

THE MATHEMATICS OF ELECTROMAGNETISM

VECTOR CALCULUS

122 GLOBAL VIEW

321 LOCAL VIEW

THIS WEEK :

\vec{E}	M
V	W
\vec{B}	θ
\vec{A}	F

FOUR MAXWELL EQUATIONS

$$\int_V \vec{E} \cdot d\vec{S} = \frac{Q}{\epsilon_0}$$

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0}$$

$$\int_V \vec{B} \cdot d\vec{S} = 0$$

$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\int_C \vec{E} \cdot d\vec{l} = - \frac{d\bar{Q}_B}{dt}$$

$$\oint \vec{\nabla} \times \vec{E} = - \frac{\partial \vec{B}}{\partial t}$$

$$\int \vec{B} \cdot d\vec{l} = \mu_0 I + \mu_0 \epsilon_0 \frac{d\phi_E}{dt}$$

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{j} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$$

STATICS

ELECTROSTATICS

T $\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0}$

HAS DIVERGENCE

F $\vec{\nabla} \times \vec{E} = 0$

NO ~~CURL~~ ZERO CURL

MAGNETOSTATICS

T $\vec{\nabla} \cdot \vec{B} = 0$

ZERO DIVERGENCE ~~IS~~

F $\vec{\nabla} \times \vec{B} = \mu_0 \vec{j}$

HAS CURL

ELECTROSTATICS

$\vec{F} = \frac{1}{4\pi\epsilon_0} \frac{Q_1 Q_2}{r^2}$

$\vec{F} = q \vec{E}$

COULOMB FORCE

ELECTRODYNAMICS

$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$

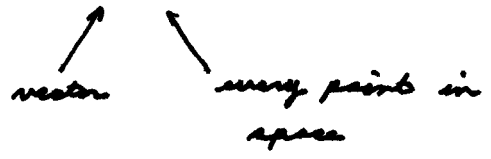
LORENTZ FORCE

13-782 500 SHEETS FILLER 5 SQUARE
42-381 50 SHEETS EYE-EASER 5 SQUARE
42-382 100 SHEETS EYE-EASER 5 SQUARE
42-383 100 SHEETS EYE-EASER 5 SQUARE
42-384 100 RECYCLED WHITE 5 SQUARE
42-385 100 RECYCLED WHITE 5 SQUARE
42-389 200 RECYCLED WHITE 5 SQUARE



Made in U.S.A.

HOW CAN WE DRAW/REPRESENT $\vec{E}(\vec{r})$



FOR 2D ~~SPACE~~

- (1) DRAW SOME SUBSET
- (2) ANIMATION
- (3) FIELD LINES
- (4) TOPOGRAPHIC MAP
 - 3d
 - contours

www.falstad.com

GAUSS' LAW

$$\int \vec{E} \cdot d\vec{S} = \frac{Q}{\epsilon_0}$$

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0}$$

GAUSS' THM

DIVERGENCE THM

$$\int_S \vec{F} \cdot d\vec{S} = \int_V \vec{\nabla} \cdot \vec{F} \, dV$$

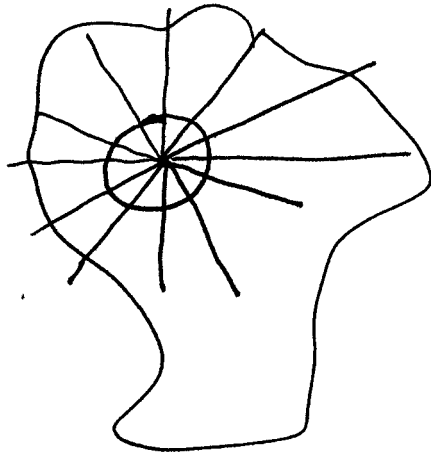
GAUSS' LAW

$$E \sim \frac{1}{r^2}$$

GAUSS' THM

ANY VECTOR FIELD

TWO INTUITIVE WAYS TO THINK
 BUT FIRST, ~~MOVE~~ ABOUT GAUSS'S LAW



Purcell's argument

$$\begin{aligned} d\Omega_1 &\longrightarrow d\Omega_2 \\ \vec{E}_1 &\longrightarrow \vec{E}_2 \end{aligned}$$

$$E \sim \frac{1}{r^2} \quad d\Omega \sim r^2$$

$$\vec{E}_2 \cdot d\vec{S}_2 = E_2 dS_2 \cos\theta$$

- ① FARADAY'S LINES OF FORCE: START ON A CHARGE
 CAN ONLY END ON A CHARGE
 OR AT ∞

if only 1 charge

N lines near charge $\Rightarrow N$ lines at ∞

$\Rightarrow N$ lines thru any closed surface

Lines are conserved.

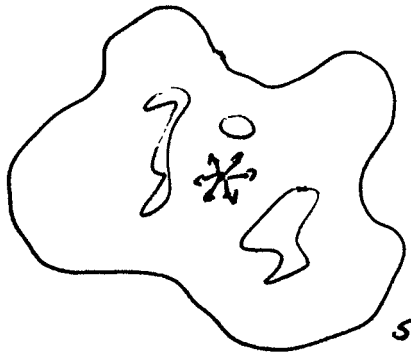
- ② WATER ~~FIELD~~

WATER FLOW THRU S

\vec{E} WATER FROM SOURCE

SOURCES

SINKS



Water is conserved

USING GAUSS' LAW

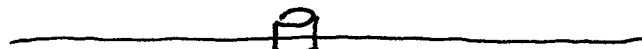
GAUSS' LAW
+
SYMMETRY } $\Rightarrow \vec{E}$

3 SYMMETRIES : PLANE

LINE

SPHERE

PLANE SHEET
UNIFORM CHARGE DENSITY σ $\frac{[\text{COULOMBS}]}{[\text{METRE}]^2}$

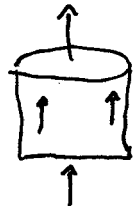


GAUSSIAN PILL BOX

SIDES L FACES A BY SYMMETRY $\vec{E} \perp$ PLANE $\vec{E} \perp \hat{n}$

$$\vec{E} \cdot \vec{A} = EA$$

$$\vec{E} \cdot \vec{S} = 0$$



$$\int \vec{E} \cdot d\vec{A} = \frac{Q}{\epsilon_0}$$

$$EA + 0 + EA = \frac{Q}{\epsilon_0} = \frac{\sigma A}{\epsilon_0}$$

$$E = \frac{1}{2} \frac{\sigma}{\epsilon_0}$$

$$E(r) = \text{constant} \propto \frac{1}{r^0}$$

17-382 500 SHEETS PER PILE 5 SQUARE
 42-381 500 SHEETS PER PILE 5 SQUARE
 42-382 100 SHEETS PER PILE 5 SQUARE
 42-383 100 SHEETS PER PILE 5 SQUARE
 42-384 200 SHEETS PER PILE 5 SQUARE
 42-385 200 SHEETS PER PILE 5 SQUARE
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 42-399 200 SHEETS PER PILE 5 SQUARE
 42-400 200 SHEETS PER PILE 5 SQUARE
 MADE IN U.S.A.

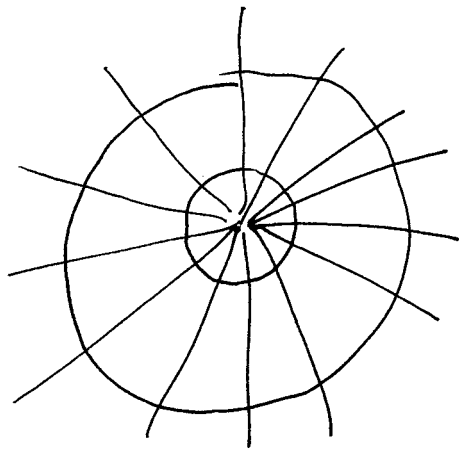


3d $\frac{1}{r^2}$

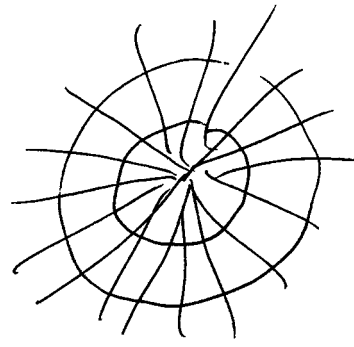
2d $\frac{1}{r}$

1d $\frac{1}{r^0}$

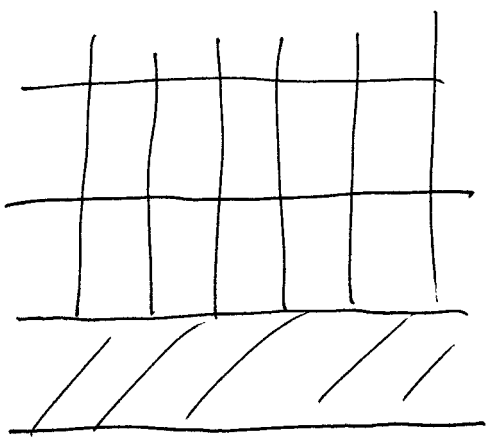
m d $r^{-(m-1)}$



lines per circumference fixed



lines per area fixed



lines fixed

ELECTRIC FIELDS VIA INTEGRATION

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(\vec{r}')}{R^2} \hat{R} d^3r'$$

$$\vec{R} = \vec{r} - \vec{r}'$$

$$R^2 = |\vec{R}|^2$$

$$\hat{R} = \frac{\vec{R}}{|\vec{R}|}$$

SHEET
~~LINE~~
~~CYLINDER~~
 SPHERE

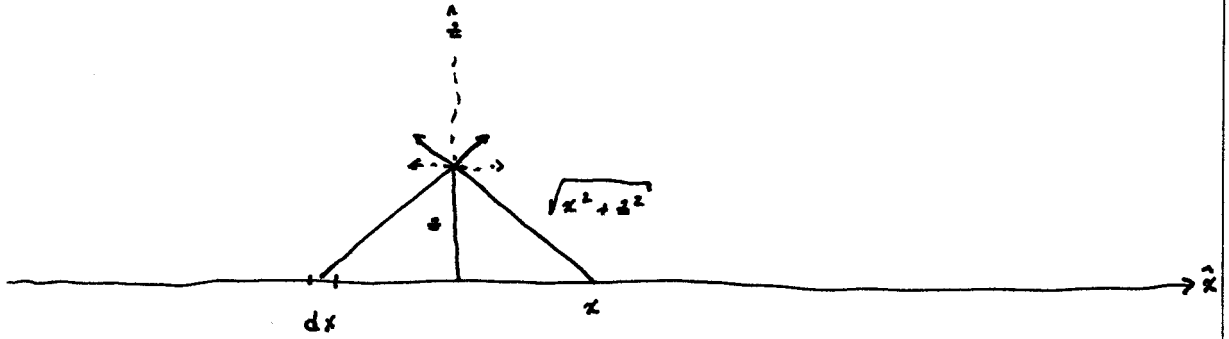
RELATIVELY
 EASY
 (STRAIGHT FORWARD)
 TO INTEGRATE

VECTOR INTEGRAL \Rightarrow TURN INTO 3 SCALAR INTEGRALS

E_x	E_n	E_p
E_y	E_θ	E_ϕ
E_z	E_φ	E_ψ

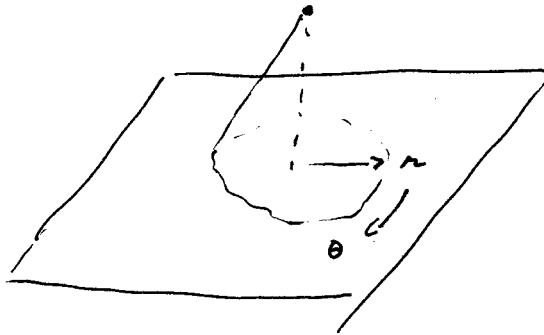
USE SYMMETRIES TO
 MAKE INTEGRALS AS
 SIMPLE AS POSSIBLE

13 782 500 SHEETS, MILLER, 5 SQUARE
 42 381 50 SHEETS, LIVE-EASE, 5 SQUARE
 42 382 100 SHEETS, LIVE-EASE, 5 SQUARE
 42 383 100 SHEETS, LIVE-EASE, 5 SQUARE
 42 384 100 SHEETS, LIVE-EASE, 5 SQUARE
 42 385 100 SHEETS, LIVE-EASE, 5 SQUARE
 42 386 100 SHEETS, LIVE-EASE, 5 SQUARE
 42 387 100 SHEETS, LIVE-EASE, 5 SQUARE
 42 388 100 SHEETS, LIVE-EASE, 5 SQUARE
 42 389 100 SHEETS, LIVE-EASE, 5 SQUARE
 42 390 100 SHEETS, LIVE-EASE, 5 SQUARE
 42 391 100 SHEETS, LIVE-EASE, 5 SQUARE
 42 392 100 SHEETS, LIVE-EASE, 5 SQUARE
 42 393 100 SHEETS, LIVE-EASE, 5 SQUARE
 42 394 100 SHEETS, LIVE-EASE, 5 SQUARE
 42 395 100 SHEETS, LIVE-EASE, 5 SQUARE
 42 396 100 SHEETS, LIVE-EASE, 5 SQUARE
 42 397 100 SHEETS, LIVE-EASE, 5 SQUARE
 42 398 100 SHEETS, LIVE-EASE, 5 SQUARE
 42 399 100 SHEETS, LIVE-EASE, 5 SQUARE
 42 400 100 SHEETS, LIVE-EASE, 5 SQUARE
 MADE IN U.S.A.



INFINITE LINE
 G FINITE LINE (CENTER)

FINITE LINE
 ANY WHERE
 HW P QUAL



INFINITE => SAME AS GAUSS' LAW

HW FINITE => DISK
 (CENTER)

HW RING

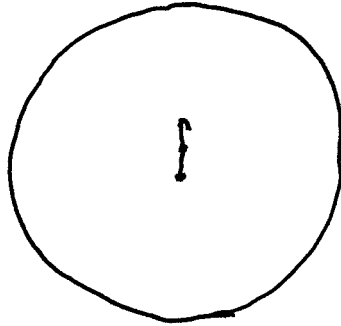
GRADED OBJECTS IN 3D (GAUSSIAN SURFACES)

SLABS $\rho(z)$

CYLINDERS $\rho(r)$ G. EXAMPLE

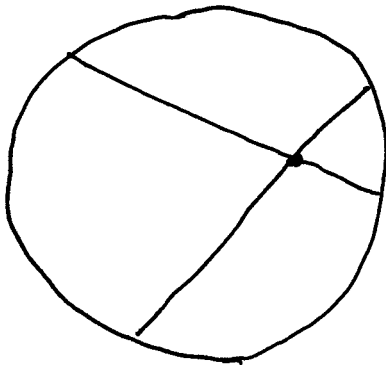
SPHERES $\rho(r)$ HW G

WHAT IS THE FIELD INSIDE A SPHERICAL SHELL?



At
the center, perfect symmetry

$$\vec{F} = 0$$



further
more

closer
fewer

NEWTON 1686 SHOWED $\vec{F} = 0$ everywhere inside

OUTSIDE, LOOKED LIKE A POINT AT THE CENTER

DIFFERENTIAL FORM OF GAUSS' LAW

$$\int \vec{E} \cdot d\vec{S} = \frac{Q}{\epsilon_0}$$

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0}$$

WHAT IS THE DIVERGENCE?

$$\vec{\nabla} \cdot \vec{E} = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right) \cdot (E_x, E_y, E_z)$$

↑ VECTOR OPERATOR
 ↑ VECTOR FIELD
 ↑ in Cartesian coords

$$= \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z}$$

↑ SCALAR FIELD

$$\text{div } \vec{E} = \vec{\nabla} \cdot \vec{E} \quad \text{divergence}$$

$$\vec{\nabla} \cdot \vec{E} \equiv \lim_{V \rightarrow 0} \int_S \vec{E} \cdot d\vec{S}$$

S is the boundary of V

$$\vec{\nabla} \cdot \vec{E} = \text{"flux out of a point"}$$

$$\vec{\nabla} \cdot \vec{E} = \rho / \epsilon_0$$

LOCAL
VERSION

$$\int \vec{E} \cdot d\vec{S} = Q / \epsilon_0$$

GLOBAL
VERSION

GAUSS' THEOREM

$$\int_S \vec{E} \cdot d\vec{S} = \int_V \vec{\nabla} \cdot \vec{E} \, dV$$

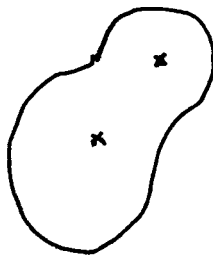
TRUE FOR ANY VECTOR FIELD!

FOR \vec{E}

$$\int_S \vec{E} \cdot d\vec{S} = \frac{Q}{\epsilon_0} = \int_V \rho(\vec{r}) \, d^3r$$

ANOTHER VIEW

$$\int_V (\text{sources}) \, dV = \int_S \text{flow thru surface}$$



fountains

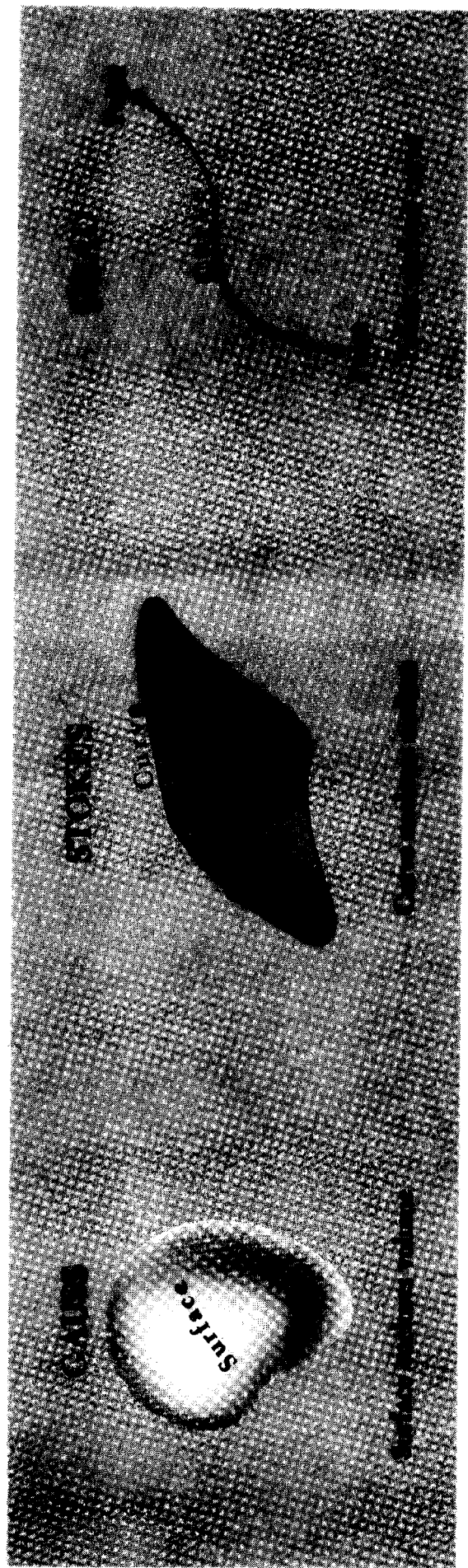
charge

⋮

water

\vec{E} flux

⋮



$$\int_{\text{surface}} \mathbf{F} \cdot d\mathbf{a} = \int_{\text{volume}} \text{div } \mathbf{F} d\mathbf{v}$$

$$\int_{\text{curve}} \mathbf{A} \cdot d\mathbf{s} = \int_{\text{surface}} \text{curl } \mathbf{A} \cdot d\mathbf{a}$$

$$\varphi_2 - \varphi_1 = \int_{\text{curve}} \text{grad } \varphi \cdot d\mathbf{s}$$

The History of Stokes' Theorem

*Let us give credit where credit is due:
Theorems of Green, Gauss and Stokes
appeared unheralded in earlier work.*

VICTOR J. KATZ

University of the District of Columbia
Washington, D.C. 20005

Most current American textbooks in advanced calculus devote several sections to the theorems of Green, Gauss, and Stokes. Unfortunately, the theorems referred to were not original to these men. It is the purpose of this paper to present a detailed history of these results from their origins to their generalization and unification into what is today called the generalized Stokes' theorem.

Origins of the theorems

The three theorems in question each relate a k -dimensional integral to a $k-1$ -dimensional integral; since the proof of each depends on the fundamental theorem of calculus, it is clear that their origins can be traced back to the late 17th century. Toward the end of the 18th century, both Lagrange and Laplace actually used the fundamental theorem and iteration to reduce k -dimensional integrals to those of one dimension less. However, the theorems as we know them today did not appear explicitly until the 19th century.

The first of these theorems to be stated and proved in essentially its present form was the one known today as Gauss' theorem or the divergence theorem. In three special cases it occurs in an 1813 paper of Gauss [8]. Gauss considers a surface (superficies) in space bounding a solid body (corpus). He denotes by PQ the exterior normal vector to the surface at a point P in an infinitesimal element of surface ds and by QX, QY, QZ the angles this vector makes with the positive x -axis, y -axis, and z -axis respectively. Gauss then denotes by $d\Sigma$ an infinitesimal element of the $y-z$ plane and erects a cylinder above it, this cylinder intersecting the surface in an even number of infinitesimal surface elements $ds_1, ds_2, \dots, ds_{2n}$. For each j , $d\Sigma = \pm ds_j \cos QX_j$, where the positive sign is used when the angle is acute, the negative when the angle is obtuse. Since if the cylinder enters the surface where QX is obtuse, it will exit where QX is acute (see FIGURE 1), Gauss obtains $d\Sigma = -ds_1 \cos QX_1 = ds_2 \cos QX_2 = \dots$ and concludes by summation that "The integral $\int ds \cos QX$ extended to the entire surface of the body is 0."

He notes further that if T, U, V are rational functions of only y, z , only x, z , and only x, y respectively, then " $\int (T \cos QX + U \cos QY + V \cos QZ) ds = 0$." Gauss then approximates the volume of the body by taking cylinders of length x and cross sectional area $d\Sigma$ and concludes in a similar way his next theorem: "The entire volume of the body is expressed by the integral $\int ds x (\cos QX)$ extended to the entire surface." We will see below how these results are special cases of the divergence theorem.

Ostrogradskyy discovered Gauss' theorem

Cauchy discovered Green's theorem

Lord Kelvin (William Thomson) discovered Stokes' theorem

**Bourbaki discovered the general case
(called the generalized Stokes' Theorem)**

Ostrogradskyy-Gauss Theorem

Cauchy-Green Theorem

Kelvin-Stokes Theorem

FLUX LINES

lines are conserved

WATER FLOW

volume is conserved

ELECTRIC FLUX

flux is conserved

Maxwell's math for Faraday's lines

$$\nabla \cdot \rho = 0$$

whenever there are no sources!

what happens if not Inverse Square Law?

$$\frac{C_1}{r} \quad \frac{1}{r}$$

falls off more slowly

central force

more water per unit solid angle

$$\frac{C_2}{r^3} \quad \frac{1}{r^2}$$

more quickly

less water per dΩ

$$C_3 \frac{e^{-\lambda r}}{r^2} \quad \frac{1}{r}$$

more quickly

less water per dΩ

Only for r^{-2} are water, lines, flux conserved!

Physics 543 Electromagnetic Theory

Autumn Quarter 2008

Professor: Larry Sorensen

Office: Physics-Astronomy Building B-435

Phone: 543-0360

e-mail: seattle@u.washington.edu

Office hours: Right after class, or by appointment

Class Website: <http://faculty.washington.edu/seattle>

Required texts: **Electricity and Magnetism** by Edward Purcell
Introduction to Electrodynamics by David Griffiths

Purcell presents a beautiful, masterful, and extremely physical treatment of EM. I expect you to master everything in Purcell---and I will do my best to assist you.

Griffiths' book is more mathematical than Purcell's. In his preface, Griffiths writes:
"Practically everything I know about electrodynamics---certainly about teaching electrodynamics---I owe to Edward Purcell."

Read Purcell for the physics. Read Griffiths for the math.

Homework: There will be 10 homework assignments, one per chapter. They will be posted on the class website on the Monday that we begin each chapter, and they will be due on the following Wednesday. See the course schedule for details.

Exams: There will be three take home exams. You will have two weeks---including two weekends---to do each exam. See the exam schedule for details.

Grades: Your course grade will be based on your homework and exam scores.

Page contents:[Description](#)[Homework](#)[Applets](#)[Lecture Notes](#)[Reading](#)**Physics 543, Autumn 2008
Electromagnetic Theory**<http://faculty.washington.edu/seattle/>

Instructor: Larry Sorensen

Email: seattle@u.washington.edu

Office: B-435 Physics-Astronomy Building

Office Hours: Right after class--or by appointment

Telephone: 543-0360

Description

Introduction to static electromagnetism:

Static electric and magnetic fields

Boundary-value problems

The electric and magnetic properties of materials

Maxwell's equations

[Course Information](#)[Exam 1](#)[Exam 2](#)[Exam 3](#)**Homework**[Homework Assignment 1](#)[Purcell Problems](#)[Example Purcell Solutions](#)[Griffiths Problems](#)[Purcell Solutions](#)[Griffiths Solutions](#)**See these solutions to learn how to solve EM problems**[Homework Assignment 2](#)[Purcell Problems](#)[Example Purcell Solutions](#)[Griffiths Problems](#)[Purcell Solutions](#)[Griffiths Solutions](#)[Homework Assignment 3](#)[Purcell Problems](#)[Example Purcell Solutions](#)[Griffiths Problems](#)[Purcell Solutions](#)[Griffiths Solutions](#)[Homework Assignment 4](#)[Purcell Problems](#)[Purcell Solutions to Chapter 4](#)[Homework Assignment 5](#)[Feynman's Version](#)[Purcell Problems](#)[Purcell Solutions to Chapter 5](#)[Purcell Solutions to Chapter 6](#)[Purcell Solutions to Chapter 7](#)[Purcell Solutions to Chapter 9](#)[Purcell Solutions to Chapter 10](#)[Purcell Solutions to Chapter 11](#)

PURCELL PROBLEMS

1.5 SEMICIRCLE

1.16 SPHERE WITH SPHERE REMOVED

1.19 PLANAR SHEET MOUNTED TO PLANAR SLAB

1.24 ROD ON AND OFF AXIS (OFF WAVE CENTER)

1.29 SPHERICAL SHELL MISSING DISK

PURCELL EXAMPLES

SPHERE, SPHERICAL SHELL

LINE

SHEET

15 789 150 SHEETS, HELIX, 5 SQUARE
15 791 150 SHEETS, CYLINDER, 5 SQUARE
42 382 100 SHEETS, 4 FT. 4.5 IN. 5 SQUARE
42 383 200 SHEETS, 4 FT. 4.5 IN. 5 SQUARE
42 384 100 SHEETS, 4 FT. 4.5 IN. 5 SQUARE
42 385 200 SHEETS, 4 FT. 4.5 IN. 5 SQUARE
42 386 200 RECYCLED WHITE 5 SQUARE
42 387 200 RECYCLED WHITE 5 SQUARE
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GAIFFITHS

~~PROBLEMS~~ EXAMPLES

- 2.1 line segment (INTEGRAL AT MIDPOINT)
- 2.2 outside uniform sphere (GAUSSIAN SURFACE)
- 2.3 radially varying cylinder (GAUSSIAN SURFACE)
- 2.4 infinite plane (using GAUSSIAN SURFACE)
- 2.5 capacitor 2 infinite parallel planes (GAUSSIAN SURFACE)

GAIFFITHS ~~PROBLEMS~~ PROBLEMS

- 2.1 polygon
- 2.5 ring
- 2.6 disc
- 2.12 inside uniform sphere
- 2.15 radially graded spherical shell inside, in shell, outside

13, 482 500 SHEETS, FILLER 5 SQUARE
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 42-387 200 RECYCLED, WHITE 5 SQUARE
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DISTRIBUTIONS OF CHARGE

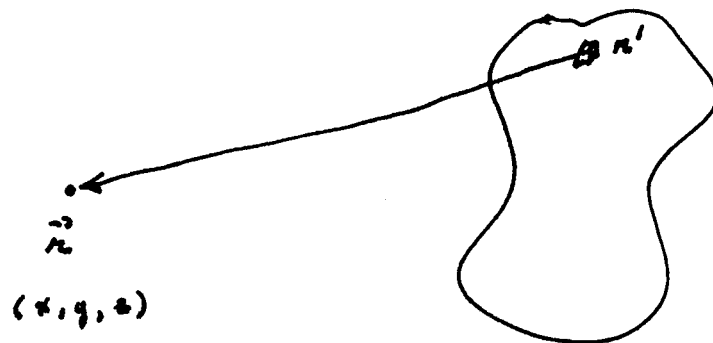
$$\vec{E}(\vec{r}) = \sum_i \frac{q_i}{|\vec{r} - \vec{r}_i|^2} \hat{r}_{oi}$$

for 10^{23} electrons

$$\Sigma \Rightarrow \int$$

$$\vec{E}(\vec{r}) = \int \frac{\rho(\vec{r}')}{r^2} \hat{r} d^3 r'$$

\hat{r} FROM \vec{r}' TO \vec{r} (x', y', z')



$$E(x, y, z) = \int \frac{\rho(x', y', z')}{r^2} \hat{r} dx' dy' dz'$$

ELECTRIC FIELDS VIA INTEGRATION

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(\vec{r}')}{R^2} \hat{R} d^3r'$$

$$\vec{R} = \vec{r} - \vec{r}'$$

$$R^2 = |\vec{R}|^2$$

$$\hat{R} = \frac{\vec{R}}{|\vec{R}|}$$

SHEET
~~LINE~~
 SPHERE

RELATIVELY
 EASY
 (STRAIGHT FORWARD)
 TO INTEGRATE

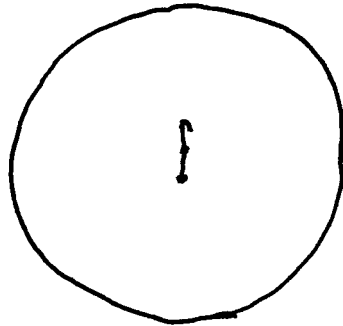
VECTOR INTEGRAL \Rightarrow TURN INTO 3 SCALAR INTEGRALS

E_x	E_y	E_z
E_y	E_x	E_z
E_z	E_x	E_y

USE SYMMETRIES TO
 MAKE INTEGRALS AS
 SIMPLE AS POSSIBLE

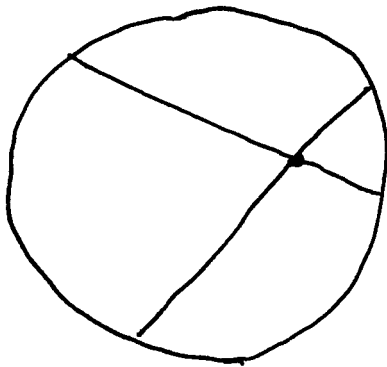
100 SHEETS FULL SIZE SQUARE
 50 SHEETS FULL SIZE SQUARE
 25 SHEETS FULL SIZE SQUARE
 100 SHEETS 1/2 SIZE SQUARE
 50 SHEETS 1/2 SIZE SQUARE
 25 SHEETS 1/2 SIZE SQUARE
 100 SHEETS 1/4 SIZE SQUARE
 50 SHEETS 1/4 SIZE SQUARE
 25 SHEETS 1/4 SIZE SQUARE
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 50 SHEETS 1/8 SIZE SQUARE
 25 SHEETS 1/8 SIZE SQUARE
 100 SHEETS 1/16 SIZE SQUARE
 50 SHEETS 1/16 SIZE SQUARE
 25 SHEETS 1/16 SIZE SQUARE
 100 SHEETS 1/32 SIZE SQUARE
 50 SHEETS 1/32 SIZE SQUARE
 25 SHEETS 1/32 SIZE SQUARE
 100 SHEETS 1/64 SIZE SQUARE
 50 SHEETS 1/64 SIZE SQUARE
 25 SHEETS 1/64 SIZE SQUARE
 100 SHEETS 1/128 SIZE SQUARE
 50 SHEETS 1/128 SIZE SQUARE
 25 SHEETS 1/128 SIZE SQUARE
 100 SHEETS 1/256 SIZE SQUARE
 50 SHEETS 1/256 SIZE SQUARE
 25 SHEETS 1/256 SIZE SQUARE
 100 SHEETS 1/512 SIZE SQUARE
 50 SHEETS 1/512 SIZE SQUARE
 25 SHEETS 1/512 SIZE SQUARE
 100 SHEETS 1/1024 SIZE SQUARE
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 100 SHEETS 1/2048 SIZE SQUARE
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 100 SHEETS 1/16384 SIZE SQUARE
 50 SHEETS 1/16384 SIZE SQUARE
 25 SHEETS 1/16384 SIZE SQUARE
 100 SHEETS 1/32768 SIZE SQUARE
 50 SHEETS 1/32768 SIZE SQUARE
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WHAT IS THE FIELD INSIDE A SPHERICAL SHELL?



At
the center, perfect symmetry

$$\vec{F} = 0$$



further
more

closer
fewer

NEWTON 1686 SHOWED $\vec{F} = 0$ everywhere inside

OUTSIDE, LOOKED LIKE A POINT AT THE CENTER