{-ik_1(0)} = C e^{ik_2(0)}$$

$$A + B = C$$

$$\psi_1'(0) = \psi_2'(0)$$

$$ik_1 A e^{0} - ik_1 B e^{0} = ik_2 C e^{0}$$

$$k_1 (A - B) = k_2 C$$

SOLVE IN TERMS OF A

$$k_1 (A - B) = k_2 C = k_2 (A + B)$$

$$(k_1 - k_2) A = (k_1 + k_2) B$$

$$\boxed{B = \frac{k_1 - k_2}{k_1 + k_2} A} \qquad \boxed{C = \frac{2k_1}{k_1 + k_2} A}$$

$B = r A$ $\qquad$ $C = t A$

$r$ = reflection amplitude

REFLECTION COEFF $R = |r|^2$

$$R = \frac{\text{intensity reflected}}{\text{incident intensity}} = \frac{|B|^2}{|A|^2}$$

$$\boxed{R = \left(\frac{k_1 - k_2}{k_1 + k_2}\right)^2} \quad \frac{|A|^2}{|A|^2}$$

TRANSMISSION ~~TRANSMISSION~~ COEFF

$t$ = transmission amplitude

$$T + R = 1 \quad \Rightarrow \quad T = 1 - R$$

$$T = 1 - \left(\frac{k_1 - k_2}{k_1 + k_2}\right)^2$$

$$= \left(\frac{k_1 + k_2}{k_1 + k_2}\right)^2 - \left(\frac{k_1 - k_2}{k_1 + k_2}\right)^2$$

$$\boxed{T = \frac{4 k_1 k_2}{(k_1 + k_2)^2}}$$

N.B., IF YOU CALCULATE $\frac{|C|^2}{|A|^2}$ YOU DO NOT

GET THE RIGHT ANSWER $(1-R)$ !

BECAUSE

FLUX = (INTENSITY) (VELOCITY)

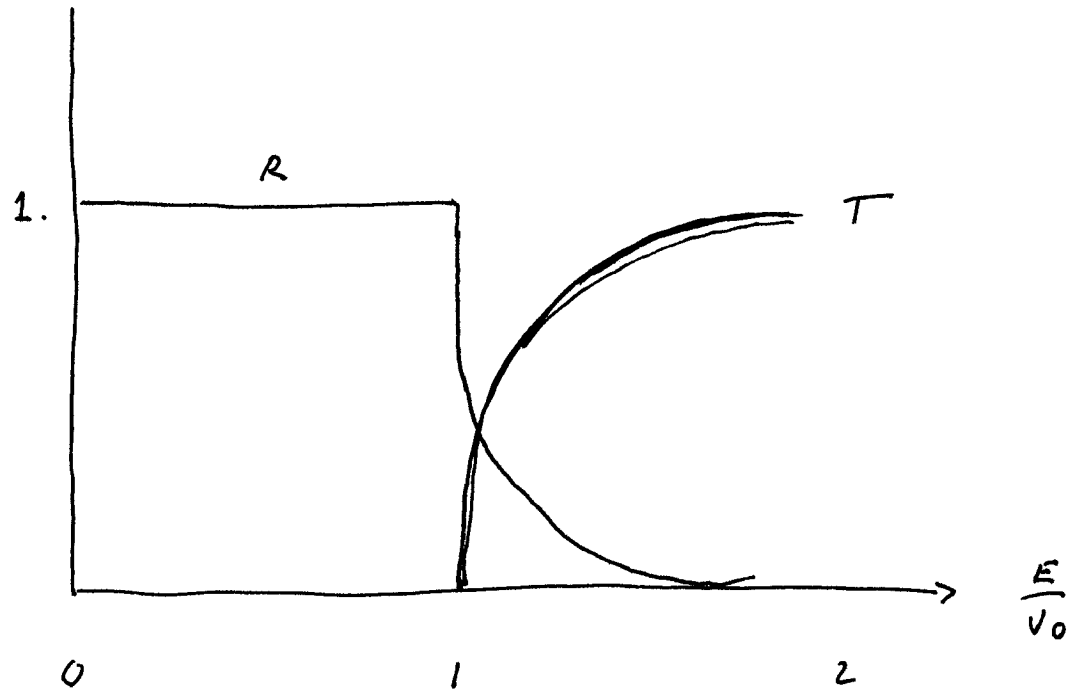
OKAY FOR R

$$R = \frac{v_1 |B|^2}{v_1 |A|^2} = \frac{|B|^2}{|A|^2}$$

BUT NOT OKAY FOR T unless you include velocity

$$T = \frac{v_2 |C|^2}{v_1 |A|^2} = \frac{\frac{\hbar k_2}{m} |C|^2}{\frac{\hbar k_1}{m} |A|^2} = \frac{k_2}{k_1}\left(\frac{2k_1}{k_1+k_2}\right)^2$$

$$T = \frac{4 k_1 k_2}{(k_1+k_2)^2}$$




$$R = 1 - T = \left[ \frac{1 - \sqrt{1 + (V_0/E)}}{1 + \sqrt{1 + (V_0/E)}} \right]^2$$

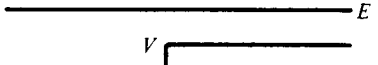
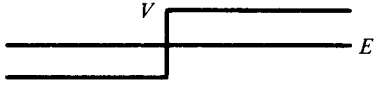
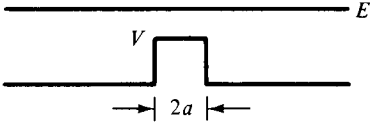
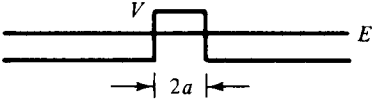
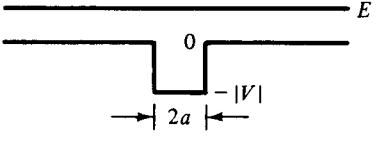
ELECTRONS $r = \dfrac{k_1 - k_2}{k_1 + k_2}$ $R = \left(\dfrac{k_1 - k_2}{k_1 + k_2}\right)^2$ $R = |r|^2$

LIGHT $r = \dfrac{n_1 - n_2}{n_1 + n_2}$

EM WAVE $r = \dfrac{z_1 - z_2}{z_1 + z_2}$ 377 OHMS

SOUND

**TABLE 7.2 Transmission coefficients for three elementary potential barriers**

| | |
|---|---|
|  | $T = \dfrac{4k_2/k_1}{[1 + (k_2/k_1)]^2}$<br>$\left(\dfrac{k_2}{k_1}\right)^2 = 1 - \dfrac{V}{E}$ |
|  | $T = 0, \quad R = 1$ |
|  | $\dfrac{1}{T} = 1 + \dfrac{1}{4}\dfrac{V^2}{E(E - V)}\sin^2(2k_2 a)$<br>$\dfrac{\hbar^2 k_2{}^2}{2m} = E - V$ |
|  | $\dfrac{1}{T} = 1 + \dfrac{1}{4}\dfrac{V^2}{E(V - E)}\sinh^2(2\kappa a)$<br>$\dfrac{\hbar^2 \kappa^2}{2m} = V - E$ |
|  | $\dfrac{1}{T} = 1 + \dfrac{1}{4}\dfrac{V^2}{E(E + \lvert V\rvert)}\sin^2(2k_2 a)$<br>$\dfrac{\hbar^2 k_2{}^2}{2m} = E - V = E + \lvert V\rvert$ |

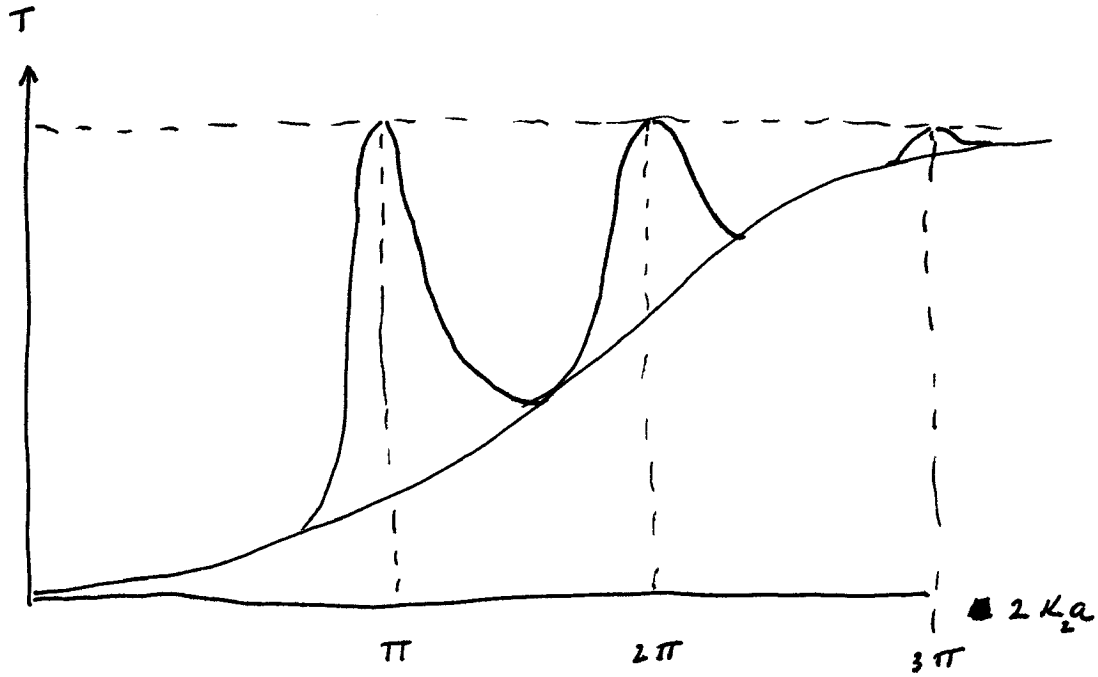
The transmission coefficients corresponding to the one-dimensional potential configurations considered above are summarized in Table 7.2.

RESONANCE EFFECTS

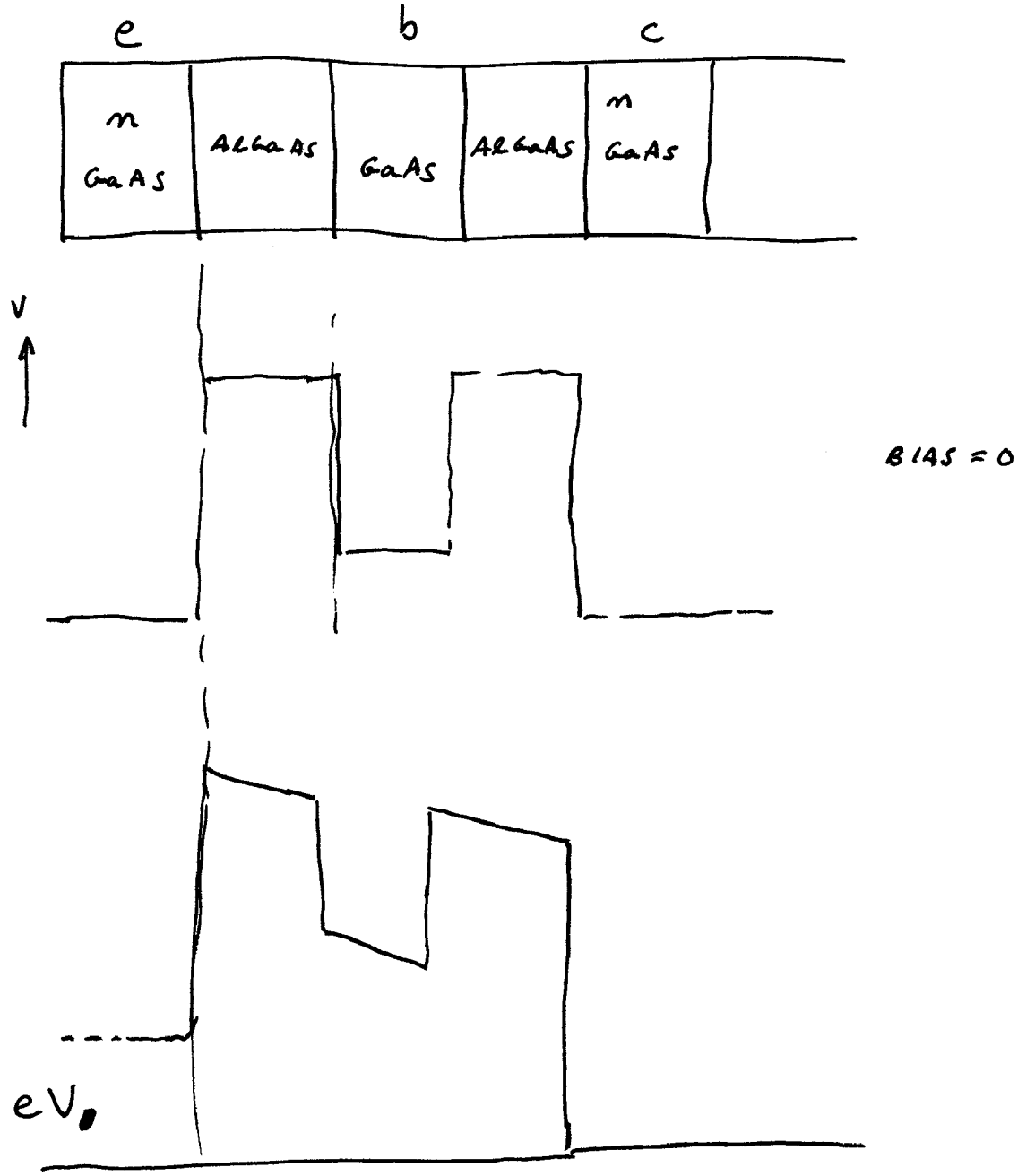
when $\sin(2k_2 a) = 0$, $T \to 1$

integral number of half wavelengths inside the well

electrons thru rare gases RAMSAUER EFFECT



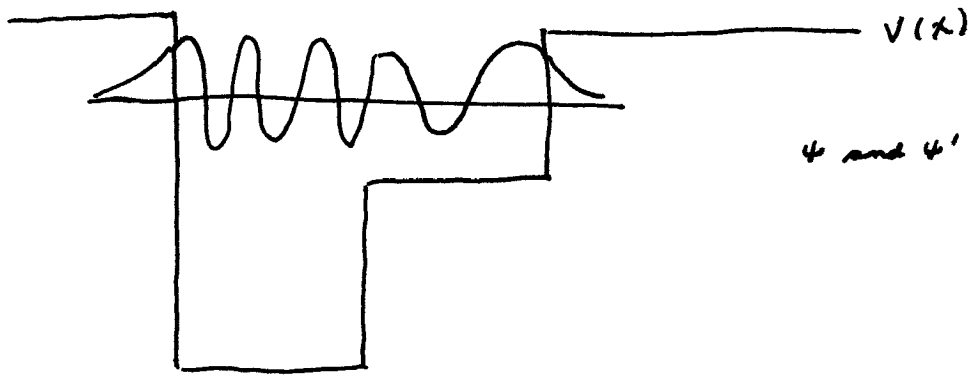
SEMICONDUCTOR EXAMPLE

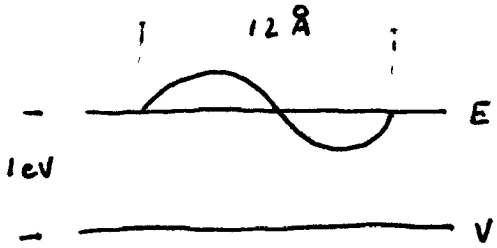
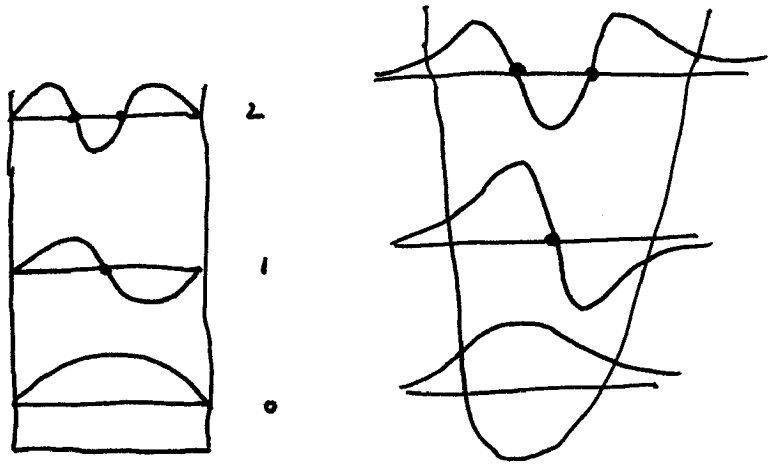


SKETCHING BOUND STATE WAVEFCN'S

$E > V$ OSCILLATING

$E < V$ DECAYING




$\psi$ and $\psi'$ CONTINUOUS

FOR ELECTRONS: $\lambda = \dfrac{12}{\sqrt{E - V}}$ Å, eV