LECTURE 6 JULY 1, 2009

TODAY : CONFORMAL MAPPING

ANALYTIC SOLUTIONS : SEPARATION OF VARIABLES

LAST TIME : TAKE ANY ANALYTIC FCN

REAL PART SOLUTION $\nabla^2 V = 0$

IMAGINARY " " "

MUTUALLY PERPENDICULAR

EQUIPOTENTIALS ⟹ FIELD LINES

FIELD LINES ⟹ EQUIPOTENTIALS

SPECIAL PAIR OF SOLUTIONS (DUAL)

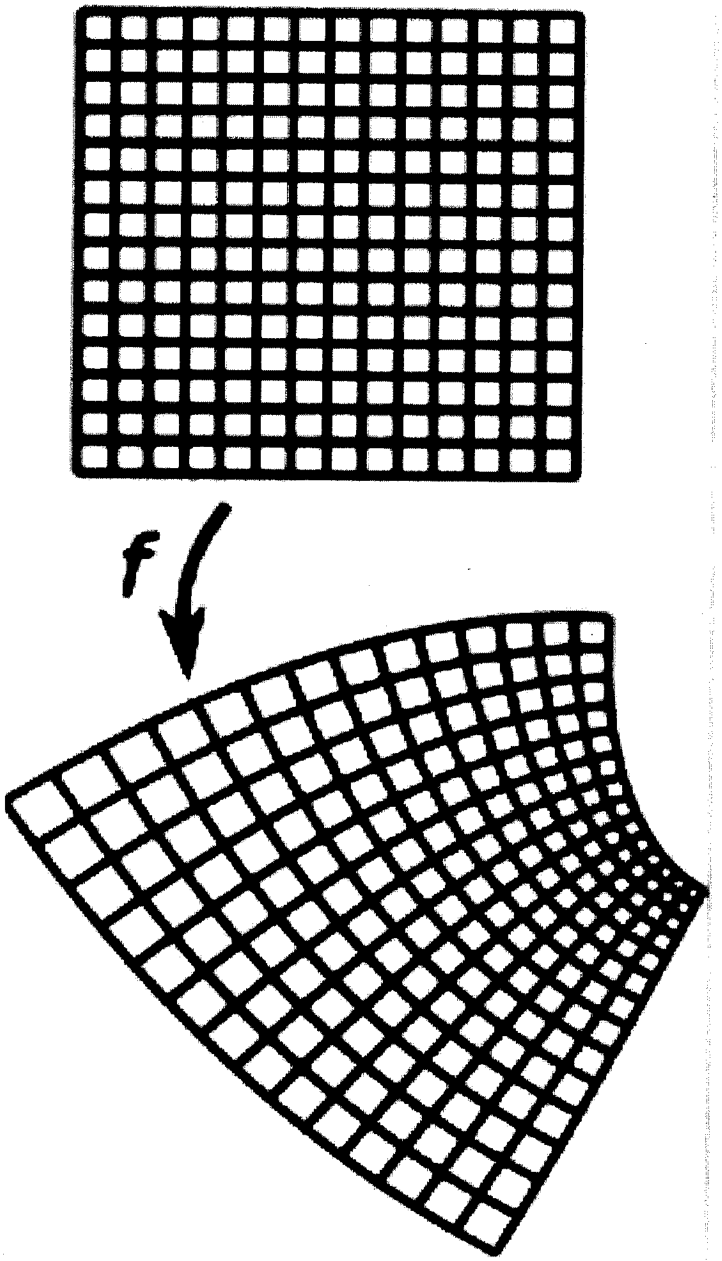
CONFORMAL MAPPING

$$\begin{pmatrix}\text{ANALYTIC}\\ \text{FCN}\end{pmatrix} \xrightarrow[\text{CONFORMAL MAP}]{} \begin{pmatrix}\text{ANOTHER}\\ \text{ANALYTIC}\\ \text{FCN}\end{pmatrix}$$

ANGLE PRESERVING MAPS

$$\begin{pmatrix}\text{ONE}\\ \text{SOLN}\end{pmatrix} \xrightarrow[\text{CONFORMAL MAP}]{} \begin{pmatrix}\text{ANOTHER}\\ \text{SOLN}\end{pmatrix}$$

A Conformal Map



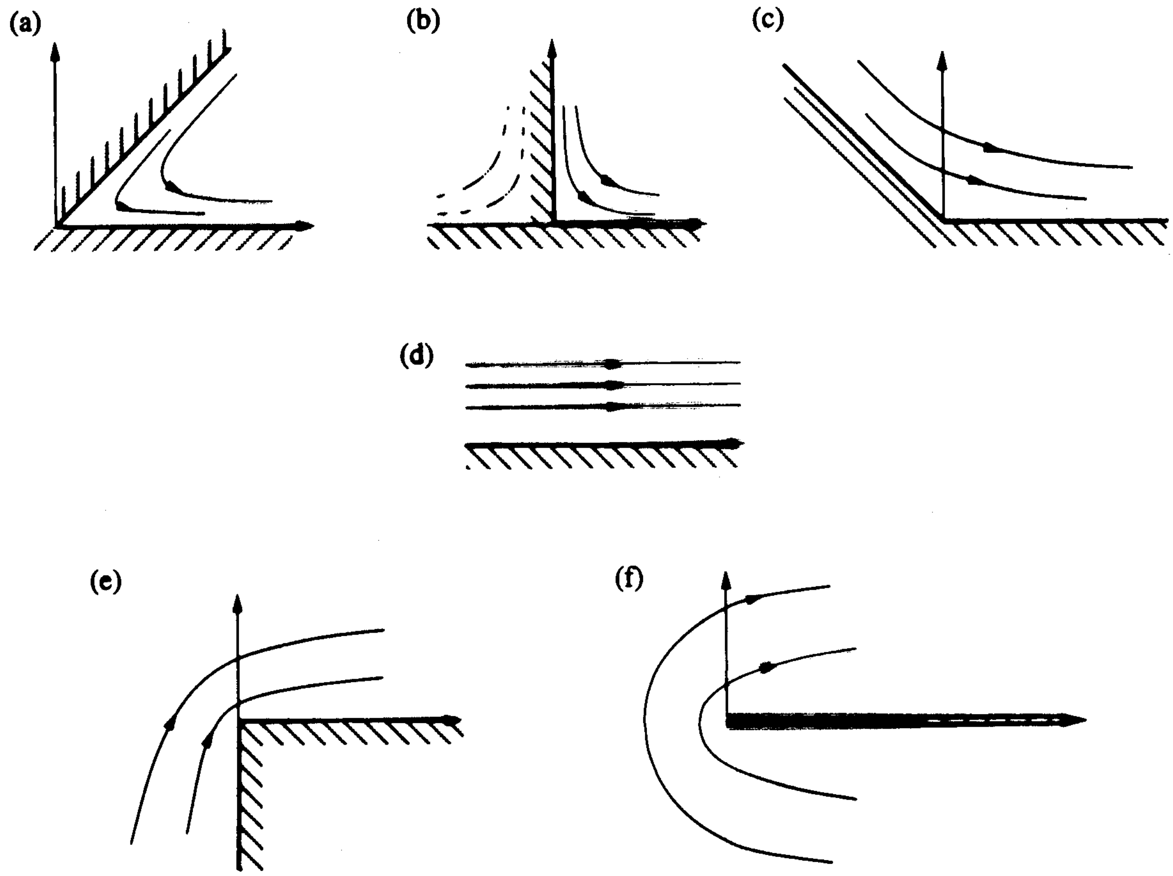
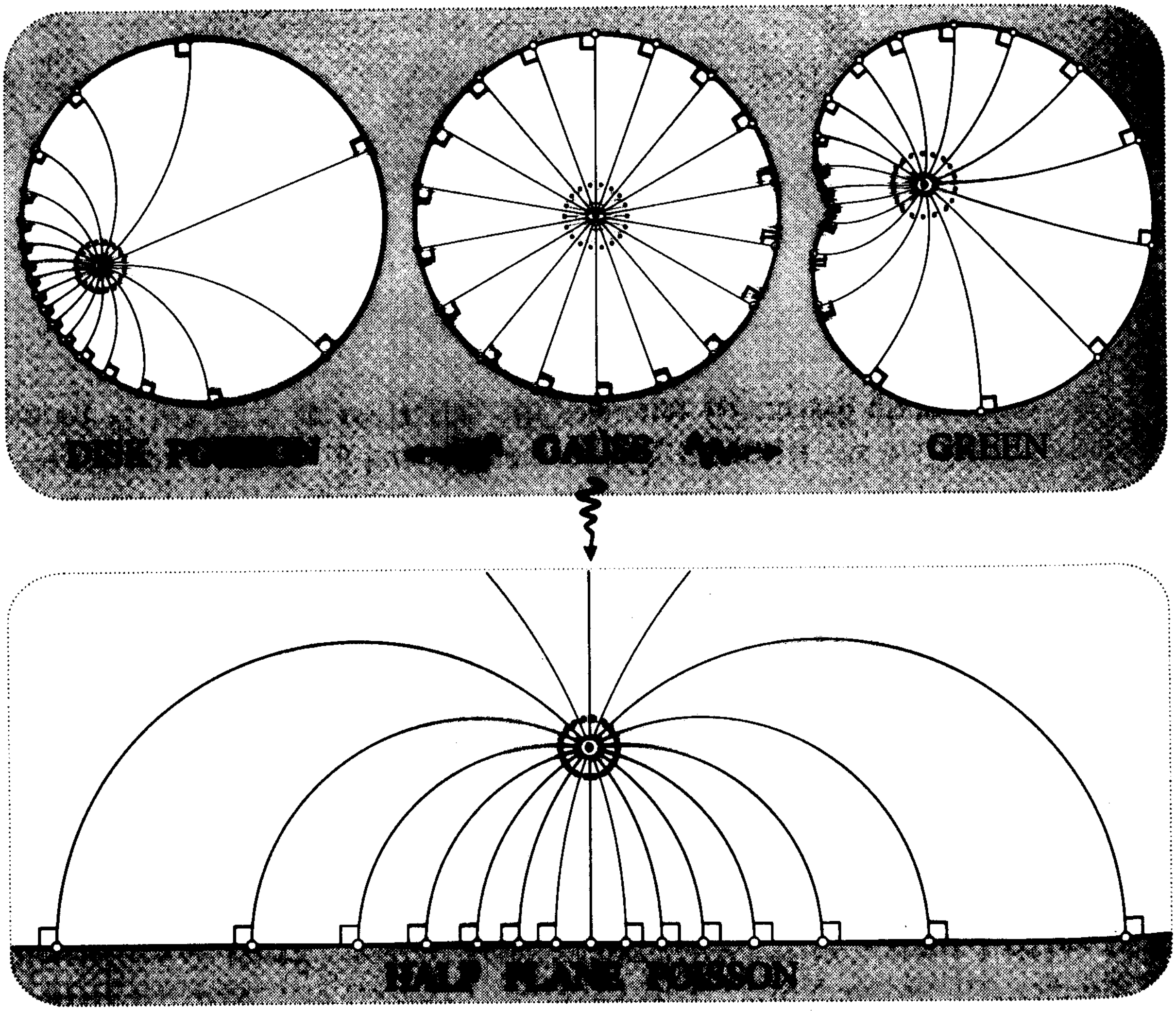
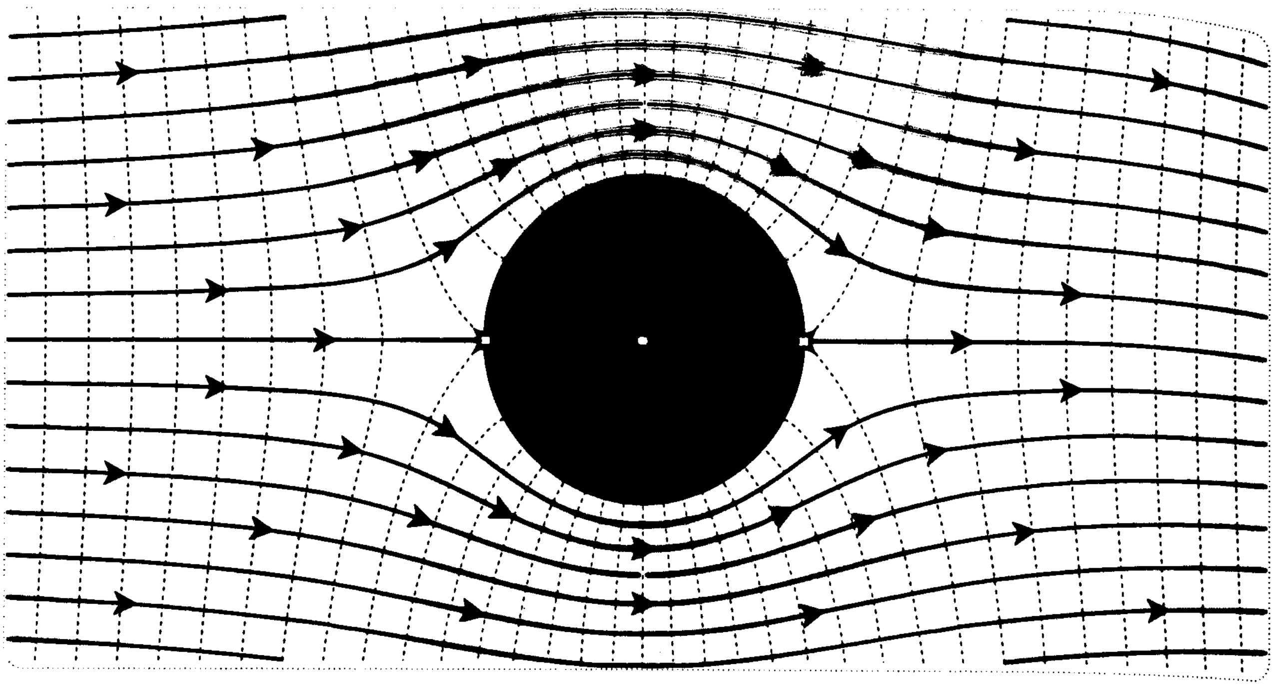


FIG. 6.24 Flows for which the velocity potential has the form $f(z) = Cz^{m+1}$. (a) $m > 1$; (b) $m = 1$; (c) $0 < m < 1$; (d) $m = 0$; (e) $-1/2 < m < 0$; (f) $m = -1/2$.


DISK POISSON
GAUSS
GREEN
HALF PLANE POISSON



CONFORMAL APPLETS

SOLVING LAPLACE'S EQUATION USING SEPARATION OF VARIABLES

$\nabla^2 V = 0$ SCALAR LAPLACE EQN

$\nabla^2 \vec{A} = 0$ VECTOR LE

$\nabla \underset{\approx}{T} = 0$ TENSOR LE

2d $\nabla^2 V(x,y) = 0$

$$\frac{\partial^2 V(x,y)}{\partial x^2} + \frac{\partial^2 V(x,y)}{\partial y^2} = 0$$

1d $\nabla^2 V(x) = 0$

$$\frac{\partial^2 V(x)}{\partial x^2} = 0 \Rightarrow \frac{d^2 V}{dx^2} = 0$$

3d $\nabla^2 V(x,y,z) = 0$

$$\frac{\partial^2 V(x,y,z)}{\partial x^2} + \frac{\partial^2 V(x,y,z)}{\partial y^2} + \frac{\partial^2 V(x,y,z)}{\partial z^2} = 0$$

STRATEGY : $\nabla^2 V = 0$ + BC's

EXPRESS ∇^2 ACCORDING TO SYMMETRY

CARTESIAN

SPHERICAL

CYLINDRICAL

SEPARATE VARIABLES

3d PDE $\Longrightarrow$ 3 SEPARATE ODE's

WRITE DOWN GENERAL SOLN

APPLY BC's TO OBTAIN UNIQUE SOLN

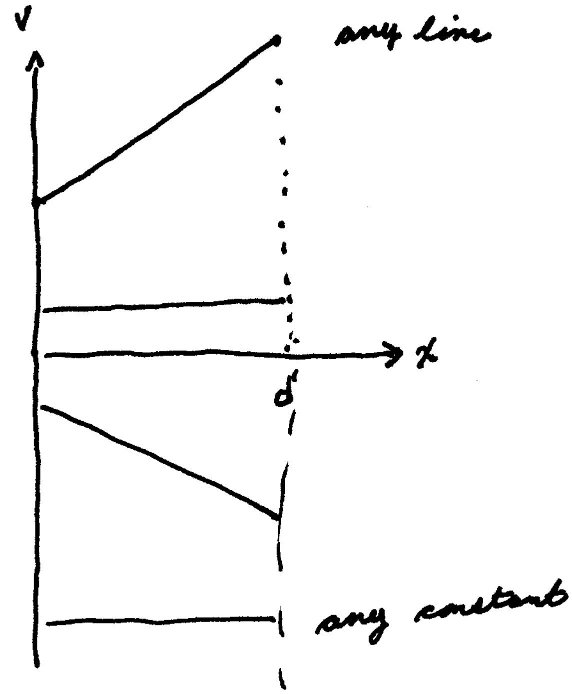
in 1d

EASY

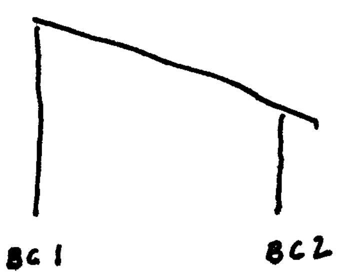
$$\frac{\partial^2 V}{\partial x^2} = 0 \qquad \text{PDE}$$

$$\frac{d^2 V}{dx^2} = 0 \qquad \text{ODE}$$

GENERAL SOLN IS $V(x) = A + Bx$



RUBBER BAND



IN 2d

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = 0$$

ANSATZ ~~via~~ $V(x,y) = X(x)\,Y(y)$

$$\frac{\partial^2 XY}{\partial x^2} + \frac{\partial^2 XY}{\partial y^2} = 0$$

$$Y \frac{\partial^2 X}{\partial x^2} + X \frac{\partial^2 Y}{\partial y} = 0$$

$$\frac{1}{XY}\left(\qquad\qquad\qquad \right) = 0$$

$$\frac{1}{X}\frac{d^2 X}{dx^2} + \frac{1}{Y}\frac{d^2 Y}{dy^2} = 0$$

$$f(x) + g(y) = 0 \qquad \forall x$$
$$c_1 + c_2 = 0 \qquad \forall y$$

$f(x) = -g(y) =$ constant ~~...~~

in 3d $\quad c_1 + c_2 + c_3 = 0$

$\nabla^2 V = 0$

SOLUTIONS ARE CALLED HARMONIC FCNS

∞ NUMBER OF SOLUTIONS

PHYSICALLY, ONLY ONE SOLUTION

DETERMINED BY THE BOUNDARY CONDITIONS (BC's)

shape of the soap bubble depends on the boundary

shape of V (inside) depends on V (boundary)

$\vdots$