LECTURE 25 8-5-09

ANGULAR MOMENTUM

(1) LADDER OPERATORS

(2) DIFF EQN

1d TISE

$$\left[-\frac{\hbar^2}{2m} \frac{d^2}{dx^2} + V(x) \right] \psi_n(x) = E_n \psi_n(x)$$

3d TISE

$$\frac{d^2}{dx^2} \longrightarrow \nabla^2$$

$$V(x) \longrightarrow V(\vec{r})$$

$$\psi_n(x) \longrightarrow \psi_n(\vec{r})$$

The Bohr Model

http://www.walter-fendt.de/ph11e/bohrh.htm

The Meaning of the Legendre Polynomials

the bowling pin and the discus

The Meaning of the Spherical Harmonics

the automobile

The Spherical Harmonics

http://oak.ucc.nau.edu/jws8/dpgraph/Yellm.html
http://www.bpreid.com/applets/poasDemo.html
http://www.du.edu/~jcalvert/math/harmonic/harmonic.htm
http://www.falstad.com/qmrotator/

Encyclopedia

http://en.wikipedia.org/wiki/Spherical_harmonics
http://en.wikipedia.org/wiki/Table_of_spherical_harmonics
http://mathworld.wolfram.com/SphericalHarmonic.html

Examples

The Mathematical Figure of the Earth---Gauss

http://cgc.rncan.gc.ca/geomag/nmp/early_nmp_e.php?p=1

The Earth's Gravitional Field

http://en.wikipedia.org/wiki/Geoid
http://www.esri.com/news/arcuser/0703/graphics/geoid1_lg.gif
http://www.geomag.us/models/pomme5.html
http://earth-info.nga.mil/GandG/images/ww15mgh2.gif
http://op.gfz-potsdam.de/champ/
http://www.gfy.ku.dk/~pditlev/annual_report/matematiker.jpg

The Earth's Magnetic Field

http://en.wikipedia.org/wiki/Earth%27s_magnetic_field
http://www.ngdc.noaa.gov/geomag/WMM/DoDWMM.shtml
http://www.geomag.us/info/Declination/magnetic_lines_2010.gif

The Universe

http://abyss.uoregon.edu/~js/21st_century_science/lectures/lec27.html
http://www.asiaa.sinica.edu.tw/~lychiang/index/node10.html
http://wmap.gsfc.nasa.gov/media/080997/080997_5yrFullSky_WMAP_4096B.tif

Computer Lighting and Games

https://buffy.eecs.berkeley.edu/PHP/resabs/images/2006//101194-5.jpg
http://www.cg.tuwien.ac.at/research/publications/2008/Habel_08_SSH/
http://www.planetlara.com/underworld/renders/lara/full.jpg
http://casuallyhardcore.com/blog/index.php?s=shader

Art

http://www.math.hawaii.edu/~dale/bleecker/bleecker.html
http://cricketdiane.files.wordpress.com/2009/04/cricketdiane-castle-in-the-sky-2006-1.jpg

The Brain
http://www.stat.wisc.edu/~mchung/research/amygdala/amyg-degree.jpg

http://en.wikipedia.org/wiki/Spherical_harmonics
http://mathworld.wolfram.com/SphericalHarmonic.html

http://www.ngdc.noaa.gov/geomag/WMM/image.shtml

http://gfdi.fsu.edu/Images/Research/04/x50s180e100.jpg

http://www.geomag.bgs.ac.uk/mercator.html

http://www.ccr.jussieu.fr/ccr/Documentation/Calcul/matlab5v11/docs/00000/00025.htm
http://www.loria.fr/~ritchied/hex/manual/hex_manual.html
http://www.asiaa.sinica.edu.tw/~lychiang/index/node10.html

http://demonstrations.wolfram.com/SphericalHarmonics/

http://demonstrations.wolfram.com/VisualizingAtomicOrbitals/

∇^2 HAS DIFF FORM IN EACH COORD SYSEM

$\nabla^2 = 0$ separates in 13 coord systems

SCHRODINGER EQN FOR HYDROGEN

only separates in 2 coord systems

SPHERICAL COORD'S

$$V(\vec{r}) \rightarrow V(r)$$

SPHERICAL SYMMETRY

MATCH COORD SYSTEM TO THE SYMMETRY

PARABOLIC COORD'S

H atom has hidden symmetry

CARTESIAN

$$\nabla^2 = \frac{p_x^2}{2m} + \frac{p_y^2}{2m} + \frac{p_z^2}{2m}$$

$$p_x = -i\hbar \frac{\partial}{\partial x}$$

$$p_y = -i\hbar \frac{\partial}{\partial y}$$

$$p_z = -i\hbar \frac{\partial}{\partial z}$$

$$\nabla^2 = \frac{-\hbar^2}{2m}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)$$

SPHERICAL

$$\nabla^2 = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right)$$

$$+ \frac{1}{r^2}\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right)$$

$$+ \frac{1}{r^2}\frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\varphi^2}$$

$$\cancel{-\frac{\hbar^2\nabla^2}{2m} = \nabla_r^2 + \nabla_\perp^2}$$

$$\nabla^2 = \nabla_r^2 + \frac{1}{r^2}\left(\frac{\partial^2}{\partial\theta^2} + \cot\frac{\partial}{\partial\theta} + \frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\varphi^2}\right)$$

$$= \nabla_r^2 + \frac{1}{r^2}L^2$$

$$L^2\, Y_{\ell m}(\theta,\varphi) = \ell(\ell+1)\hbar^2\, Y_{\ell m}(\theta,\varphi)$$

$H\,\psi = E\,\psi$ TISE

$$-\frac{\hbar^2}{2m}\left[\nabla_r^2 + \frac{L^2}{r} + V(\vec{r})\right]\psi_{n\ell m} = E_n\,\psi_{n\ell m}$$

$$\cancel{\frac{1}{r^2}\left(r\frac{\partial}{\partial r}\right)^2 + \frac{1}{r}\frac{\partial}{\partial r}} \qquad \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right)$$

$V(\vec{r})$ IS A CENTRAL POTENTIAL

$V(\vec{r})$ only depends on $|\vec{r}| = r$

$$V(\vec{r}) = V(r, \theta, \varphi) = V(r)$$

if not central, could not separate r from θ and φ

$$\psi_{n\ell m}(r, \theta, \varphi) = R_n(r)\, Y_{\ell m}(\theta, \varphi)$$

SEPARATE $Y_{\ell m}(\theta, \varphi)$

$$\left[-\frac{\hbar^2}{2m} \nabla_r^2 + \frac{\ell(\ell+1)\hbar^2}{2 m r^2} - \frac{e}{r} \right] R_n(r) = E_n R_n(r)$$

$$\left[\frac{p_r^2}{2m} + \frac{L^2}{2 m r^2} - \frac{e}{r} \right]$$

RADIAL MOMENTUM — ANGULAR MOMENTUM

LINEAR MASS — ROTATIONAL MASS

3d $\rightarrow$ 1d PROBLEM

RADIAL EQUATION

$$\left[-\frac{\hbar^2}{2m} \nabla_r^2 + V_{eff} \right] R_n(r) = E_n(r)$$

$$V_{eff} = -\frac{e^2}{r} + \frac{l(l+1)\hbar^2}{2mr^2}$$

ATTRACTIVE COULOMB POTENTIAL

REPULSIVE CENTRIFUGAL BARRIER