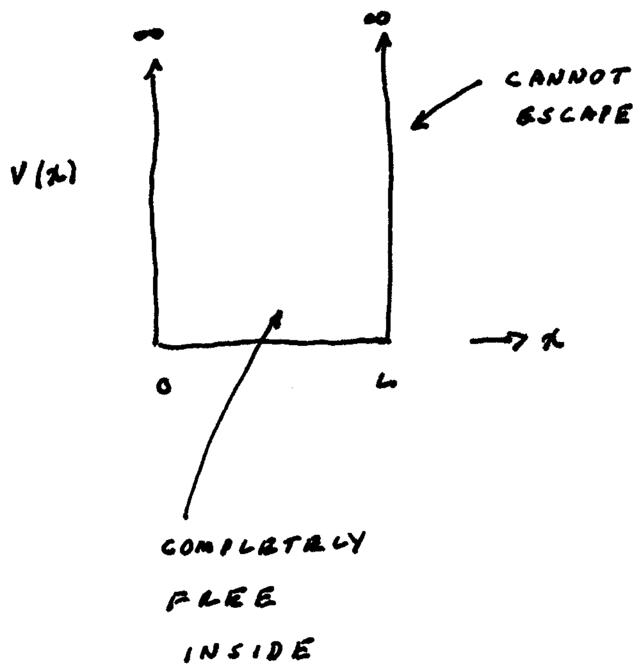
LECTURE 9 7-8-09

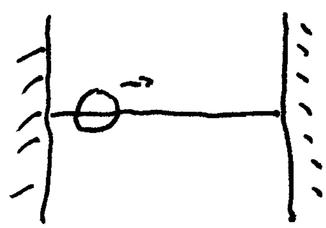
PARTICLE IN A BOX

INFINITE SQUARE WELL



MOTION IS 1d

CM: LIKE BEAD ON A WIRE



QM: BEAD HAS WAVE-LIKE PROPERTIES

ACTUALLY POINT-LIKE BEAD

SCHRODINGER EQN

$$\left[-\frac{\hbar^2}{2m} \frac{d^2}{dx^2} + V(x) \right] \psi(x) = E\,\psi(x)$$

KE | PE | TE

KE $\propto$ CURVATURE

PE inside well is zero

IN SQUARE WELL

$$\left(\frac{d^2}{dx^2} + \frac{2mE}{\hbar^2} \right) \psi(x) = 0$$

$$\left(\frac{d^2}{dx^2} + k^2 \right) \psi(x) = 0$$

$\sin(kx)$ REAL VALUED
$\cos(kx)$ OSCILLATING
SOLNS

$$\left(\frac{d^2}{dt^2} + \omega^2 \right) q(t) = 0$$

$\sin(\omega t)$
$\cos(\omega t)$

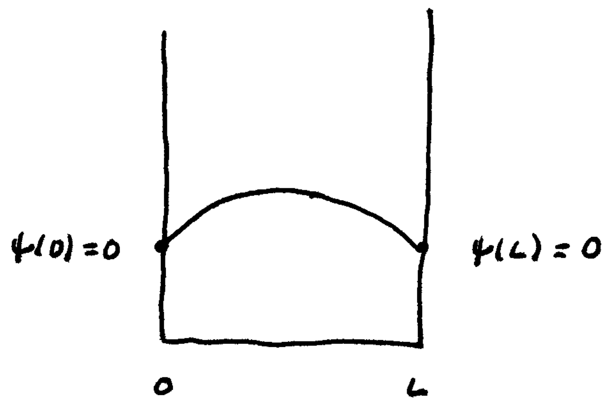
$$\left(\frac{d^2}{dx^2} + k^2 \right) f(x) = 0$$

$$\frac{d}{dx} (\sin kx) = k \cos kx$$

$$\frac{d}{dx} (k \cos kx) = -k^2 \sin kx$$

$\Rightarrow \sin kx$ is a solution

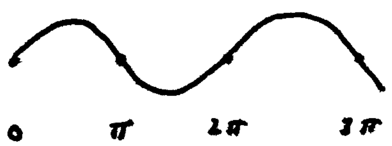
DOES IT MATCH THE BOUNDARY CONDITIONS?



$\psi(0) = 0 \qquad \sin(0) = 0$

$\psi(L) = 0 \qquad \sin kL = 0 \qquad$ NOT TRUE FOR ANY k

TRUE FOR $k_n = n \dfrac{\pi}{L}$

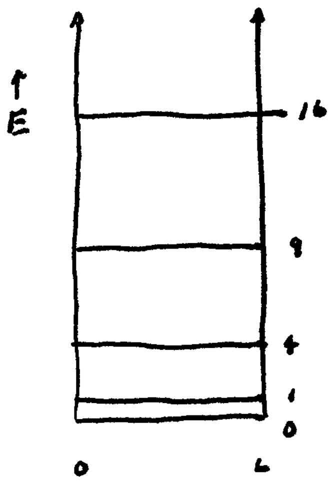


SOLN'S THAT FIT B.C.'S

$$\sin(k_n x)$$

$$E_n = \frac{\hbar^2 k_n^2}{2m} = \underbrace{\left(\frac{\hbar^2 \pi^2}{2mL^2}\right)}_{= E_1} n$$

$$E_n = n^2 E_1$$



QUANTIZED SET OF STATES

WITH QUANTIZED ~~ENERGIES~~ ENERGYS

IN QM, THERE ARE SUPERPOSITION STATES

$$\psi(x) = A\,\psi_1(x) + B\,\psi_2(x) + C\,\psi_3(x) + \cdots$$

EXAMPLE

CM: $\bullet \longrightarrow R$

OR

$L \longleftarrow \bullet$

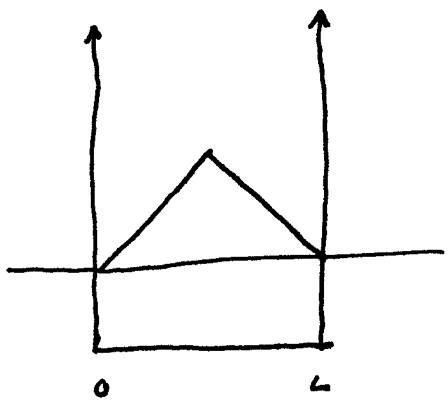
QM: $\bullet \longrightarrow R$

AND

$L \longleftarrow \bullet$

SUPERPOSITION
OF OPPOSITELY
MOVING STATES

TIME EVOLUTION:



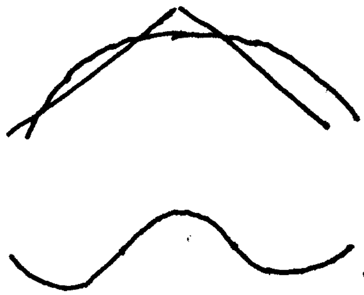
START WITH Δ WAVE FCN

WHAT HAPPENS?

DECOMPOSE INTO QUANTUM STATES

QUANTUM STATES ARE SINE STATE

DECOMPOSE = FIND FOURIER SERIES !



1 +

$$\int_0^L \psi_1(x) \wedge \, dx = a_1$$

$$\int_0^L \psi_2(x) \wedge \, dx = 0 = a_2$$

3 −

$$\int_0^L \psi_3(x) \wedge \, dx = a_3$$

$$a_m = \int_0^L \psi_m(x) \, F(x) \, dx$$

5 +

$$\psi(x) = \frac{8}{\pi^2}\left[\sin(k_1 x) - \frac{1}{9}\sin(k_3 x) + \frac{1}{25}\sin(k_5 x) + \cdots\right.$$

ONLY ODD HARMONICS $m = 1, 3, 5, \ldots$

AMPLITUDES DECAY LIKE $\frac{1}{m^2}$

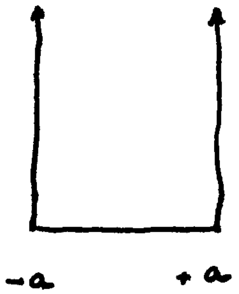
SIGNS ALTERNATE

1	3	5	7	9
+	−	+	−	+

$$k_m = \frac{m\pi}{L}$$

THREE CONVENTIONS FOR THE SQUARE WELL

(1)

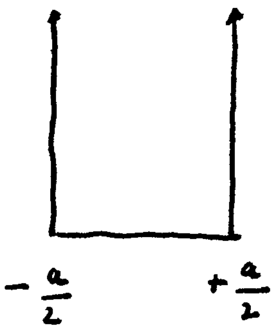


$$\psi_n(x) = \sqrt{\frac{1}{a}} \cos\left(\frac{n\pi}{2a} x\right) \qquad n \text{ ODD}$$

$$= \sqrt{\frac{1}{a}} \sin\left(\frac{n\pi}{2a} x\right) \qquad n \text{ EVEN}$$

$$E_n = \left(\frac{\hbar^2 \pi^2}{8 m a^2}\right) n^2 \qquad L = 2a$$

(2)

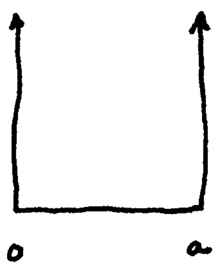


$$\psi_n(x) = \sqrt{\frac{2}{a}} \cos\left(\frac{n\pi}{a} x\right) \qquad n \text{ odd}$$

$$= \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi}{a} x\right) \qquad n \text{ even}$$

$$E_n = \left(\frac{\hbar^2 \pi^2}{2 m a^2}\right) n^2$$

(3)



$$\psi_n(x) = \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi}{a} x\right)$$

$$E_n = \left(\frac{\hbar^2 \pi^2}{2 m a^2}\right) n^2$$

TIME EVOLUTION

$$\psi(x,t) = \frac{8}{\pi^2}\Big[\sin(k_1 x)\, e^{-iE_1 t/\hbar}$$

$$- \frac{1}{9}\sin(k_3 x)\, e^{-iE_3 t/\hbar}$$

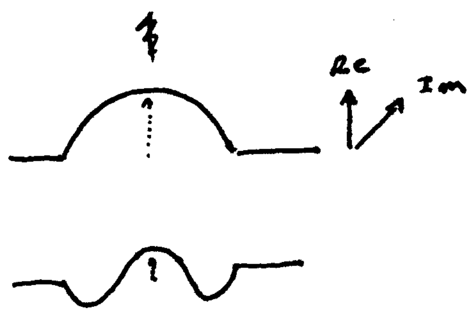
$$+ \frac{1}{25}\sin(k_5 x)\, e^{-iE_5 t/\hbar}$$

$$+ \ \ldots \Big]$$

$$E_n = n^2 E_1$$

$$\frac{E_1}{\hbar} = \omega_1$$

$$\omega_n = n^2 \omega_1$$



$$e^{-i\omega_1 t}$$

$$e^{-i\omega_3 t} = e^{-i9\omega_1 t}$$

$$e^{-i(25\omega_1)t}$$

The Addition of Phasors

http://ptolemy.eecs.berkeley.edu/eecs20/berkeley/phasors/demo/phasors.html

Falstad's One-Dimensional QM

http://www.falstad.com/qm1d/

More about Phasors

http://www.jhu.edu/signals/phasorlecture2/indexphasorlect2.htm

Wolfram Demonstrations

http://demonstrations.wolfram.com/LaplacesEquationOnASquare/
http://demonstrations.wolfram.com/LaplacesEquationOnACircle/
http://demonstrations.wolfram.com/PoissonEquationOnACircularMembrane/
http://demonstrations.wolfram.com/InstabilityOfLaplacesEquation/