Lecture 2 January 9, 2012

Your Academic Geneology
Synopsis
Some Stories
Some Math

Quantum Mechanics Books

Principles of Quantum Mechanics
R. Shankar
The book I was using when we wrote the virtual book
Mathematics and Postulates are from the First edition

Introduction to Quantum Mechanics
D. Griffiths
The most popular junior level QM book
in UW bookstore under Physics 325

Introductory Quantum Mechanics
R. Liboff
More detailed than Griffiths'
Excellent junior level QM book

Quantum Mechanics
Cohen-Tannoudji, Diu, Laloe
Encyclopedic
Previous students have found it exhausting to read

QM Is a Linear Theory

Study Linear Operators

If s_1 and s_2 are solutions, then so are $a \cdot s_1$, $b \cdot s_2$, $a \cdot s_1 + b \cdot s_2$, etc.

Usually nonlinear effects occur as the solutions get large

Linear Algebra

matrices and vectors

eigenvalues and eigenvectors

Live in a vector space

Functional Analysis

differential operators and functions

eigenvalues and eigenfunctions

Live in a Hilbert space

Hilbert spaces are the generalization of finite-dimensional vector spaces to the infinite dimensional limit. Much of the important and extremely useful machinery of the finite-dimensional case still applies in the infinite dimensional limit.

Linear Algebra	**Functional Analysis**
Linear vector spaces	Hilbert spaces
numbers	numbers
vectors	functions
matrices	operators
tensors	tensor operators
Basis of vectors	Basis of functions
change of basis	change of basis
diagonalization	diagonalization
eigenvalues	eigenvalues
eigenvectors	eigenfunctions

To learn more about Hilbert spaces:

Read Shankar's Mathemetics Chapter

To learn more details, read Paul Halmos' appendix---it's on the class website:

What's the difference between a Hilbert space and a finite-dimensional vector space?

And consult the following web resources:

http://en.wikipedia.org/wiki/Hilbert_space

http://mathworld.wolfram.com/HilbertSpace.html

http://terrytao.wordpress.com/2009/01/17/254a-notes-5-hilbert-spaces/

http://en.wikipedia.org/wiki/Terence_Tao   (also check out his homepage and his blog)

APPENDIX

HILBERT SPACE

Probably the most useful and certainly the best developed generalization of the theory of finite-dimensional inner product spaces is the theory of Hilbert space. Without going into details and entirely without proofs we shall now attempt to indicate how this generalization proceeds and what are the main difficulties that have to be overcome.

The definition of Hilbert space is easy: it is an inner product space satisfying one extra condition. That this condition (namely, completeness) is automatically satisfied in the finite-dimensional case is proved in elementary analysis. In the infinite-dimensional case it may be possible that for a sequence (x_n) of vectors $\| x_n - x_m \| \to 0$ as $n, m \to \infty$, but still there is no vector x for which $\| x_n - x \| \to 0$; the only effective way of ruling out this possibility is explicitly to assume its opposite. In other words: a Hilbert space is a complete inner product space. (Sometimes the concept of Hilbert space is restricted by additional conditions, whose purpose is to limit the size of the space from both above and below. The most usual conditions require that the space be infinite-dimensional and separable. In recent years, ever since the realization that such additional restrictions do not pay for themselves in results, it has become customary to use "Hilbert space" for the concept we defined.)

It is easy to see that the space $\mathcal{P}$ of polynomials with the inner product defined by $(x, y) = \int_0^1 x(t)\overline{y(t)}\, dt$ is not complete. In connection with the completeness of certain particular Hilbert spaces there is quite an extensive mathematical lore. Thus, for instance, the main assertion of the celebrated Riesz-Fischer theorem is that a Hilbert space manufactured out of the set of all those functions x for which $\int_0^1 |x(t)|^2\, dt < \infty$ (in the sense of Lebesgue integration) is a Hilbert space (with formally the same definition of inner product as for polynomials). Another popular Hilbert space, reminiscent in its appearance of finite-dimensional coordinate space, is the space of all those sequences (ξ_n) of numbers (real or complex, as the case may be) for which $\sum_n |\xi_n|^2$ converges.

Using completeness in order to discuss intelligently the convergence of some infinite sums, one can proceed for quite some time in building the theory of Hilbert spaces without meeting any difficulties due to infinite-dimensionality. Thus, for instance, the notions of orthogonality and of complete orthonormal sets can be defined in the general case exactly as we defined them. Our proof of Bessel's inequality and of the equivalence of the various possible formulations of completeness for orthonormal sets have to undergo slight verbal changes only. (The convergence of the various infinite sums that enter is an automatic consequence of Bessel's inequality.) Our proof of Schwarz's inequality is valid, as it stands, in the most general case. Finally, the proof of the existence of complete orthonormal sets parallels closely the proof in the finite case. In the unconstructive proof Zorn's lemma (or transfinite induction) replaces ordinary induction, and even the constructive steps of the Gram-Schmidt process are easily carried out.

In the discussion of manifolds, functionals, and transformations the situation becomes uncomfortable if we do not make a concession to the topology of Hilbert space. Good generalizations of all our statements for the finite-dimensional case can be proved if we consider *closed* linear manifolds, *continuous* linear functionals, and *bounded* linear transformations. (In a finite-dimensional space every linear manifold is closed, every linear functional is continuous, and every linear transformation is bounded.) If, however, we do agree to make these concessions, then once more we can coast on our finite-dimensional proofs without any change most of the time, and with only the insertion of an occasional ϵ the rest of the time. Thus once more we obtain that $\mathcal{V} = \mathfrak{M} \oplus \mathfrak{M}^{\perp}$, that $\mathfrak{M} = \mathfrak{M}^{\perp\perp}$, and that every linear functional of x has the form (x, y); our definitions of self-adjoint and of positive transformations still make sense, and all our theorems about perpendicular projections (as well as their proofs) carry over without change.

The first hint of how things can go wrong comes from the study of orthogonal and unitary transformations. We still call a transformation U orthogonal or unitary (according as the space is real or complex) if $UU^* = U^*U = 1$, and it is still true that such a transformation is isometric, that is, that $\| Ux \| = \| x \|$ for all x, or, equivalently, $(Ux, Uy) = (x, y)$ for all x and y. It is, however, easy to construct an isometric transformation that is not unitary; because of its importance in the construction of counterexamples we shall describe one such transformation. We consider a Hilbert space in which there is a countable complete orthonormal set, say $\{x_0, x_1, x_2, \cdots\}$. A unique bounded linear transformation U is defined by the conditions $Ux_n = x_{n+1}$ for $n = 0, 1, 2, \cdots$. This U is isometric ($U^*U = 1$), but, since $UU^*x_0 = 0$, it is not true that $UU^* = 1$.

It is when we come to spectral theory that the whole flavor of the development changes radically. The definition of proper value as a number λ for which $Ax = \lambda x$ has a non-zero solution still makes sense, and our theorem about the reality of the proper values of a self-adjoint transformation is still true. The notion of proper value loses, however, much of its significance. Proper values are so very useful in the finite-dimensional case because they are a handy way of describing the fact that something goes wrong with the inverse of $A - \lambda$, and the only thing that can go wrong is that the inverse refuses to exist. Essentially different things can happen in the infinite-dimensional case; just to illustrate the possibilities, we mention, for example, that the inverse of $A - \lambda$ may exist but be unbounded. That there is no useful generalization of determinant, and hence of the characteristic equation, is the least of our worries. The whole theory has, in fact, attained its full beauty and maturity only after the slavish imitation of such finite-dimensional methods was given up.

After some appreciation of the fact that the infinite-dimensional case has to overcome great difficulties, it comes as a pleasant surprise that the spectral theorem for self-adjoint transformations (and, in the complex case, even for normal ones) does have a very beautiful and powerful generalization. (Although we describe the theorem for bounded transformations only, there is a large class of unbounded ones for which it is valid.) In order to be able to understand the analogy, let us re-examine the finite-dimensional case.

Let A be a self-adjoint linear transformation on a finite-dimensional inner product space, and let $A = \sum_j \lambda_j F_j$ be its spectral form. If M is an interval in the real axis, we write $E(M)$ for the sum of all those F_j for which λ_j belongs to M. It is clear that $E(M)$ is a perpendicular projection for each M. The following properties of the projection-valued interval-function E are the crucial ones: if M is the union of a countable collection $\{M_n\}$ of disjoint intervals, then

$$E(M) = \sum_n E(M_n), \tag{1}$$

and if M is the improper interval consisting of all real numbers, then $E(M) = 1$. The relation between A and E is described by the equation

$$A = \sum_j \lambda_j E(\{\lambda_j\}),$$

where, of course, $\{\lambda_j\}$ is the degenerate interval consisting of the single number λ_j. Those familiar with Lebesgue-Stieltjes integration will recognize the last written sum as a typical approximating sum to an integral of the form $\int \lambda \, dE(\lambda)$ and will therefore see how one may expect the generalization to go. The algebraic concept of summation is to be replaced by the analytic concept of integration; the generalized relation between A and E is described by the equation

$$A = \int \lambda \, dE(\lambda). \tag{2}$$

Except for this formal alteration, the spectral theorem for self-adjoint transformations is true in Hilbert space. We have, of course, to interpret correctly the meaning of the limiting operations involved in (1) and (2). Once more we are faced with the three possibilities mentioned in § 91. They are called uniform, strong, and weak convergence respectively, and it turns out that both (1) and (2) may be given the strong interpretation. (The reader deduces, of course, from our language that in an infinite-dimensional Hilbert space the three possibilities are indeed distinct.)

We have seen that the projections F_j entering into the spectral form of A in the finite-dimensional case are very simple functions of A (§ 82). Since the $E(M)$ are obtained from the F_j by summation, they also are functions of A, and it is quite easy to describe what functions. We write $g_M(\zeta) = 1$ if ζ is in M and $g_M(\zeta) = 0$ otherwise; then $E(M) = g_M(A)$. This fact gives the main clue to a possible proof of the general spectral theorem. The usual process is to discuss the functional calculus for polynomials, and, by limiting processes, to extend it to a class of functions that includes all the functions g_M. Once this is done, we may define the interval-function E by writing $E(M) = g_M(A)$; there is no particular difficulty in establishing that E and A satisfy (1) and (2).

After the spectral theorem is proved, it is easy to deduce from it the generalized versions of our theorems concerning square roots, the functional calculus, the polar decomposition, and properties of commutativity, and, in fact, to answer practically every askable question about bounded normal transformations.

The chief difficulties that remain are the considerations of non-normal and of unbounded transformations. Concerning general non-normal transformations, it is quite easy to describe the state of our knowledge; it is non-existent. No even unsatisfactory generalization exists for the tri-ngular form or for the Jordan canonical form and the theory of elementary ivisors. Very different is the situation concerning normal (and par-icularly self-adjoint) unbounded transformations. (The reader will ympathize with the desire to treat such transformations if he recalls hat the first and most important functional operation that most of us earn is differentiation.) In this connection we shall barely hint at the main obstacle the theory faces. It is not very difficult to show that if a self-adjoint linear transformation is defined for all vectors of Hilbert space, then it is bounded. In other words, the first requirement concerning transformations that we are forced to give up is that they be defined everywhere. The discussion of the precise domain on which a self-adjoint transformation may be defined and of the extent to which this domain may be enlarged is the chief new difficulty encountered in the study of unbounded transformations.

http://scidiv.bellevuecollege.edu/Math/halmos.html

http://zalafilms.com/films/halmospsynopsis.html

Stories about Physicists

Frauenfelder was a great story teller

Feynman

Dirac

Schrodinger

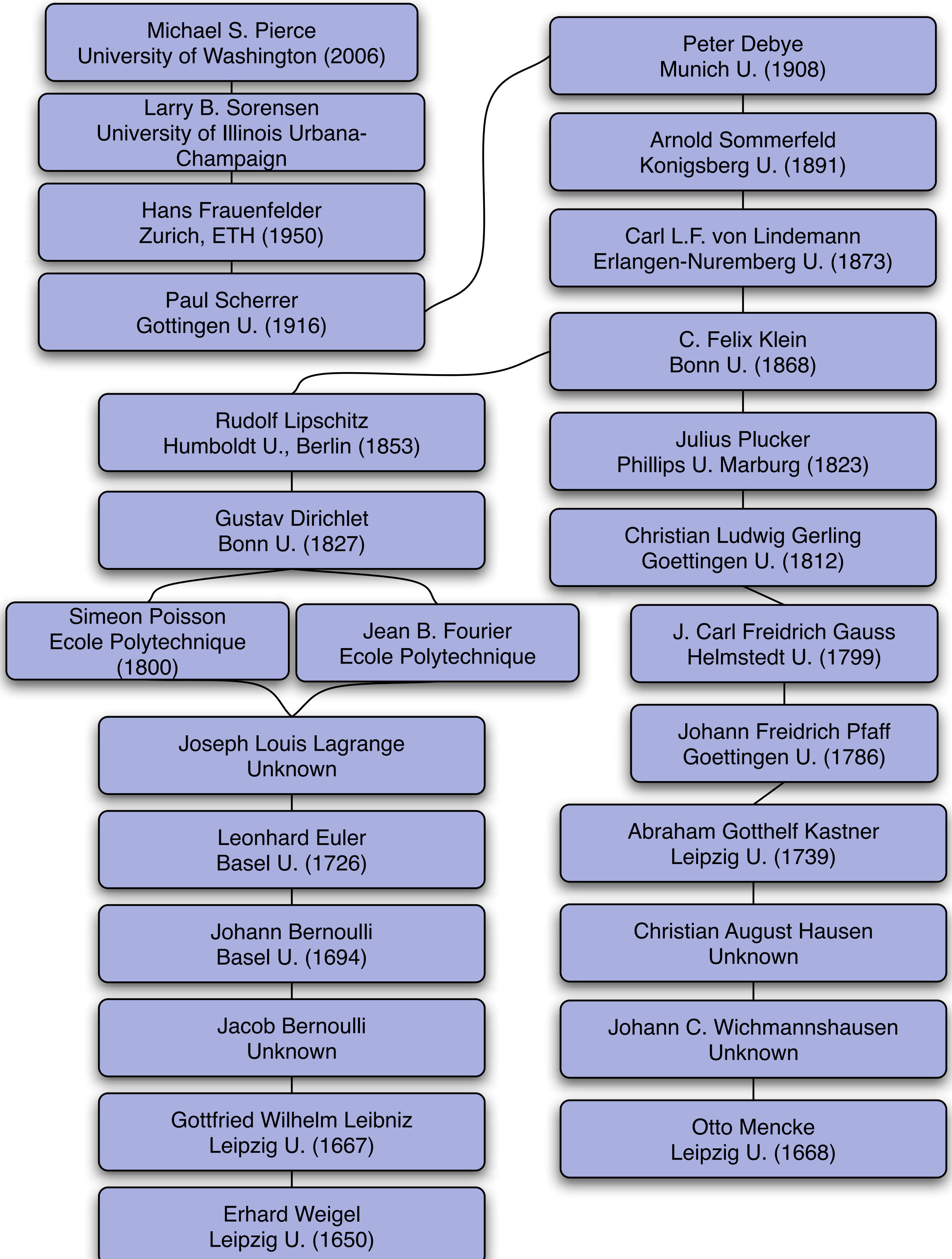
Heisenberg

Four Degrees of Separation

One Branch of your Academic Family Tree

Wolfgang Pauli (your great-grand-teacher)
Hans Frauenfelder (your grand-teacher)
Larry Sorensen (your teacher)
{ your name here goes here }

http://en.wikipedia.org/wiki/Six_degrees_of_separation
http://en.wikipedia.org/wiki/Six_Degrees_of_Kevin_Bacon
http://en.wikipedia.org/wiki/Erdos_number


Michael S. Pierce
University of Washington (2006)
Larry B. Sorensen
University of Illinois Urbana-Champaign
Hans Frauenfelder
Zurich, ETH (1950)
Paul Scherrer
Gottingen U. (1916)
Peter Debye
Munich U. (1908)
Arnold Sommerfeld
Konigsberg U. (1891)
Carl L.F. von Lindemann
Erlangen-Nuremberg U. (1873)
C. Felix Klein
Bonn U. (1868)
Julius Plucker
Phillips U. Marburg (1823)
Christian Ludwig Gerling
Goettingen U. (1812)
J. Carl Freidrich Gauss
Helmstedt U. (1799)
Johann Freidrich Pfaff
Goettingen U. (1786)
Abraham Gotthelf Kastner
Leipzig U. (1739)
Christian August Hausen
Unknown
Johann C. Wichmannshausen
Unknown
Otto Mencke
Leipzig U. (1668)
Rudolf Lipschitz
Humboldt U., Berlin (1853)
Gustav Dirichlet
Bonn U. (1827)
Simeon Poisson
Ecole Polytechnique (1800)
Jean B. Fourier
Ecole Polytechnique
Joseph Louis Lagrange
Unknown
Leonhard Euler
Basel U. (1726)
Johann Bernoulli
Basel U. (1694)
Jacob Bernoulli
Unknown
Gottfried Wilhelm Leibniz
Leipzig U. (1667)
Erhard Weigel
Leipzig U. (1650)

Four Degrees of Separation

One Branch of your Academic Family Tree

Enrico Fermi (your great-grand-teacher)

Darragh Nagle (your grand-teacher)

Larry Sorensen (your teacher)

{ your name here goes here }

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http://en.wikipedia.org/wiki/Six_Degrees_of_Kevin_Bacon
http://en.wikipedia.org/wiki/Erdos_number

The Four Primary Formulations of Quantum Mechanics

Matrix Mechanics
Heisenberg (1925; age 23)

Wave Mechanics
Schrodinger (1926; age 38)

Transformation Theory
Dirac (1925; age 23)

Path Integral
Feynman (1941; age 23)

Matrix Mechanics

Heisenberg (1925; age 23)

$$xp - px \neq 0$$

$$xp - px = i\hbar$$

$$H\psi = -i\hbar \frac{d\psi}{dt}$$

$$H\psi_n = E_n \psi_n$$

Matrix formulation

H, x, and p are matrices

ψ is a vector

E_n is a number

1. The Birth of Quantum Mechanics

It was June 7, 1925, that Werner Heisenberg left for the North Sea island of Helgoland wanting to find some rest after a bad attack of hay fever. Heisenberg was working at that time on the spectral lines of hydrogen, trying to find a manner to calculate these lines in a consistent way. In Helgoland, although he went there to rest, he got completely obsessed by the problem, and he hardly slept, deviding his time between working on his problem, engaging in mountain climbing and learning by heart poems from Goethe's West Osticher Divan.

It was one of these nights that Heisenberg 'invented' modern quantum mechanics. He wrote later in his book 'Der Teil und das Ganze' [1]: "It was about three o' clock at night when the final result of the calculation lay before me. At first I was deeply shaken. I was so excited that I could not think of sleep. So I left the house and awaited the sunrise on the top of a rock."

On June 9 Heisenberg returned to Gottingen and sent a copy of his results to Wolfgang Pauli, commenting in the accompanying letter: "Everything is still vague and unclear to me, but it seems as if the electrons will nomore move on orbits".

On July 25, Heisenberg's paper announcing the invention of quantum mechanics is received by the Zeitschrift fur Physik [2]. Before that he had also given a copy of the paper to Max Born commenting "that he had written a crazy paper and did not dare to send it in for publication, and that Born should read it and advice him on it."

Born mentiones that at first he was completely astonished by the strangeness of the calculations that Heisenberg proposes in the paper. But then, one morning, on July 10, Born suddenly realized that the type of calculation that Heisenberg proposes corresponds exactly to the matrix calculation that had been invented by mathematicians a long time before. Then Born reformulates, together with one of his students Pascual Jordan, Heisenberg's results in formal matrix language, to give rise to the first formal formulation of the new quantum mechanics [3].

It is amazing to know that shortly after Born received a copy of a paper written by a young British physicist that he did not know, Paul Adrien Dirac, which contained many of the results that he and Jordan just derived from Heisenberg's calculations [4]. Dirac had already in this first paper on quantum mechanics introduced a much more abstract mathematical language than matrix mechanics, it were the first steps finally leading to von Neumann's abstract Hilbert space formulation.

When Heisenberg wrote the first paper on quantum mechanics he had not known about matrix mathematics, but rapidly caught up, and started to work together with Born and Jordan on elaborating further the mathematical aspects of the theory, and also Pauli got caught up in the new physics. In the fall of 1925 he derived for the first time the complete Balmer formula for the hydrogyn atom (the set of discrete energy levels of an electron bound in hydrogen) [5].

Erwin Schroedinger did not know anything of all these happenings. He was also working on the problem of the hydrogyn atom but starting from a completely different approach. Schroedinger was, already long before he came with the 'second' invention' of quantum mechanics, actively interested in the problem of the description of the atom. He was inspired in his approach by work of Louis de

Broglie and Albert Einstein, considering the wave aspects of quantum particles. His goal was to formulate quantum mechanics as a part of classical wave mechanics, where the particle behavior of quantum entities would correspond to the behavior of singularities of the waves.

And so Schrodinger indeed manages to present a wave model of the atom and to also derive the complete Balmer formula, as Pauli did at the same time by means of matrix mechanics: the foundations of Schrodinger's wave mechanics was laid [6]. During the next half year, Schrodingers paper on the foundations of wave mechanics was followed by three other papers, containing elaborations of the mathematical aspects of the formalism and applications to new problems [7, 8, 9]. It became clear that wave mechanincs and matrix mechanics gave identical results, also in problems other than the description of the hydrogen atom. And of course the question arose: what do these theories, founded on completely different conceptual assumptions, have in common? Schrodinger [10] investigated the similarities of matrix mechanics and wave mechanics, and could show that indeed they will lead to similar results in all conceivable situations.

The mystery of how such conceptually completely different theories, expressed in formalism formulated by means of a completely different mathematical appratus, could give rise to identical results was only completely understood however after John von Neumann formulated the operator algebra version of quantum mechanics in 1932 [11]. That is also the place where Hilbert space, as a mathematical structure, was introduced into the formulation of quantum mechanics.

References

1.

2. W. Heisenberg, "Der Teil un das Ganze", Piper, Munich, (1969).
3.

4. W. Heisenberg, *Zeitschr. Phys.*, **33**, 879, (1925).
5.

6. M. Born and P. Jordan, *Zeitschr. Phys.*, **34**, 858, (1925).
7.

8. P.A.M. Dirac, *Proc. Roy. Soc.* A, **109**642, (1925).
9.

10. W. Pauli, *Zeitschr. Phys.*, **36**, 336, (1926).
11.

12. E. Schrodinger, *Ann. der Phys.*, **79**361, (1926).
13.

Wave Mechanics

Schrodinger (1926; age 38)

The Schrodinger Equation

$$H\psi = -i\hbar\frac{d\psi}{dt}$$

$$H\psi_n = E_n\psi_n$$

Differential equation formulation

H is an operator

ψ is a function

E_n is a number

Schrodinger: Life and Thought

By Walter Moore

A few days before Christmas, 1925, Schrodinger, a Viennese-born professor of physics at the University of Zurich, took off for a two-and-a-half-week vacation at a villa in the Swiss Alpine town of Arosa. Leaving his wife in Zurich, he took along de Broglie's thesis, an old Viennese girlfriend (whose identity remains a mystery) and two pearls. Placing a pearl in each ear to screen out any distracting noise, and the woman in bed for inspiration, Schrodinger set to work on wave mechanics. When he and the mystery lady emerged from the rigors of their holiday on Jan. 9, 1926, the great discovery was firmly in hand.

Schrodinger's wave equation, published only a few weeks later, was immediately accepted as "a mathematical tool of unprecedented power in dealing with problems of the structure of matter," according to Mr. Moore. By 1960, more than 100,000 scientific papers had appeared based on the application of the equation. Schrodinger lavishly thanked his physicist friend Hermann Weyl for his help with the mathematics. (He was perhaps indebted to Weyl for an even greater favor: Weyl regularly bedded down Schrodinger's wife, Anny, so that Schrodinger was free to seek elsewhere the erotic inspiration he needed for his work.) Three more papers followed in quick succession, each an arrow to the hearts of the likes of Heisenberg, Born and Bohr, who had labored so long and so unsuccessfully on the problem. Schrodinger's equations were easy for physicists to solve. More important, for the first time, one could visualize what was happening to particles in the atom.

The physical basis of Schrodinger's theory was this: Ordinarily, one can think of a particle as a dot; but one should really visualize it as a little clump of waves, a "standing wave" in today's parlance. Don't bother thinking of electrons as particles, Schrodinger said, and forget about this quantum-leap business. Just apply rules of wave interactions. Beyond constructing a mechanism for particle interactions, Schrodinger linked the quantum world of the microscopic to the classical world of macroscopic objects. Waves now existed, figuratively speaking, in atoms as well as in oceans. Physicists could understand waves, which they had endlessly studied. Schrodinger's wave mechanics saved quantum theory and at the same time threatened its underpinnings. It utilized continuous phenomena, waves, to explain the discontinuous quantum world of the atom.

For this, Schrodinger earned the Nobel Prize in Physics (in 1933) and the undying enmity of the great Werner Heisenberg. Schrodinger had destroyed Heisenberg's precious matrices. Schrodinger was old. He was an outsider from Zurich, not part of the Gottingen-Copenhagen quantum clique. Worst of all, he was right. The clique felt compelled to retaliate. Pauli referred to Schrodinger's views as "Zurich superstitions." Heisenberg was less charitable, calling the theory "abominable" and worse. Heisenberg would later eat his words. In 1927 he incorporated Schrodinger's wave functions as an integral part of his uncertainty principle.

How does one explain Schrodinger's sudden burst of genius, uncommon even in that post-World War I era of uncommon geniuses? The man appears to have been extraordinarily common. The picture of Schrodinger that emerges from Mr. Moore's book is one of a conceited, selfish, childish, hopelessly middle-class nerd, one who worried about his awards and medals and was obsessed with his pension and salary.

The New York Times
nytimes.com

January 7, 1990

THE LONE RANGER OF QUANTUM MECHANICS

By DICK TERESI; Dick Teresi is the co-author of ''The Three-Pound Universe'' and the forthcoming ''Would the Buddha Wear a Walkman?,'' about Eastern mysticism and Western technology.

SCHRODINGER

Life and Thought.

By Walter Moore.

Illustrated. 513 pp. New York:

Cambridge University Press. $39.50.

Theoretical physicists are the shooting stars of science. They do their best work in their 20's, then seemingly burn out. Theorists commonly retire, intellectually speaking, by their 30's to become "elder statesmen" of physics. Four of the giants of quantum mechanics - Paul Dirac, Werner Heisenberg, Wolfgang Pauli and Niels Bohr - all crafted their greatest theories as very young men. (Dirac and Heisenberg, in fact, were accompanied by their mothers to Stockholm to accept their Nobel Prizes.) Dirac summed up the phenomenon in a poem he once wrote, the sentiment of which is that a physicist is better off dead once past his 30th birthday.

How, then, does one explain Erwin Schrodinger? At the age of 38, positively geriatric for a theorist, Schrodinger changed forever the face of physics with four exquisite papers, all written and published in a six-month period of theoretical research that is without parallel in the history of science. During this time he discovered wave mechanics, which greatly accelerated the progress of quantum theory. J. Robert Oppenheimer called Schrodinger's theory "perhaps one of the most perfect, most accurate, and most lovely man has discovered," and the great physicist and mathematician Arnold Sommerfeld said wave mechanics "was the most astonishing among all the astonishing discoveries of the twentieth century."

Yet, until this flurry of activity, Schrodinger was nothing more than a competent, undistinguished physicist who had revealed no hint of his extraordinary brilliance early in his career. After his great discovery, he never again exhibited this brilliance. And in the 1920's world of theoretical physics in which collaboration was the norm, Schrodinger chose to work alone. Moreover, he had no love for the branch of physics he had saved. He was the Lone Ranger of quantum mechanics - a stranger who rode into town, saw a problem, solved it, then virtually rode away from it all.

Walter Moore has written an admirable book about this intriguing man. Mr. Moore, an emeritus professor of physical chemistry at the University of Sydney, Australia, and the author of the textbook "Physical Chemistry," sets out to do more than chronicle Schrodinger's life and work. He attempts to find the roots of genius in a man's life; in this case, he is searching for the secret behind the greatest six-month burst of creativity in scientific history.

By 1925 quantum theory had already modified, if not supplanted, the classical Newtonian view that everything was a continuum: that energy could be emitted in an infinite range of amounts, that light undulated in continuous waves, and so forth. Quantum theory, on the other hand, held that everything is quantized, or expressed in multiples of a basic unit. Energy and matter are distributed in discrete amounts; you must have multiples of certain minimum quantities. The universe is lumpy - a pile of rice as opposed to a scoop of mashed potatoes. Niels Bohr had extrapolated this theory to the arena of the atom. The electrons in an atom, he said, occupy quantized orbits. They can leap from one fixed orbit to another, but may not rest between these states. This made the theorists uneasy. Where, for instance, do the electrons go between orbits? And what are the rules that govern their quantum leaps?

Enter Werner Heisenberg, at the age of 24 already considered, next to Einstein, the most brilliant physicist in the world. Heisenberg, with help from Max Born and Pascual Jordan, came up with a matrix theory, which supposedly explained the travels of the electron by a complex form of mathematics called matrices. There remained some problems, however. Heisenberg's solution did not allow one to visualize what was happening inside the atom. Also, the smartest physicists in the world found the equations impossible to solve.

Along came Louis de Broglie. This young French physicist presented a most unusual thesis for his doctoral degree at the University of Paris. He put forth the proposition that, at certain velocities, an electron behaves more like a wave than a particle. De Broglie's thesis examiners couldn't make head or tail out of this concept and neither could most theorists, with the exception of two: Albert Einstein, who applauded it, and Erwin Schrodinger, who exploited it.

A few days before Christmas, 1925, Schrodinger, a Viennese-born professor of physics at the University of Zurich, took off for a two-and-a-half-week vacation at a villa in the Swiss Alpine town of Arosa. Leaving his wife in Zurich, he took along de Broglie's thesis, an old Viennese girlfriend (whose identity remains a mystery) and two pearls. Placing a pearl in each ear to screen out any distracting noise, and the woman in bed for inspiration, Schrodinger set to work on wave mechanics. When he and the mystery lady emerged from the rigors of their holiday on Jan. 9, 1926, the great discovery was firmly in hand.

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For this, Schrodinger earned the Nobel Prize in Physics (in 1933) and the undying enmity of the great Werner Heisenberg. Schrodinger had destroyed Heisenberg's precious matrices. Schrodinger was old. He was an outsider from Zurich, not part of the Gottingen-Copenhagen quantum clique. Worst of all, he was right. The clique felt compelled to retaliate. Pauli referred to Schrodinger's views as "Zurich superstitions." Heisenberg was less charitable, calling the theory "abominable" and worse. Heisenberg would later eat his words. In 1927 he incorporated Schrodinger's wave functions as an integral part of his uncertainty principle.

How does one explain Schrodinger's sudden burst of genius, uncommon even in that post-World War I era of uncommon geniuses? The man appears to have been extraordinarily common. The picture of Schrodinger that emerges from Mr. Moore's book is one of a conceited, selfish, childish, hopelessly middle-class nerd, one who worried about his awards and medals and was obsessed with his pension and salary. (He didn't accept an offer from Princeton University because it wouldn't give him parity with Einstein.) He even drove a BMW. Mr. Moore is exhaustive in his research of Schrodinger's life, but, as in a scientific paper, he is heavy on data and parsimonious in his explanation of that data. Mr. Moore is a chemist and - if you'll forgive the cheap shot - the book is more of a quantitative analysis than a deep psychological portrait. On the other hand, his objectivity allows him to study candidly, and nonjudgmentally, two major obsessions of Schrodinger's life - the Eastern philosophy of Vedanta and sex.

Mr. Moore informs us that Schrodinger kept a series of "little black books" in which he recorded the names of all his loves with a code to indicate "the denouement," as the author puts it, of each affair. He unbuttons Schrodinger's code and reveals a life of stunning promiscuity. Schrodinger admitted he detested his wife, Anny, sexually, and took on a series of mistresses, three of whom bore him illegitimate daughters. Immediately after his triumph in wave mechanics, he agreed to tutor 14-year-old twin girls named Withi and Ithi Junger. Schrodinger called the latter "Ithy-bitty" and regularly fondled her during their math lessons. He finally seduced her when she was 17, assuring her she wouldn't get pregnant. She did, Schrodinger immediately lost interest in her, and the girl underwent a disastrous abortion that left her sterile. He then took on Hilde March, the wife of his assistant Arthur March, as his mistress, and she bore him a daughter. March, ever the dutiful assistant, agreed to be named the father, while his wife moved eventually into the Schrodinger household to serve as Schrodinger's "second wife." Well, the great man's sordid affairs go on and on, and Mr. Moore faithfully serves up all of the titillating details. He concludes that Schrodinger needed "tempestuous sexual adventures" to inspire his great discoveries. Unfortunately, the notebook for the critical year 1925 has disappeared, so the woman who erotically guided Schrodinger to his famous wave equation, "like the dark lady who inspired Shakespeare's sonnets," the biographer tells us, "may remain forever mysterious."

As for Vedanta, the recent rash of new-age physics writers will be chagrined to learn that Schrodinger himself rejected the idea that philosophical conclusions can be drawn from wave mechanics or any work in theoretical physics. But Mr. Moore believes that Vedanta - which holds that through the Self one can comprehend the essence of the universe - may have been instrumental in Schrodinger's discovery of wave mechanics. Much has been written about Schrodinger's insistence that the electron is not a particle; it doesn't just behave like a wave, he said, but rather is a wave, as real as a radio wave or an ocean wave. This belief of Schrodinger's, soon discarded by other physicists, is played down by Mr. Moore, who points out that Schrodinger actually wavered on this point very early on.

After wave mechanics, Schrodinger attempted, and failed (as did Einstein), to forge a unified field theory, but he did write a bizarre and wonderful book entitled "What Is Life?" in which he was the first to suggest that a chromosome is nothing more than a message written in code. The book inspired at least two young scientists to seek careers in biology - James Watson and Francis Crick, who eventually were given the Nobel Prize in Physiology or Medicine for decoding DNA.

Schrodinger never accomplished his greatest dream, to reinstate classical physics with its almost Vedantic continuity over the lumpiness of quantum mechanics. Perhaps as a revenge against his quantum enemies, he did leave behind a paradox that torments scientists to this day. The paradox of Schrodinger's cat links the squishy quantum microworld, with its statistical probabilities that replace cause and effect, to the Newtonian macroworld of everyday objects that obey hard-and-fast rules of causality. Put a cat in a box, Schrodinger said, with a flask of lethal acid. In a Geiger tube, place a small quantity of radioactive material, so little that in the course of an hour one atom has a 50-50 chance of disintegrating, setting off the Geiger counter, which will trigger a hammer that shatters the flask of acid that will kill the cat. So, after one hour is the cat dead or alive? Schrodinger said that if one used the quantum wave function to describe the entire system, "the living and the dead cat" would be "smeared out (pardon the expression) in equal parts." Schrodinger intended his paradox as a sarcastic comment on quantum probability or "blurred variables." One can resolve the uncertainty, he explained, by looking in the box.

Schrodinger himself, however, must always remain somewhat blurred, despite Walter Moore's heroic efforts in this important book about the century's most enigmatic scientist. For the average reader, "Schrodinger" may be tough going, but it serves up a wonderfully frank and unglamorized, albeit narrow, history of the development of quantum mechanics. Much of the science in this book is only opaquely explained, but explaining science is not the book's main function. It is an attempt to analyze a soul, and in that respect, it surpasses even "The Double Helix" by James Watson in its examination of the most visceral drives of a great scientist.

Erwin Schrodinger in 1911 during his one-year military service inthe army. (From "Schrodinger)

Transformation Theory

Dirac (1925; age 23)

$$H \mid \psi > = -i\hbar \frac{d \mid \psi >}{dt}$$

$$H \mid \psi_n > = E_n \mid \psi_n >$$

$$< bra \mid ket >$$

$$< a \mid \mid b > \mid c > < d \mid$$

Abstract geometric formulation

H is an abstract operator

$|\psi>$ is an abstract vector

E_n is a number

Dirac at the University of Wisconsin

Paul Adrien Maurice Dirac (1902-1984)
From: **Dirac: A Scientific Biography** by Helge Kragh

Dirac's introversive style and his interest in abstract theory were rather foreign to the scientists at the University of Wisconsin. They recognized his genius but had difficulties in comprehending his symbolic version of quantum theory. The Americans also found him a bit of a strange character. A local newspaper, the Wisconsin State Journal, wanted to interview the visiting physicist from Europe and assigned this task to a humorous columnist known as 'Roundy'. His encounter with Dirac is quoted here in extenso because it not only reveals some characteristic features of Dirac's personality but also is an amusing piece of journalism:

I been hearing about a fellow they have up at the U. this spring–a mathematical physicist, or something, they call him–who is pushing Sir Isaac Newton, Einstein and all the others off the front page. So I thought I better go up and interview him for the benefit of the State Journal readers, same as I do all the other top notchers. His name is Dirac and he is an Englishman. He has been giving lectures for the intelligensia of the math and physics department–and a few other guys who got in by mistake.

So the other afternoon I knocks at the door of Dr. Dirac's office in Sterling Hall and a pleasant voice says, "Come in." And I want to say here and now that this sentence "come in" was about the longest one emitted by the doctor during our interview. He sure is all for efficiency in conversation. It suits me. I hate a talkative guy.

I found the doctor a tall youngish-looking man, and the minute I see the twinkle in his eye I knew I was going to like him. His friends at the U. say he is a real fellow too and good company on a hike – if you can keep him in sight, that is.

The thing that hit me in the eye about him was that he did not seem to be at all busy. Why if I went to interview an American scientist of his class–supposing I could find one–I would have to stick around an hour first. Then he would blow in carrying a big briefcase, and while he talked he would be pulling lecture notes, proof, reprints, books, manuscripts, or what have you, out of his bag. But Dirac is different. he seems to have all the time there is in the world and his heaviest work is looking out the window. If he is a typical Englishman it's me for England on my next vacation!

Then we sat down and the interview began. "Professor," says I, "I notice you have quite a few letters in front of your last name. Do they stand for anything in particular?"

"No." says he.

"You mean I can write my own ticket?"

"Yes," says he.

"Will it be all right if I say that P. A. M. stands for Poincare Aloysius Mussolini?"

"Yes," says he.

“Fine,” says I, “We are getting along great! Now doctor will you give me in a few words the low-down on all your investigations?”

“No,” says he.

“Good,” says I. “Will it be all right if I put it this way–‘Professor Dirac solves all the problems of mathematical physics, but is unable to find a better way of figuring out Babe Ruth’s batting average’?”

“Yes,” says he.

“What do you like best in America?” says I.

“Potatoes,” says he.

“Same here,” says I. “What is your favorite sport?”

“Chinese chess,” says he.

That knocked me cold! It sure was a new one to me! Then I went on: “Do you go to the movies?”

“Yes,” says he.

“When?” says I.

“In 1920–perhaps also 1930,” says he.

“Do you like to read the Sunday comics?”

“Yes,” says he, warming up a bit more than usual.

“This is the most important thing yet Doctor,” says I. “It shows that me and you are more alike than I thought. And now I want to ask you something more: They tell me that you and Einstein are the only two real sure-enough high-brows and the only ones who can really understand each other. I won’t ask you if this is straight stuff for I know you are too modest to admit it. But I want to know this–Do you ever run across a fellow that even you can’t understand?”

“Yes,” says he.

“This will make great reading for the boys down at the office,” says I. “do you mind releasing to me who he is?”

“Weyl,” says he.

The interview came to a sudden end just then for the doctor pulled out his watch and I dodged and jumped for the door. But he let loose a smile as we parted and I knew that all the time he had been talking to me he was solving some problem no one else could touch.

But if that Professor Weyl ever lectures in this town again I sure am going to take a try at understanding him! A fellow ought to test his intelligence once in a while.

A true story about the physcist/mathematician Paul Dirac.

Dirac was apparently a very hard person to get along with. Soon after he was awarded the Nobel Prize in Physics, Dirac went on a speaking tour of the country, visiting different universities and talking about his research. In those days, it was more convenient for him to travel by car, so he had a big car and a driver who took him from one speaking engagement to the next.

Dirac and his driver got to be very good friends after awhile and at one point, his driver remarked, "You know, I am so sick and tired of hearing the same lecture over and over again. I easily give it myself!"

Dirac thought about this for a moment, and then decided that his driver could give the next speaking engagement at U. Michigan in Ann Arbor, Michigan. Before reaching the university, Dirac and his driver switched clothes. When the reached the university, the driver went up to the podium and delivered Dirac's seminar flawlessly. After he was finished, an upstart graduate student asked a question, snottily pointing out a perceived mistake in the talk.

The Driver gave the student a long look of contempt and then exclaimed, "That question is so stupid that even my driver could answer it!", and Dirac stepped forward and proceeded to do so.

Today, if you go to U Mich and see a picture on the wall of Dirac and his driver, you would have to know this story to realize that the two are switched.

Path Integral Formulation
Sum over Histories Formulation
Lagrangian Formulation
Amplitude Formulation

Feynman (1941; age 23)

The probability to go from a to b is the square of an amplitude

$$P(b,a) = \mid Amp(b,a) \mid^2$$

The amplitude is the weighted sum over all possible ways to go to b from a

$$Amp(b,a) = \text{constant} \sum_{\text{all paths}} \exp(\mathrm{i}S/\hbar)$$

S is the classical action

I went to a beer party in the Nassau Tavern in Princeton. There was a gentleman, newly arrived from Europe (Herbert Jehle) who came and sat next to me. Europeans are much more serious than we are in America because they think a good place to discuss intellectual matters is a beer party. So he sat by me and asked, "What are you doing" and so on, and I said, "I'm drinking beer." Then I realized that he wanted to know what work I was doing and I told him I was struggling with this problem, and I simply turned to him and said "Listen, do you know any way of doing quantum mechanics starting with action--where the action integral comes into the quantum mechanics?" "No," he said, "but Dirac has a paper in which the Lagrangian, at least, comes into quantum mechanics. I will show it to you tomorrow."

Next day we went to the Princeton Library (they have little rooms on the side to discuss things) and he showed me this paper. Dirac's short paper in the *Physikalische Zeitschrift der Sowjetunion* claimed that a mathematical tool which governs the time development of a quantal system was "analogous" to the classical Lagrangian.

Professor Jehle showed me this; I read it; he explained it to me, and I said, "What does he mean, they are analogous; what does that mean, *analogous?* What is the use of that?" He said, "You Americans! You always want to find a use for everything!" I said that I thought that Dirac must mean that they were equal. "No," he explained, "he doesn't mean they are equal." "Well," I said, "let's see what happens if we make them equal."

So, I simply put them equal, taking the simplest example . . . but soon found that I had to put a constant of proportionality *A* in, suitably adjusted. When I substituted . . . and just calculated things out by Taylor-series expansion, out came the Schrödinger equation. So I turned to Professor Jehle, not really understanding, and said, "Well you see Professor Dirac meant that they were proportional." Professor Jehle's eyes were bugging out -- he had taken out a little notebook and was rapidly copying it down from the blackboard and said, "No, no, this is an important discovery."

Feynman's thesis advisor, John Archibald Wheeler (age 30), was equally impressed. He believed that the amplitude formulation of quantum mechanics--although mathematically equivalent to the matrix and wave formulations--was so much more natural than the previous formulations that it had a chance of convincing quantum mechanics's most determined critic. Wheeler writes:

Visiting Einstein one day, I could not resist telling him about Feynman's new way to express quantum theory. "Feynman has found a beautiful picture to understand the probability amplitude for a dynamical system to go from one specified configuration at one time to another specified configuration at a later time. He treats on a footing of absolute equality every conceivable history that leads from the initial state to the final one, no matter how crazy the motion in between. The contributions of these histories differ not at all in amplitude, only in phase. And the phase is nothing but the classical action integral, apart from the Dirac factor h. This prescription reproduces all of standard quantum theory. How could one ever want a simpler way to see what quantum theory is all about!

Doesn't this marvelous discovery make you willing to accept the quantum theory, Professor Einstein?"

Einstein replied in a serious voice, "I still cannot believe that God plays dice. But maybe", he smiled, "I have earned the right to make my mistakes."

John Wheeler

Thus it would have pleased Richard to know (and perhaps he did know, without my being aware of it) that there are now some indications that his PhD dissertation may have involved a really basic advance in physical theory and not just a formal development. The path integral formulation of quantum mechanics may be more fundamental than the conventional one, in that there is a crucial domain where it may apply and the conventional one may fail. That domain is quantum cosmology.

For Richard's sake (and Dirac's too), I would rather like it to turn out that the path integral method is the real foundation of quantum mechanics and thus of physical theory. This is true despite the fact that, having an algebraic turn of mind, I have always personally preferred the operator approach, and despite the added difficulty, in the absence of a Hilbert-space formalism, of interpreting the wavefunction or density matrix of the universe (already a bit difficult to explain in any case, as anyone attending my classes will attest). If notions of transformation theory, unitarity and causality really emerge from the mist only after a fairly clear background metric appears (that metric itself being the result of a quantum mechanical probabilistic process), then we may have a little more explaining to do. Here Dick Feynman's talents and clarity of thought would have been a help.

Murray Gell-Mann

Thirty-one years ago, Dick Feynman told me about his "sum over histories" version of quantum mechanics.

"The electron does anything it likes," he said. "It just goes in any direction at any speed, . . . however it likes, and then you add up the amplitudes and it gives you the wave function."

I said to him, "You're crazy." But he wasn't.

Freeman Dyson

The Five Stages to Learning Quantum Mechanics

You don’t know how to calculate
You don’t know what it means
It doesn’t bother you (now)

You don’t know how to calculate
You don’t know what it means
It bothers you (soon)

You know how to calculate
You don’t know what it means
It bothers you (end of this class)

You know how to calculate
You don’t know what it means
It doesn’t bother you (most physicists)

You know how to calculate
You know what it means
It doesn’t bother you (Nirvana)

or maybe it still does (Samsara)

from the Preface to the Virtual Book

Nine formulations of quantum mechanics

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(Received 18 July 2001; accepted 29 November 2001)

Nine formulations of nonrelativistic quantum mechanics are reviewed. These are the wavefunction, matrix, path integral, phase space, density matrix, second quantization, variational, pilot wave, and Hamilton–Jacobi formulations. Also mentioned are the many-worlds and transactional interpretations. The various formulations differ dramatically in mathematical and conceptual overview, yet each one makes identical predictions for all experimental results.
[DOI: 10.1119/1.1445404]

I. WHY CARE ABOUT VARIOUS FORMULATIONS?

A junior-level classical mechanics course devotes a lot of time to various formulations of classical mechanics—Newtonian, Lagrangian, Hamiltonian, least action, and so forth (see Appendix A). But not a junior-level quantum mechanics course! Indeed, even graduate-level courses emphasize the wavefunction formulation almost to the exclusion of all variants. It is easy to see why this should be so—learning even a single formulation of quantum mechanics is difficult enough—yet at the same time students must wonder why it is so important to learn several formulations of classical mechanics but not of quantum mechanics. This article surveys nine different formulations of quantum mechanics. It is a project of the Spring 2001 offering of Oberlin College's Physics 412, "Applied Quantum Mechanics."

Why should one care about different formulations of mechanics when, in the end, each provides identical predictions for experimental results? There are at least three reasons. First, some problems are difficult in one formulation and easy in another. For example, the Lagrangian formulation of classical mechanics allows generalized coordinates, so it is often easier to use than the Newtonian formulation. Second, different formulations provide different insights.[1] For example, the Newtonian and least action principles provide very different pictorializations of "what's really going on" in classical mechanics. Third, the various formulations are variously difficult to extend to new situations. For example, the Lagrangian formulation extends readily from conservative classical mechanics to conservative relativistic mechanics, whereas the Newtonian formulation extends readily from conservative classical mechanics to dissipative classical mechanics. In the words of the prolific chemist E. Bright Wilson:[2]

> "I used to go to [J. H. Van Vleck] for quantum mechanical advice and found him always patient and ready to help, sometimes in a perplexing flow of mixed wave mechanical, operator calculus, and matrix language which often baffled this narrowly Schrödinger-equation-oriented neophyte. I had to learn to look at things in these alternate languages and, of course, it was indispensable that I do so."

Any attempt to enumerate formulations must distinguish between "formulations" and "interpretations" of quantum mechanics. Our intent here is to examine only distinct mathematical formulations, but the mathematics of course influences the conceptual interpretation, so this distinction is by no means clear cut,[3] and we realize that others will draw boundaries differently. Additional confusion arises because the term "Copenhagen interpretation" is widely used but poorly defined: For example, of the two primary architects of the Copenhagen interpretation, Werner Heisenberg maintained that[4] "observation of the position will alter the momentum by an unknown and undeterminable amount," whereas Niels Bohr[5] "warned specifically against phrases, often found in the physical literature, such as 'disturbing of phenomena by observation.' "

II. CATALOG OF FORMULATIONS

A. The matrix formulation (Heisenberg)

The matrix formulation of quantum mechanics, developed by Werner Heisenberg in June of 1925, was the first formulation to be uncovered. The wavefunction formulation, which enjoys wider currency today, was developed by Erwin Schrödinger about six months later.

In the matrix formulation each mechanical observable (such as the position, momentum, or energy) is represented mathematically by a matrix (also known as "an operator"). For a system with N basis states (where in most cases $N=\infty$) this will be an $N\times N$ square Hermitian matrix. A quantal state $|\psi\rangle$ is represented mathematically by an $N\times 1$ column matrix.

Connection with experiment. Suppose the measurable quantity $\mathcal{A}$ is represented by the operator $\hat{A}$. Then for any function $f(x)$ the expectation value for the measurement of $f(\mathcal{A})$ in state $|\psi\rangle$ is the inner product

$$\langle\psi|f(\hat{A})|\psi\rangle. \tag{1}$$

Because the above statement refers to $f(\mathcal{A})$ rather than to $\mathcal{A}$ alone, it can be used to find uncertainties [related to $f(\mathcal{A})=\mathcal{A}^2$] as well as expectation values. Indeed, it can even produce the eigenvalue spectrum, as follows:[6] Consider a set of real values $a_1, a_2, a_3, \ldots$, and form the non-negative function

$$g(x)\equiv(x-a_1)^2(x-a_2)^2(x-a_3)^2\cdots. \tag{2}$$

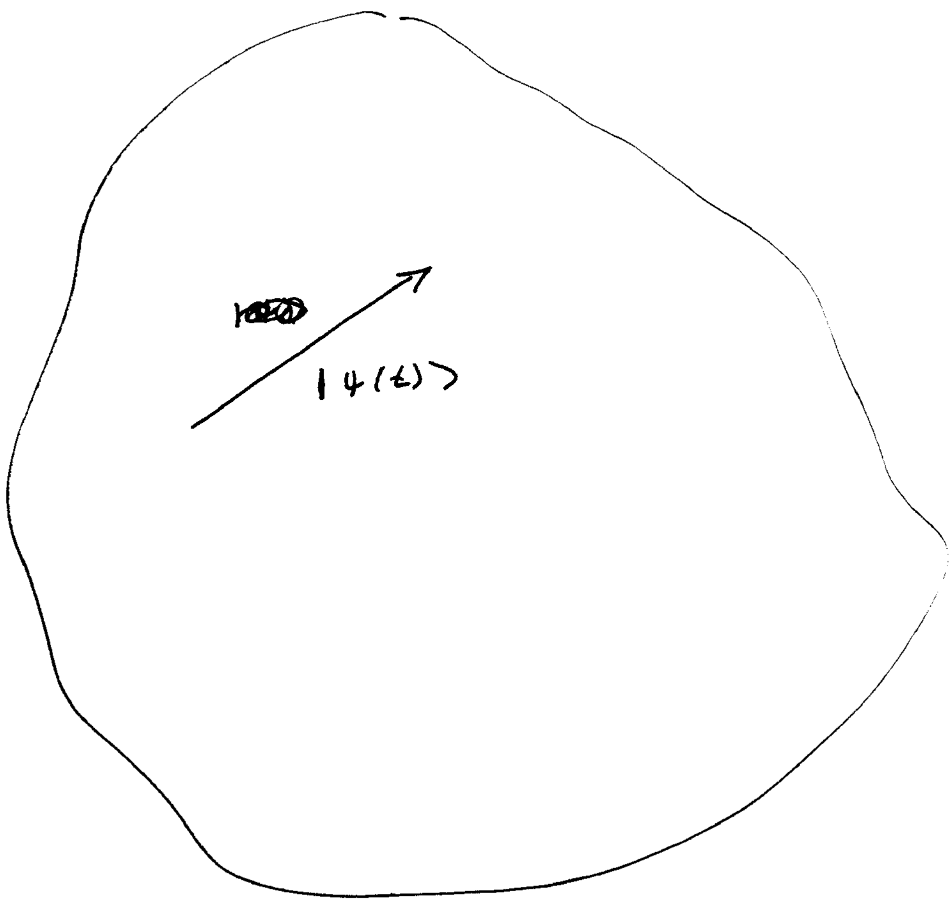
Oberlin's 9 Formulations

1 Matrix Mechanics (Heisenberg)
2 Wave Mechanics (Schrodinger)
Transformation Theory (Dirac)
3 Path Integral (Feynman)
4 Phase Space (Wigner)
5 Density Matrix
6 Second Quantization
7 Variational
8 Pilot Wave (Bohm)
9 Hamilton-Jacoby

THE GEOMETRY OF QM

QM TAKES PLACE IN HILBERT SPACE,

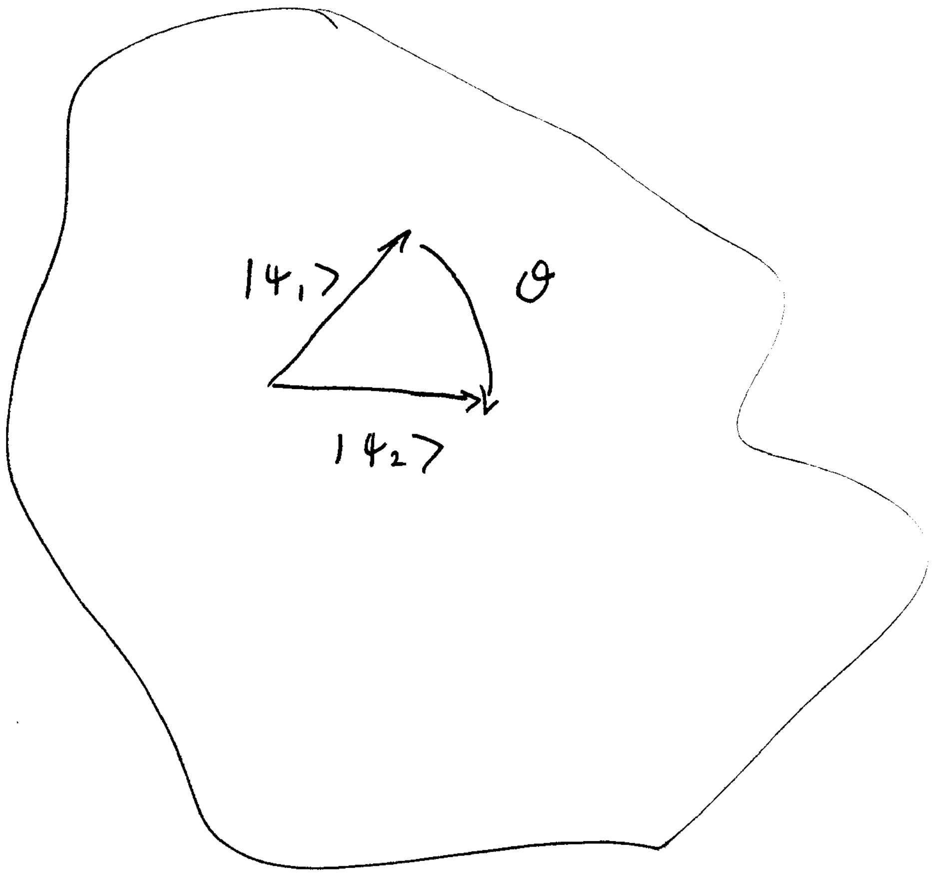
AN INFINITE DIMENSIONAL VECTOR SPACE.



STATES OF THE SYSTEM $\Rightarrow$ VECTORS

$|\psi(t)\rangle$ = A VECTOR IN THE HILBERT SPACE

ALSO HAVE OPERATORS $\mathcal{O}|\psi_1\rangle = |\psi_2\rangle$



EIGENVALUES OF HERMITEAN

PHYSICAL OBSERVABLES => OPERATORS

DYNAMICS OF THE SYSEM => $\mathcal{H}$ via TDSE

Time-Dependent Schrodinger Equation

$$H \, | \psi(t) \rangle = i \hbar \frac{d}{dt} | \psi(t) \rangle$$

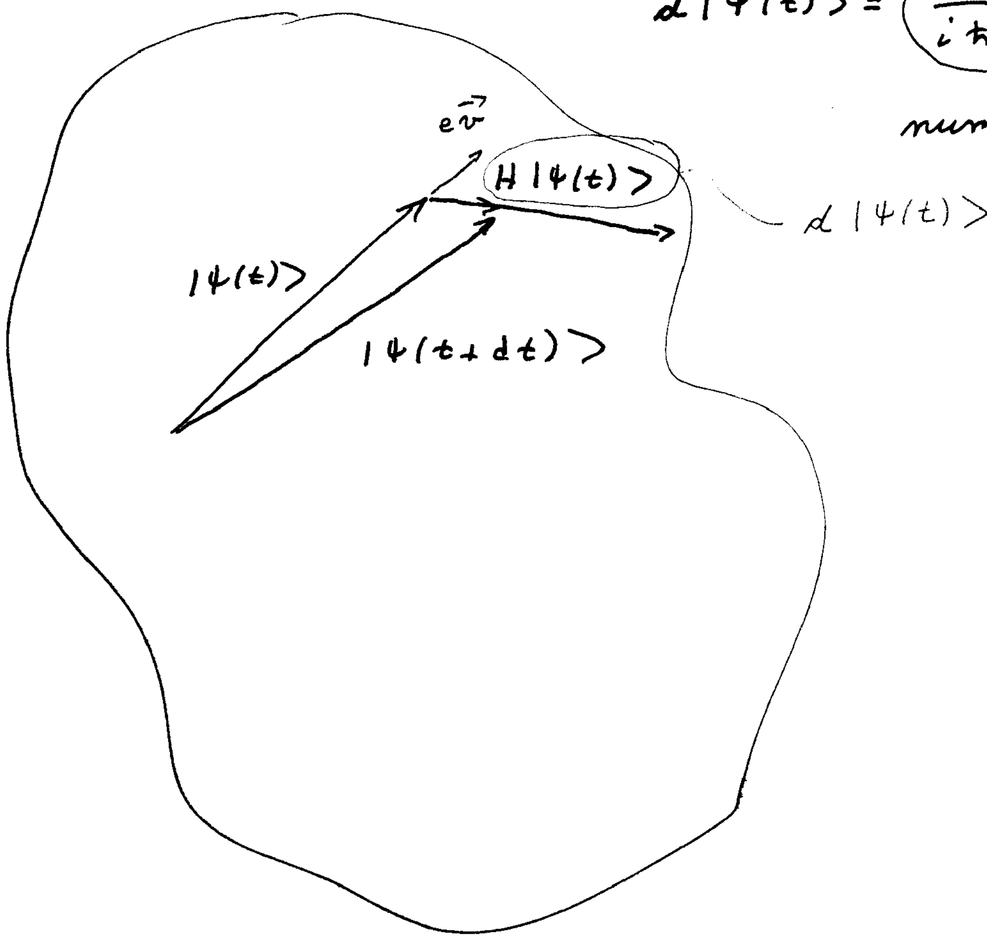
FIRST-ORDER DIFFEQ

SIMPLER THAN CM $\quad F = m \dfrac{d^2 x}{dt^2}$ SECOND ORDER

GEOMETRY OF THE TIME EVOLUTION

$$d | \psi(t) \rangle = \left(\frac{dt}{i \hbar} \right) \left(H \, | \psi(t) \rangle \right)$$

number



WHY MUST WE USE AN INFINITE DIMENSIONAL HILBERT SPACE? Because we live in a continuous world. (we think)

OBSERVABLES: POSITION, MOMENTUM, ENERGY ⇒ OPERATORS

LINE

HAS AN INFINITE NUMBER OF POINTS

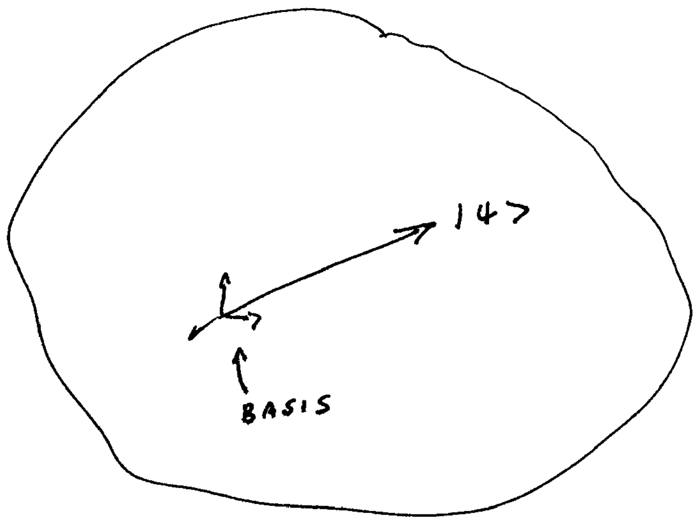
SO WE NEED AN INFINITE DIMENSIONAL VECTOR TO SPECIFY EVERY POINT ON THE LINE

infinite dimensional vector ⇒ function

10 d of STRING THEORY ARE LIKE THE ~~4d~~ 3d OF SPACETIME ...

change of basis

THE HILBERT SPACE IS AN ABSTRACT SPACE



ABSTRACT STATE VECTOR

TO GET THE USUAL ~~POSITION SPACE~~ WAVEFCNS,

WE MUST INTRODUCE A BASIS

$|\psi\rangle$ ABSTRACT, NO BASIS, GEOMETRIC OBJECT

$$\langle x|\psi\rangle = \psi(x)$$ POSITION-SPACE WAVEFCN

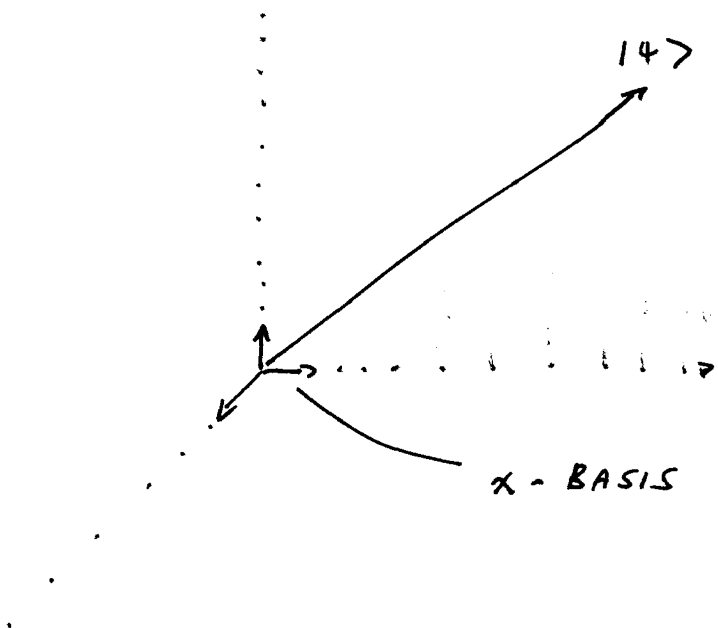
$$\langle p|\psi\rangle = \hat{\psi}(p)$$ MOMENTUM-SPACE WAVEFCN

$$\langle E|\psi\rangle = \tilde{\psi}(E)$$ ENERGY-SPACE WAVEFCN

inner product in DIRAC notation

GEOMETRY OF MEASUREMENT

MEASURE x



an ev x - OPERATOR

RESULT IS $e\vec{v}$ OF THE x - BASIS

STATE IS AN $e\vec{v}$ OF THE x - OPERATOR

The QM professor's escape:

I have taught graduate courses in quantum mechanics at Columbia, Stanford, Oxford, and Yale, and for almost all of them have dealt with measurement in the following manner. On beginning the lectures I told the students, "You must first learn the rules of calculation in quantum mechanics, and then I will discuss the theory of measurement and discuss the meaning of the subject." Almost invariably, the time allotted to the course ran out before I had to fulfill my promise.

Willis Lamb

Dedication

To all students of quantum mechanics past, present, and future, but especially to the first students who will learn quantum mechanics not as a mystery which cannot be understood, but as reality which must be experienced, explored, and harnessed.