http://www.scribd.com/doc/26811145/Cohen-Tannoudji-quantum-Mechanics-Vol-1

Free on screen viewer. There are also "free downloads"

http://www.amazon.com/Quantum-Mechanics-2-vol-set/dp/0471569526

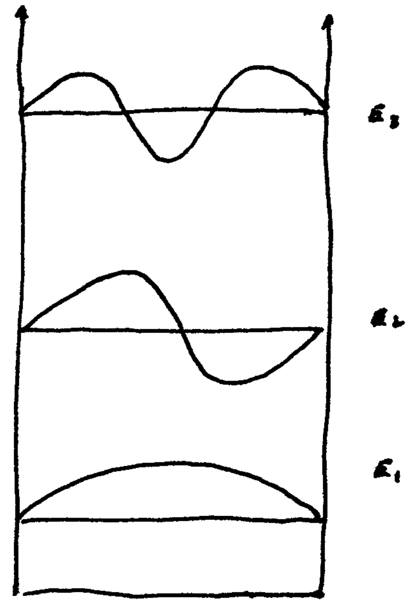
This didactically unrivalled textbook and timeless reference by Nobel Prize Laureate Claude Cohen-Tannoudji separates essential underlying principles of quantum mechanics from specific applications and practical examples and deals with each of them in a different section. Chapters emphasize principles; complementary sections supply applications. The book provides a qualitative introduction to quantum mechanical ideas; a systematic, complete and elaborate presentation of all the mathematical tools and postulates needed, including a discussion of their physical content and applications.
The book is recommended on a regular basis by lecturers of undergraduate courses.

LECTURE 6 JUNE 30, 2009

FINITE DIMENSIONAL $\longrightarrow$ INFINITE DIMENSIONAL

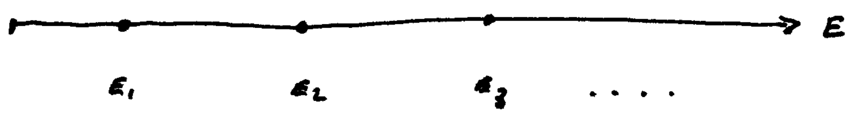
MATRICES	OPERATORS
VECTORS	FUNCTIONS
(DISCRETE EV'S)	(CONTINUOUS OR DISCRETE EV'S)

DISCRETE ~~ef's~~ en's



INFINITE SQUARE WELL

DISCRETE SPECTRUM



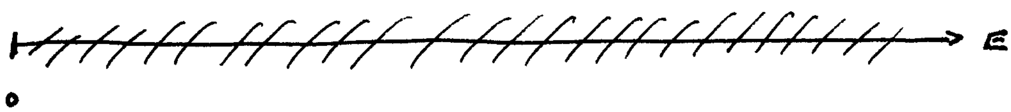
CAN BUILD ANY STATE INSIDE THE WELL OUT OF ef's

FOURIER SERIES EXPANSION

CONTINUOUS ev's

FREE PARTICLE

CONTINUOUS SPECTRUM



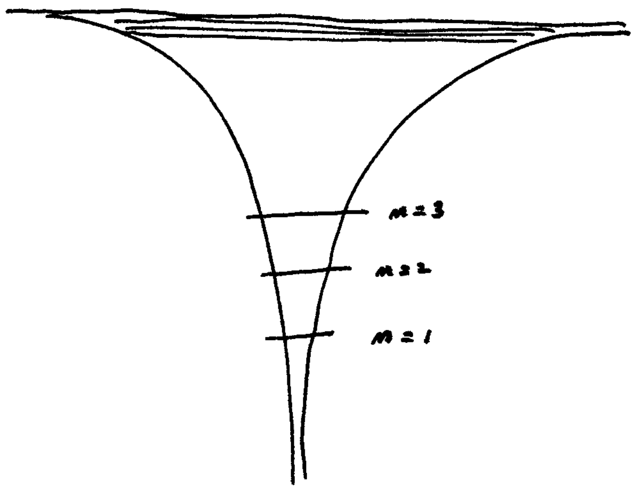
CAN BUILD ANY STATE in space OUT OF ef's

e^{ikx}

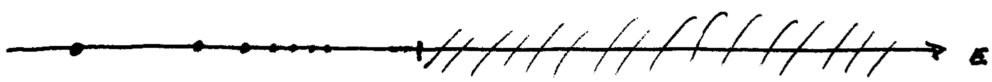
FOURIER TRANSFORM EXPANSION

BOTH

HYDROGEN



SPECTRUM



CAN BUILD ANY STATE OUT OF ef's

HYDROGEN EIGENFCN EXPANSION

FREE PARTICLE

$$\Omega |\omega\rangle = \omega |\omega\rangle$$

OPERATOR ef's ev's

FORM IN ANY BASIS

POSITION SPACE = POSITION BASIS

MOMENTUM SPACE = MOMENTUM BASIS

~~FOR~~ IN POSITION SPACE

TWO CLASSICAL OBSERVABLES

POSITION	X_{op}	ev's	ef's
MOMENTUM	P_{op}	ev's	ef's

$$X_{op}\, f(x) = x\, f(x)$$

~~ef's~~

$$P_{op}\, f(x) = -i\hbar \frac{\partial}{\partial x} f(x)$$

DO X AND P COMMUTE?

IS $[X, P] = 0$?

$XP - PX$

$$op = (x)\left(-i\hbar \frac{\partial}{\partial x}\right) - \left(-i\hbar \frac{\partial}{\partial x}\right)(x)$$

DOES $x \dfrac{\partial}{\partial x} = \dfrac{\partial}{\partial x} x$

$$x \frac{\partial f}{\partial x} \qquad \frac{\partial}{\partial x}(x f) = \frac{\partial x}{\partial x} f + x \frac{\partial f}{\partial x}$$

NO! THEY DO NOT COMMUTE!

$x_{op}\, p_{op} - p_{op}\, x_{op} = i\hbar I$

X AND P ARE COMPLETELY INCOMPATIBLE

$[\ ,\ ] \sim I$ COMP INCOMPATIBLE

$[\ ,\ ] = 0$ COMP COMPATIBLE

IN MOMENTUM SPACE

$$q(p)$$

$$X_{op}\, q(p) = i\hbar \frac{\partial}{\partial p} q(p)$$

$$P_{op}\, q(p) = p\, q(p)$$

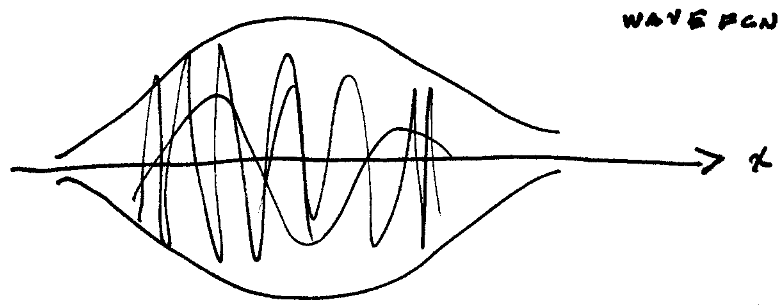
DO THEY COMMUTE?

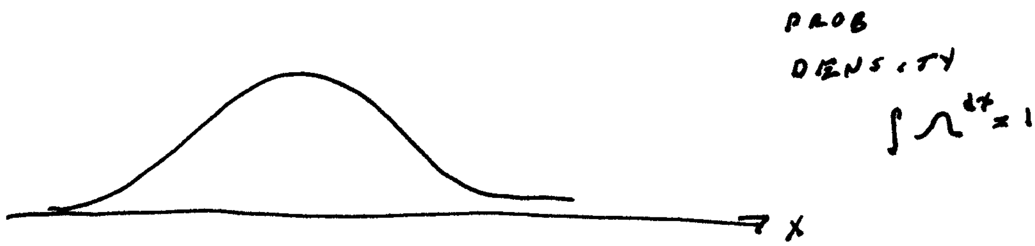
NO

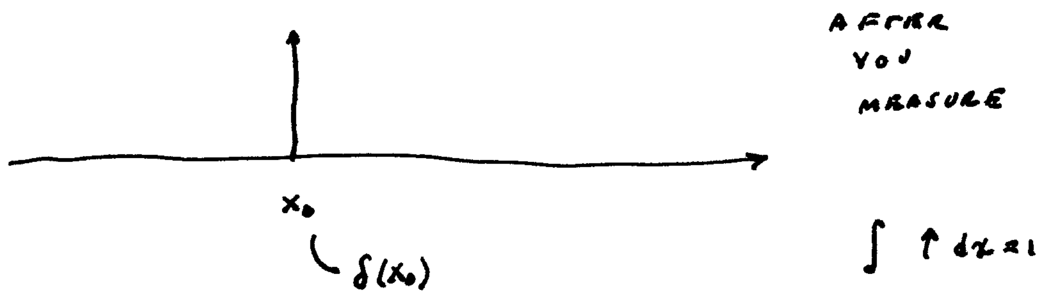
$$\left(p \frac{\partial}{\partial p}\right) q \neq \left(\frac{\partial}{\partial p} p\right) q$$

$$[X_{op}, P_{op}] = i\hbar I$$ COMPLETELY INCOMPATIBLE

WHAT ARE THE ef's OF x_{op} FOR THE FREE PARTICLE







THIS IS CALLED A DIRAC DELTA FCN

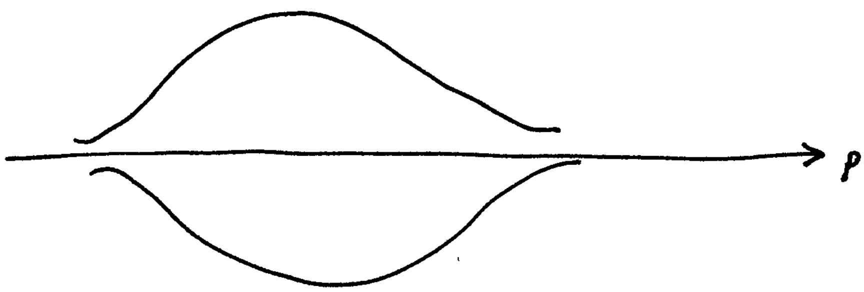
∞ HIGH

∞ NARROW

$\int \uparrow \, dx = 1$

WHAT ARE THE ef's OF THE P_{op}

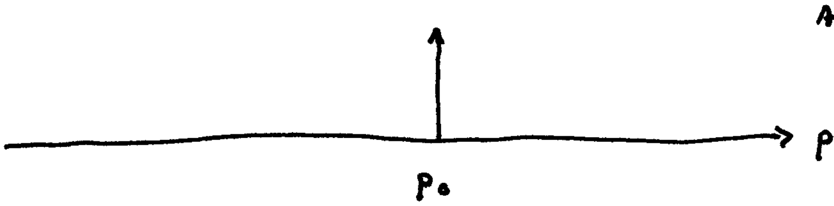
WAVEFCN



PROB DENSITY

$\int \sim \, dp = 1$

AFTER YOU MEASURE



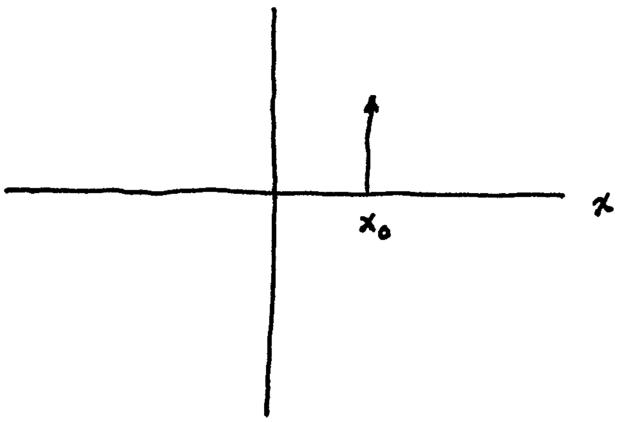
$\int \uparrow \, dp = 1$

$\delta(p_0)$

MEASURE X   ALWAYS $\delta(x_0)$

MEASURE p   ALWAYS $\delta(p_0)$

WHAT DOES $\delta(x_0)$ LOOK LIKE IN p-space?



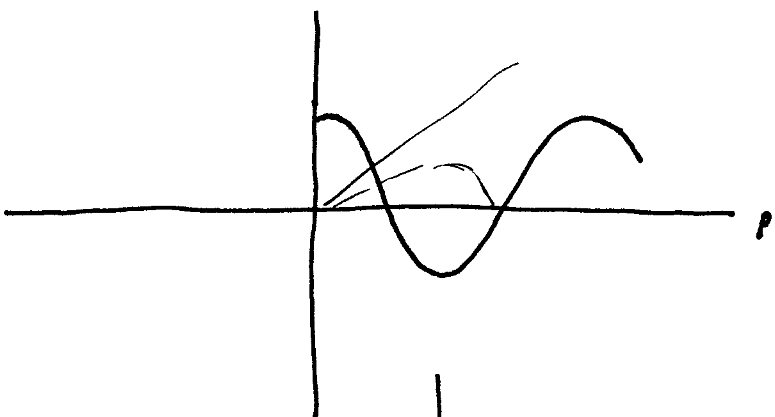
SPATIAL FREQ   MOMENTUM

$\hbar k = p$

$k = p/\hbar$

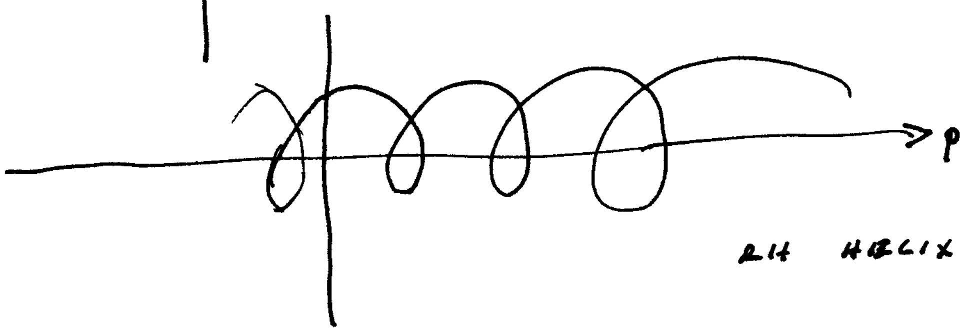
$e^{ipx_0/\hbar}$

$\cos(px/\hbar) + i\sin(px/\hbar)$



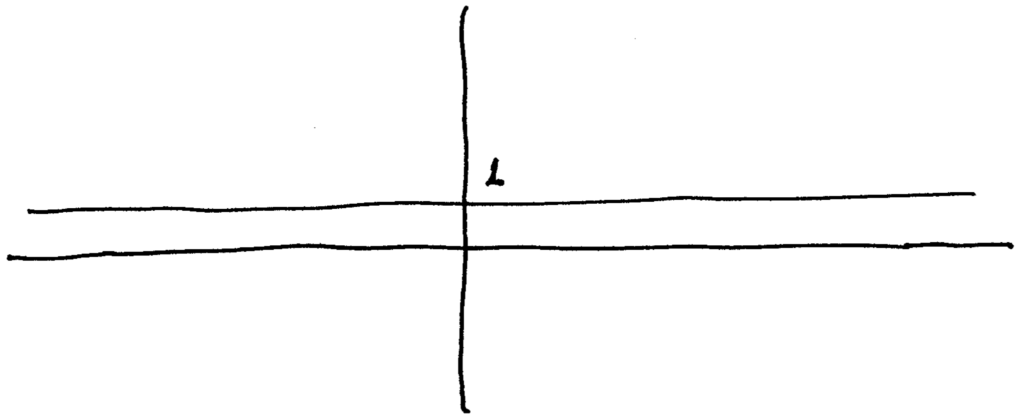
real part ~ cos

imag part ~ sin



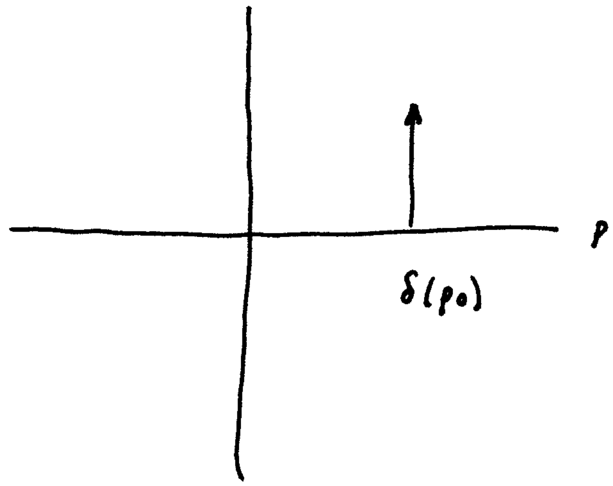
RH HELIX

PROB DENSITY $\left| e^{ipx_0/\hbar} \right|^2$

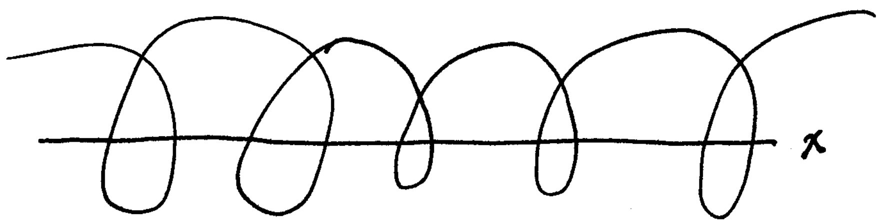


equal probability to find with any p!

MEASURE p

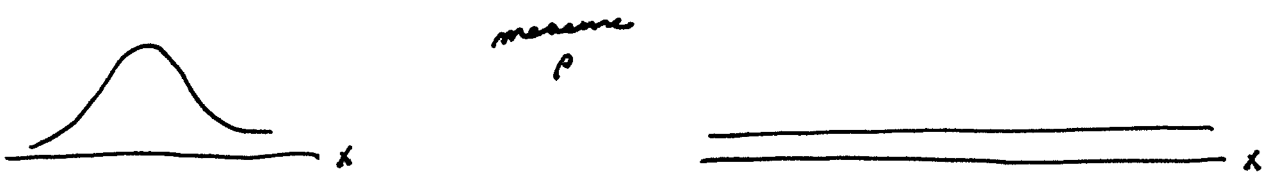


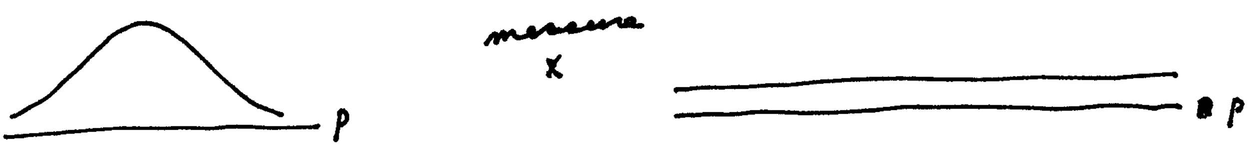
$$e^{i p_0 x/\hbar}$$



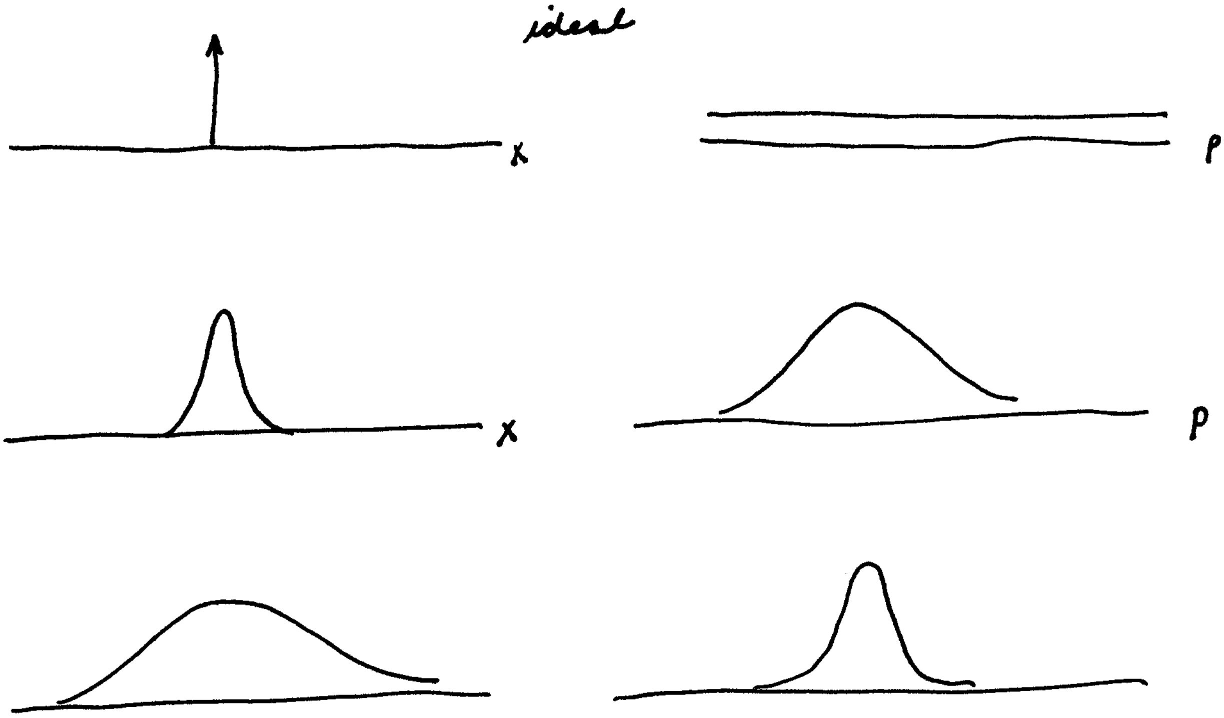
EQUAL PROBABILITY TO FIND

AT EVERY X





CANNOT REALLY DO A PERFECT x OR p MEASUREMENT



$$\Delta x \, \Delta p \geq \hbar/2$$

LECTURE 7

JULY 1, 2009

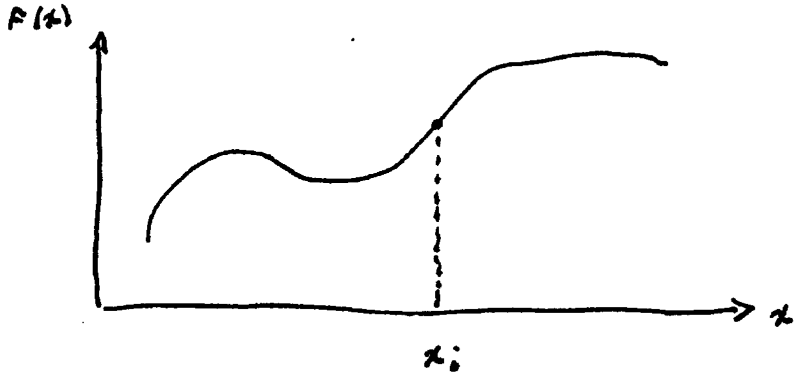
DIRAC DELTA FCN

FOURIER TRANSFORMS

THEIR RELATIONSHIP

LECTURE 6 — JUNE 29, 2007

DIRAC DELTA FUNCTION



(THING) THAT PICKS OUT $f(x_i)$

DIRAC:

$$\int \underbrace{(\text{THING})}_{\delta(x-x_i)} f(x)\, dx = f(x_i)$$

NUMBER

KRONECKER:

$$\sum \underbrace{(\text{THING})}_{\delta_{ij}}\, a_i = a_j$$

DIRAC THING — MAP: FUNCTION → NUMBER

KRONECKER THING — MAP: VECTOR → NUMBER

OLD WISDOM: DIRAC δ-FCNS ONLY MAKE SENSE INSIDE AN INTEGRAL

NEW WISDOM: FUNCTIONAL not a FUNCTION

TWO KINDS OF FUNCTIONS

GOOD OLD FUNCTIONS

~~R.~~ ~~PHYSICAL~~ PHYSICAL FCNS

P: PROPER FCNS

M: SQUARE INTEGRABLE L^2

$$\langle f | f \rangle \quad \text{FINITE}$$

GENERALIZED FCNS

$$\langle x | x' \rangle \quad \text{INFINITE} \rightarrow \text{NOT SQUARE INTEGRABLE}$$

$$\langle x | f \rangle \quad \text{FINITE}$$

P: IMPROPER FCNS

M: DISTRIBUTIONS

DIRAC DELTA FCN

$$\langle x | g \rangle = \int (\text{THING})\, g(x')\, dx' = g(x)$$

$$\delta(x - x')$$

work this one!

$$\delta(x - x') = \langle x | x' \rangle = \langle x' | x \rangle$$

$$\delta(p - p') = \langle p | p' \rangle = \langle p' | p \rangle$$

PROPERTIES OF DIRAC DELTA FCN

(1) $\delta(x-x') = 0$ WHEN $x \neq x'$

(2) $\displaystyle\int_a^b \delta(x-x')\, dx' = 1$ WHEN $a \leq x \leq b$

(3) $\delta(x-x') = \delta(x'-x)$ EVEN FCN

GAUSSIAN REPRESENTATION

$$g_\Delta(x-x') = \frac{1}{\sqrt{\pi}\,\Delta}\, e^{-(x-x')^2/\Delta^2}$$

$$\delta(x-x') = \lim_{\Delta \to 0} g_\Delta(x-x')$$

ACTION

$$\int \delta(x-x')\, f(x')\, dx' = f(x)$$

OLD WISDOM: DELTA FCNS ONLY MAKE SENSE INSIDE INTEGRALS

NEW WISDOM: DIRAC'S δ IS NOT A FCN, it is a FCNal

FCN : number → number

FCNAL : number → FCN

CAN ALSO DEFINE DERIVATIVES OF DELTA

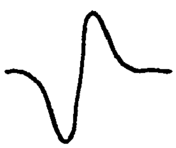
$$\delta'(x-x') = \frac{d}{dx}\left[\delta(x-x')\right]$$

$$= -\frac{d}{dx'}\left[\delta(x-x')\right]$$

$\Downarrow$

ODD FCN

$\delta^{\bullet}$



δ'

ACTION OF δ'

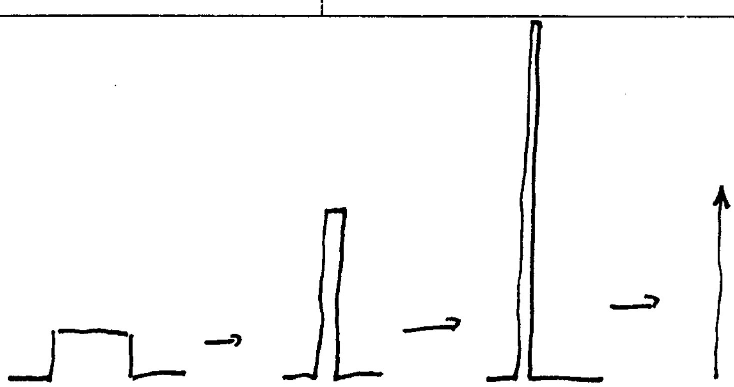
$$\int \delta'(x-x')\, f(x')\, dx' = \frac{d\, f(x)}{dx}$$

GENERALIZE

$$\frac{d^n\left[\delta(x-x')\right]}{dx^n} = \delta(x-x')\, \frac{d^n}{dx'^n}$$

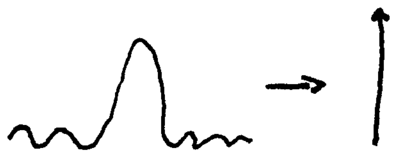
OTHER REPS

RECTANGLE FCN



SINC FCN

$$\text{SINC}(x) = \frac{\sin x}{x}$$



EXP FCN

$$\delta(x - x') = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-i(x-x')\xi} \, d\xi$$

Representations of the delta function

The delta function can be viewed as the limit of a sequence of functions

$$\delta(x) = \lim_{a\to 0} \delta_a(x),$$

where $\delta_a(x)$ is sometimes called a *nascent delta function*. This limit is in the sense that

$$\lim_{a\to 0} \int_{-\infty}^{\infty} \delta_a(x) f(x) dx = f(0)$$

for all continuous *f*.

The term *approximate identity* has a particular meaning in harmonic analysis, in relation to a limiting sequence to an identity element for the convolution operation (also on groups more general than the real numbers, e.g. the unit circle). There the condition is made that the limiting sequence should be of positive functions.

Some nascent delta functions are:

$$\delta_a(x) = \frac{1}{a\sqrt{\pi}} \mathrm{e}^{-x^2/a^2}$$

Limit of a Normal distribution

$$\delta_a(x) = \frac{1}{\pi} \frac{a}{a^2 + x^2} = \frac{1}{2\pi} \int_{-\infty}^{\infty} \mathrm{e}^{ikx - |ak|}\, dk$$

Limit of a Cauchy distribution

$$\delta_a(x) = \frac{e^{-|x/a|}}{2a} = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{e^{ikx}}{1 + a^2 k^2}\, dk$$

Cauchy φ(see note below)

$$\delta_a(x) = \frac{\mathrm{rect}(x/a)}{a} = \frac{1}{2\pi} \int_{-\infty}^{\infty} \mathrm{sinc}\left(\frac{ak}{2\pi}\right) e^{ikx}\, dk$$

Limit of a rectangular function

$$\delta_a(x) = \frac{1}{\pi x} \sin\left(\frac{x}{a}\right) = \frac{1}{2\pi} \int_{-1/a}^{1/a} \cos(kx)\, dk$$

rectangular function φ(see note below)

$$\delta_a(x) = \partial_x \frac{1}{1 + \mathrm{e}^{-x/a}} = -\partial_x \frac{1}{1 + \mathrm{e}^{x/a}}$$

Derivative of the sigmoid (or Fermi-Dirac) function

$$\delta_a(x) = \frac{a}{\pi x^2} \sin^2\left(\frac{x}{a}\right)$$

$$\delta_a(x) = \frac{1}{a} A_i\left(\frac{x}{a}\right)$$

Limit of the Airy function

$$\delta_a(x) = \frac{1}{a} J_{1/a}\left(\frac{x+1}{a}\right)$$ Limit of a Bessel function

Note: If $\delta(a, x)$ is a nascent delta function which is a probability distribution over the whole real line (i.e. is always non-negative between $-\infty$ and $+\infty$) then another nascent delta function $\delta_\varphi(a, x)$ can be built from its characteristic function as follows:

$$\delta_\varphi(a, x) = \frac{1}{2\pi} \frac{\varphi(1/a, x)}{\delta(1/a, 0)}$$

where

$$\varphi(a, k) = \int_{-\infty}^{\infty} \delta(a, x) e^{-ikx}\, dx$$

is the characteristic function of the nascent delta function $\delta(a, x)$. This result is related to the localization property of the continuous Fourier transform.

The Dirac comb

Main article: Dirac comb

A so-called uniform "pulse train" of Dirac delta measures, which is known as a Dirac comb, or as the shah distribution, creates a sampling function, often used in digital signal processing (DSP) and discrete time signal analysis.

See also

- Kronecker delta
- Dirac comb
- Logarithmically-spaced Dirac comb
- Green's function
- Dirac measure

External links

- Delta Function (http://mathworld.wolfram.com/DeltaFunction.html) on MathWorld
- Dirac Delta Function (http://planetmath.org/encyclopedia/DiracDeltaFunction.html) on PlanetMath
- The Dirac delta measure is a hyperfunction (http://www.osaka-kyoiku.ac.jp/~ashino/pdf/chinaproceedings.pdf)
- We show the existence of a unique solution and analyze a finite element approximation when the source term is a Dirac delta measure (http://epubs.siam.org/sam-bin/dbq/article/43178)
- Non-Lebesgue measures on R. Lebesgue-Stieltjes measure, Dirac delta measure. (http://www.mathematik.uni-muenchen.de/~lerdos/WS04/FA/content.html)

FOURIER TRANSFORMS

$\boxed{\text{TIME}}$

$$F(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(t)\, e^{-i\omega t}\, dt$$

$$f(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} F(\omega)\, e^{i\omega t}\, d\omega$$

SYMMERIC CONVENTION

ω TEMPORAL FREQUENCY

$\boxed{\text{SPACE}}$

K SPATIAL FREQUENCY

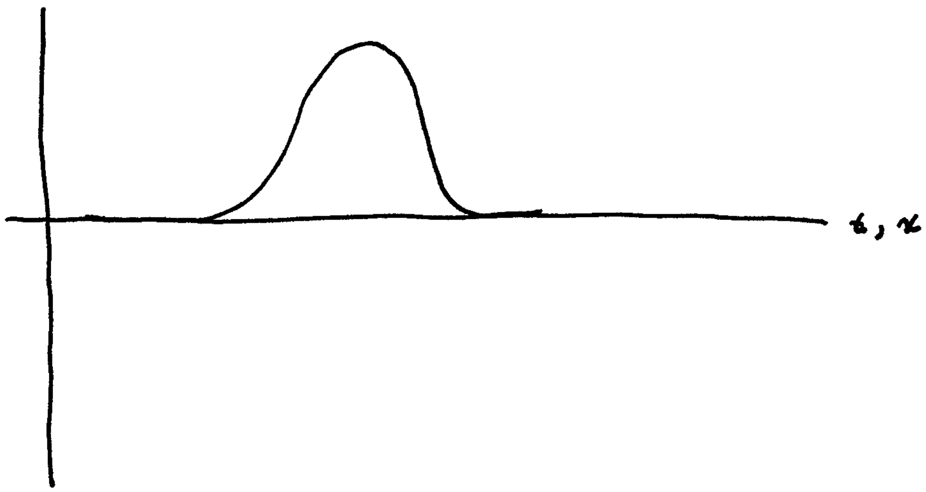
$$F(K) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x)\, e^{-iKx}\, dx$$

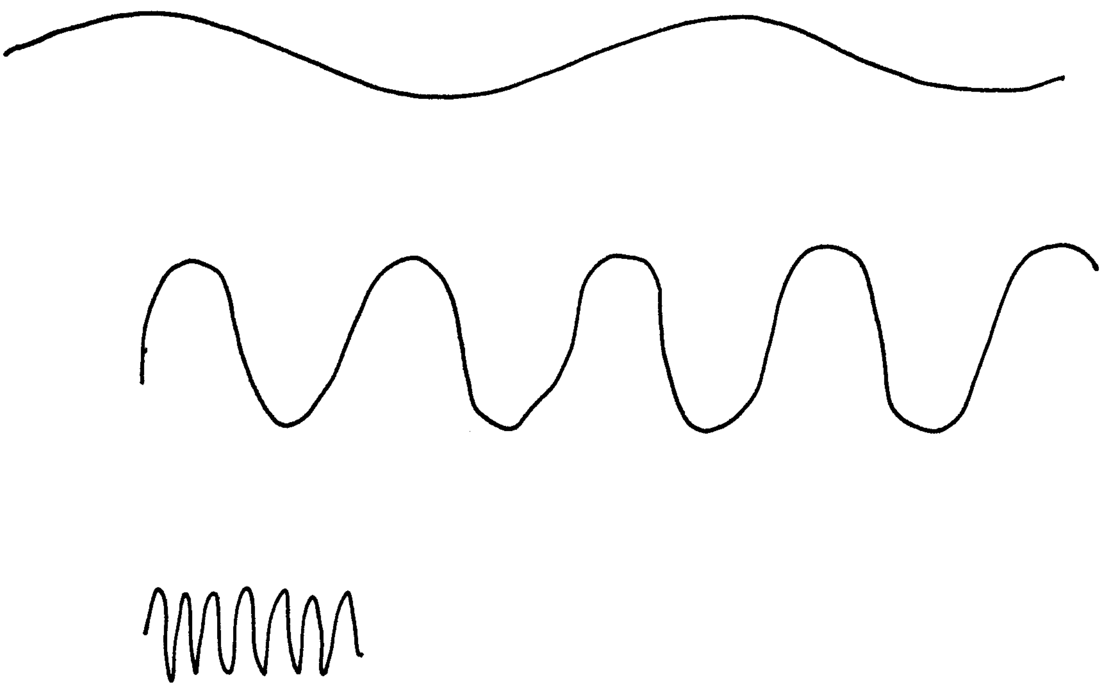
$$f(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} F(K)\, e^{iKx}\, dK$$

$E = \hbar \omega$

$p = \hbar K \qquad dp = \hbar\, dK$

CARTOON VERSION





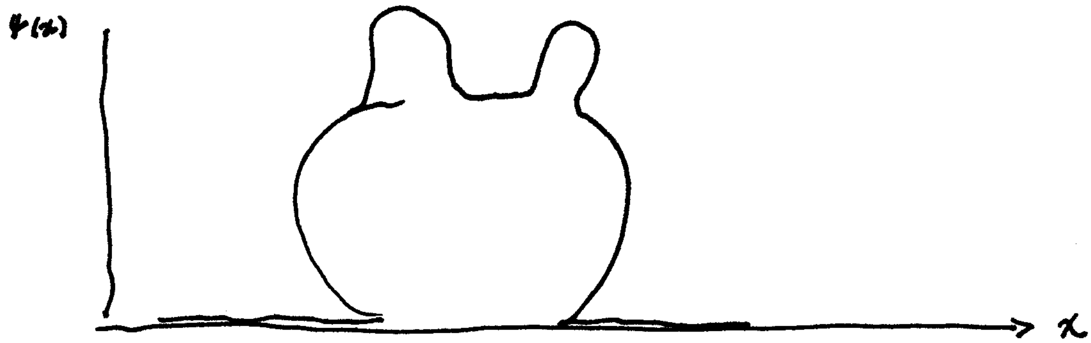
N.B.,
THESE HAVE
INFINITE
EXTENT

if complex => HELICIES

ANY FUNCTION?

YES

HOW ABOUT THIS WAVEFCN?



NO MICKEY MOUSE WAVEFCNS!

$$\hat{q}(p) = \frac{1}{\sqrt{2\pi\hbar}} \int_{-\infty}^{\infty} q(x)\, e^{-ipx/\hbar}\, dx$$

$$q(x) = \frac{1}{\sqrt{2\pi\hbar}} \int_{-\infty}^{\infty} \hat{q}(p)\, e^{ipx/\hbar}\, dp$$

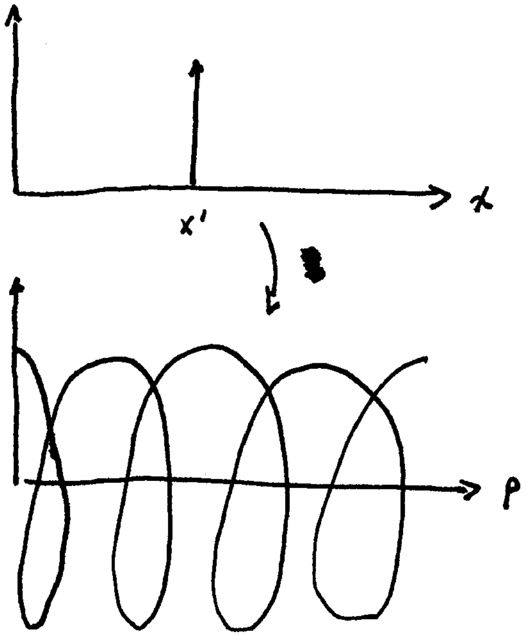
FT OF DELTA FCNS

$$\hat{f}(k) = (2\pi)^{-1/2} \int_{-\infty}^{\infty} \delta(x - x') \, e^{-ikx} \, dx$$

$$= (2\pi)^{-1/2} \, e^{-ikx'}$$

$$f(x) = (2\pi)^{-1/2} \int_{-\infty}^{\infty} \delta(k - k') \, e^{ikx} \, dk$$

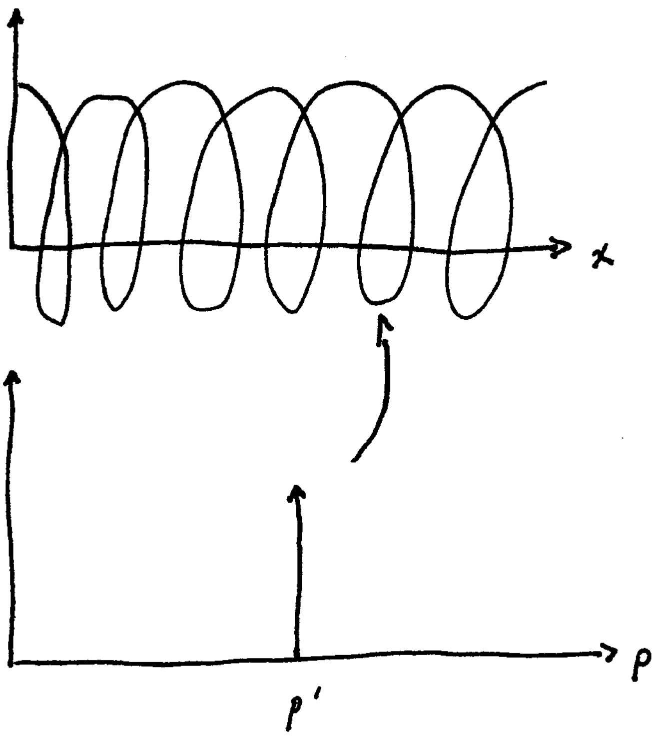
$$= (2\pi)^{-1/2} \, e^{ik'x}$$



$\delta(x - x')$

$e^{-ipx'/\hbar}$

$e^{-i\kappa x'}$



$e^{ip'x/\hbar}$

$e^{i\kappa' x}$

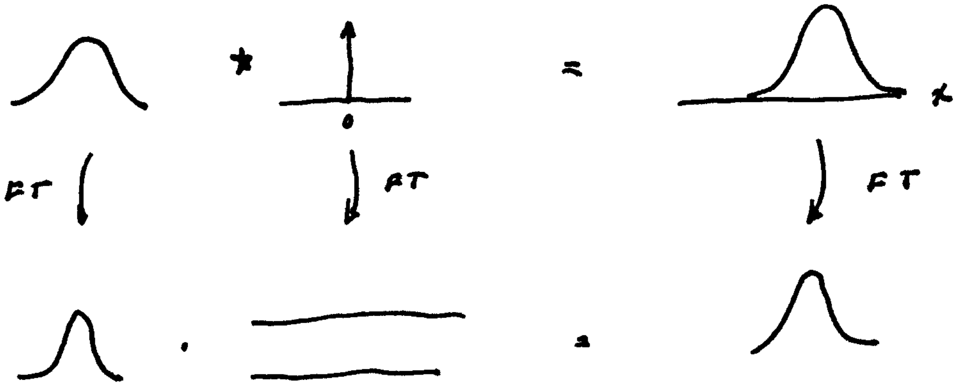
$\delta(p - p')$

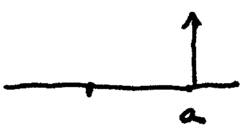
CONVOLUTION THM

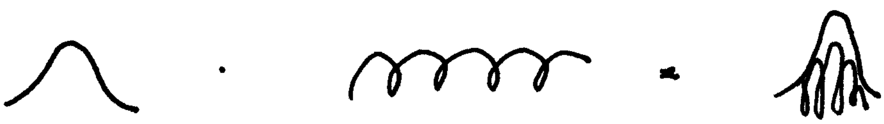
CONVOLUTION IN ONE SPACE $\Longleftrightarrow$ MULTIPLICATION IN THE OTHER SPACE

$f * g$	$\hat{f} \cdot \hat{g}$
$f \cdot g$	$\hat{f} * \hat{g}$

$$FT\{ f(x) * g(x) \} = \hat{f}(k) \cdot \hat{g}(k)$$

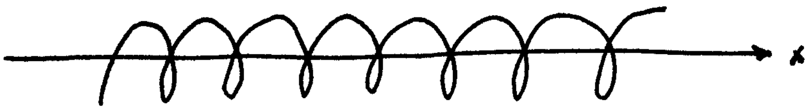




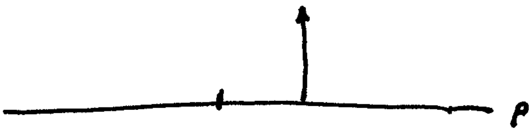


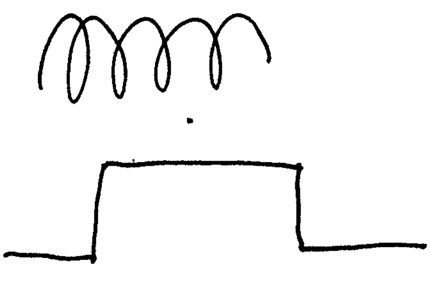
CONVOLUTION INTEGRAL

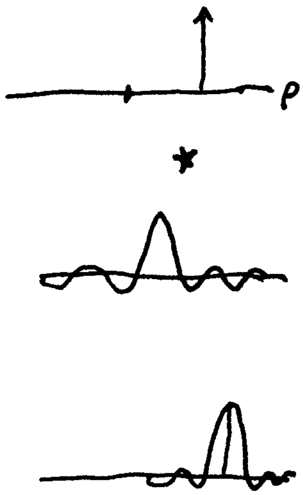
$$h(x) = \int_{-\infty}^{\infty} f(u)\, g(x-u)\, du$$



FT







SINC(p)

$\dfrac{\sin p}{p}$

FTS AND DIRAC NOTATION

$$|q\rangle = |q\rangle$$

$$= I\,|q\rangle$$ INSERT A COMPLETE SET OF STATES

$$= \int dK\, |K\rangle\langle K|q\rangle$$

$$= \int dK\, |K\rangle\ \hat{q}(K)$$

$$\langle x|q\rangle = \langle x| \int dK\, |K\rangle\, \hat{q}(K)$$

$$q(x) = \int \underbrace{\langle x|K\rangle}_{(2\pi)^{-1/2} e^{iKx}}\, \hat{q}(K)\, dK$$

$$= (2\pi)^{-1/2} \int \underbrace{\hat{q}(K)}_{G(K)}\, e^{iKx}\, dK$$

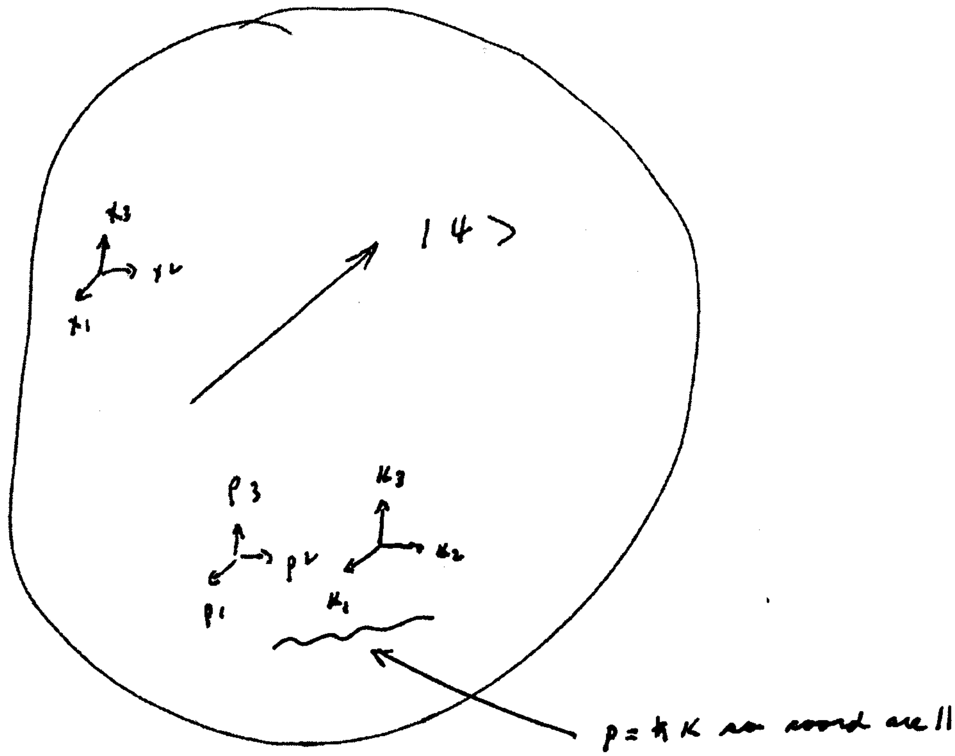
EXERCISE:

$$I = \int dx\, |x\rangle\langle x|$$

$$\hat{q}(K) = (2\pi)^{-1/2} \int q(x)\, e^{-iKx}\, dx$$

FOURIER TRANSFORMS ARE JUST

A CHANGE OF BASIS



$$\left.\begin{aligned} \langle x|\psi\rangle &= \psi(x) \\ \langle k|\psi\rangle &= \hat{\psi}(k) \\ \langle p|\psi\rangle &= \hat{\psi}(p) \\ \langle E|\psi\rangle &= \tilde{\psi}(E) \end{aligned}\right\}$$

all have all of the info

precisely the same info

$|\psi\rangle$ is the real thing

FT's are just a change in basis in QM!

ALL OBSERVABLES ...