LECTURE 2 COMPLEX NUMBERS IN CM $\{CM, EM, QM\}$

~~THEME~~ SIMPLE HARMONIC OSCILLAR

DAMPED HARMONIC OSCILLATOR

DAMPED DRIVEN HARMONIC OSCILLATOR

COUPLED OSCILLATORS AND NORMAL MODES

FRESHMAN PHYSICS $\sin(\omega t) \quad \cos(\omega t)$

$\sin(\omega t + \varphi) \quad \cos(\omega t + \varphi)$

MUCH EASIER TO USE $e^{i\omega t}$

$$x(t) = A e^{i\omega t} = A\left(\cos(\omega t) + i \sin(\omega t)\right)$$

NONSENSE CM
EM

CORRECT
REQUIRED
IN QM

WHEN WE WRITE

$x(t) = A e^{i\omega t}$ WE MEAN $x(t) = Re\left\{\hat{A} e^{i\omega t}\right\}$

$$\vec{E}(t) = \hat{A}\vec{E}_0 \, e^{i(\vec{k}\cdot\vec{r} - \omega t)}$$

$$= Re\left\{\hat{A}E_0 \, e^{i(\vec{k}\cdot\vec{r} - \omega t)}\right\}$$

OSCILLATOR PROBLEMS

UNDAMPED SHO

$$F = ma = m \frac{d^2x}{dt^2}$$

$$-kx = m \frac{d^2x}{dt^2}$$

$$\frac{dx^2}{dt^2} + \left(\frac{k}{m}\right)x = 0$$

$$\cos(\omega t) \qquad\qquad e^{i\omega t} \qquad\qquad \cos(\omega t)$$

$$\frac{d}{dt} \downarrow$$

$$-\omega \sin(\omega t) \qquad\qquad i\omega e^{i\omega t} \qquad\qquad -\omega \sin(\omega t)$$

$$\frac{d}{dt} \downarrow$$

$$-\omega^2 \cos(\omega t) \qquad\qquad (i\omega)(i\omega)\, e^{i\omega t} = -\omega^2 e^{i\omega t}$$

$$-\omega^2 \cos \omega t$$

FEYNMAN: WHY ARE EXP'S BETTER THAN SIN, COS?

BECAUSE THE ALGEBRA IS EASIER!

SHO

$$\frac{d^2 \hat{x}}{dt^2} + \frac{K}{m}\hat{x} = 0 \qquad x = \hat{x}\, e^{i\omega_0 t}$$

$$(i\omega)^2 \hat{x}\, e^{i\omega t} + \frac{K}{m}\hat{x}\, e^{i\omega t} = 0$$

$$\left(\omega_0^2 - \frac{K}{m}\right)\hat{x} = 0$$

$$\omega_0 = \sqrt{\frac{K}{m}}$$

MAKING X COMPLEX

CONVERTS DIFF EQ

INTO AN ALGEBRAIC

EQUATION

MUCH EASIER TO SOLVE!

REALLY FT for a single Fourier component

DRIVEN SHO

UNDAMPED

$$x = \hat{x} e^{i\omega t}$$

$$\frac{d^2 \hat{x} e^{i\omega t}}{dt^2} + \left(\frac{k}{m}\right) \hat{x} e^{i\omega t} = \frac{\hat{F}}{m} e^{i\omega t}$$

$$(i\omega)^2 \hat{x} + \left(\frac{k}{m}\right) \hat{x} = \frac{\hat{F}}{m}$$

$$(-\omega^2 + \omega_0^2)\, \hat{x} = \frac{\hat{F}}{m}$$

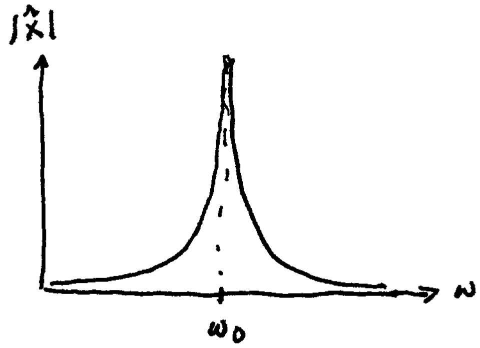
$$\hat{x} = \frac{\hat{F}}{m(\omega_0^2 - \omega^2)}$$

$$|\hat{x}| = \frac{1}{m(\omega_0^2 - \omega^2)} |\hat{F}|$$

as $\omega \to \omega_0$, AMP OF THE RESPONSE DIVERGES

$\omega < \omega_0$ $\hat{x}$ AND $\hat{F}$ are in phase

$\omega > \omega_0$ $\hat{x}$ AND $\hat{F}$ are 180° out of phase



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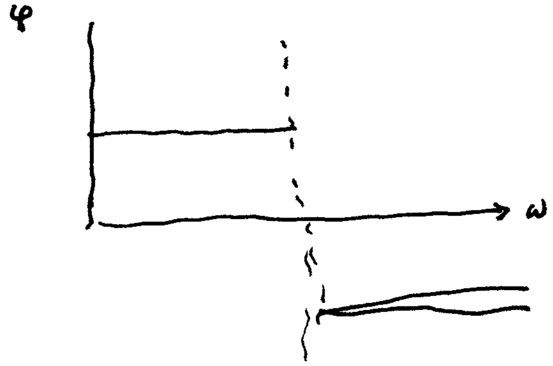
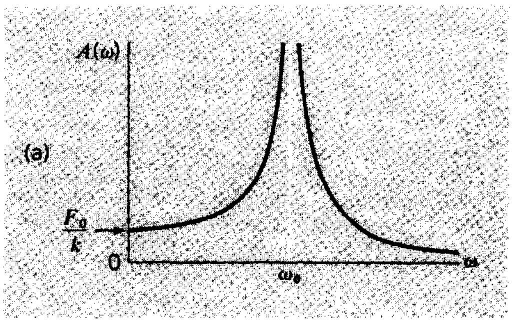
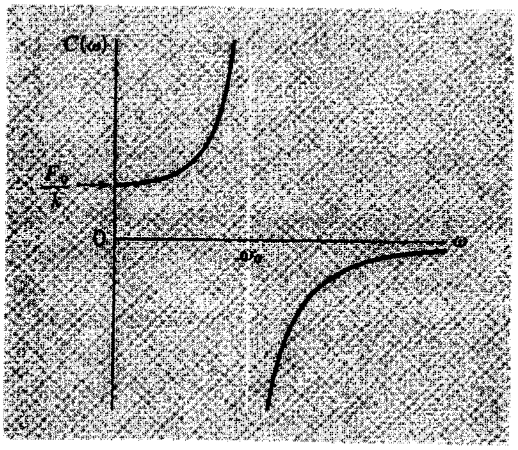


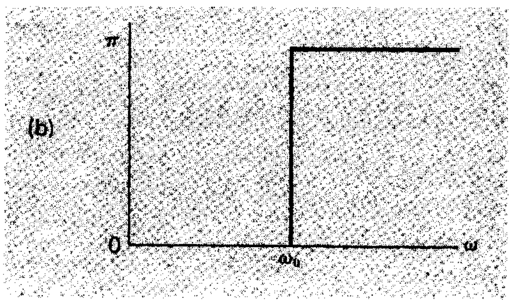


$|\hat{x}|$



$Re[\hat{x}]$

$\varphi[\hat{x}]$



DAMPED FREE OSCILLATOR

NOT DRIVEN

$F = ma$

$F = -kx - bv$

$$m\frac{d^2 \hat{x} e^{i\omega t}}{dt^2} + \gamma \frac{d\hat{x}e^{i\omega t}}{dt} + k\hat{x}e^{i\omega t} = 0$$ DIFF EQN

$$\left(m(i\omega)^2 + b(i\omega) + k\right)\hat{x} = 0$$ ALGEBRAIC EQN

$$\omega^2 + i b\,\omega + \omega_0^2 = 0$$

~~ω^2 is negative~~

$\omega^2 + i\gamma\omega = \omega_0^2$

IF b IS SMALL

$$\omega = \sqrt{\left(\frac{k}{m}\right) - \left(\gamma\right)^2}$$

$$\left(\frac{b}{2m} \equiv \gamma\right)$$

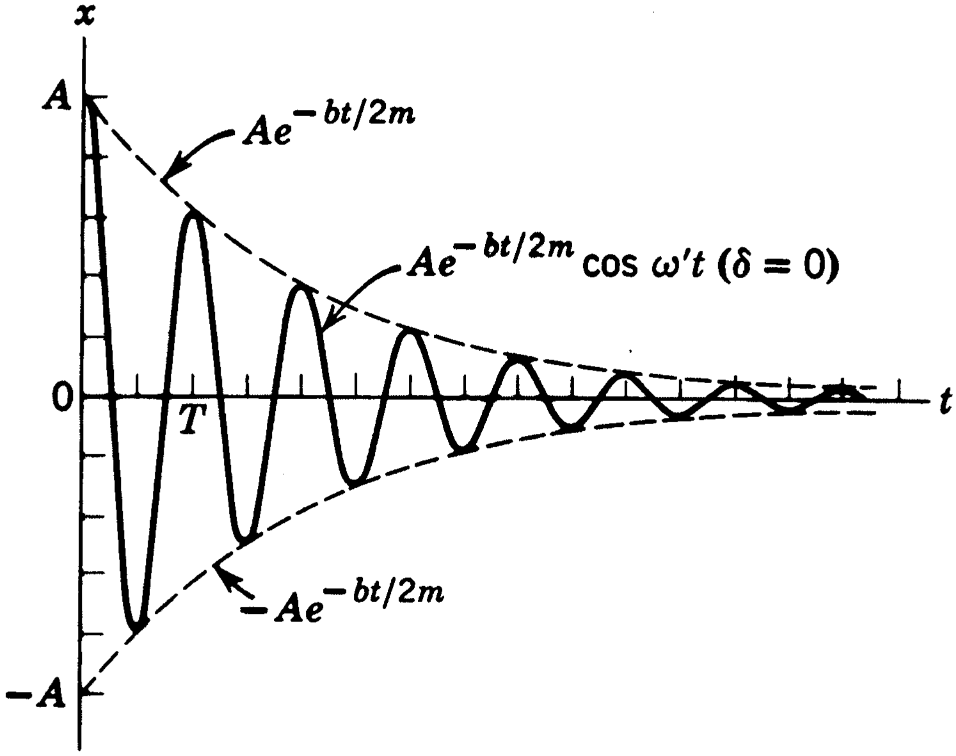
SHOW THIS

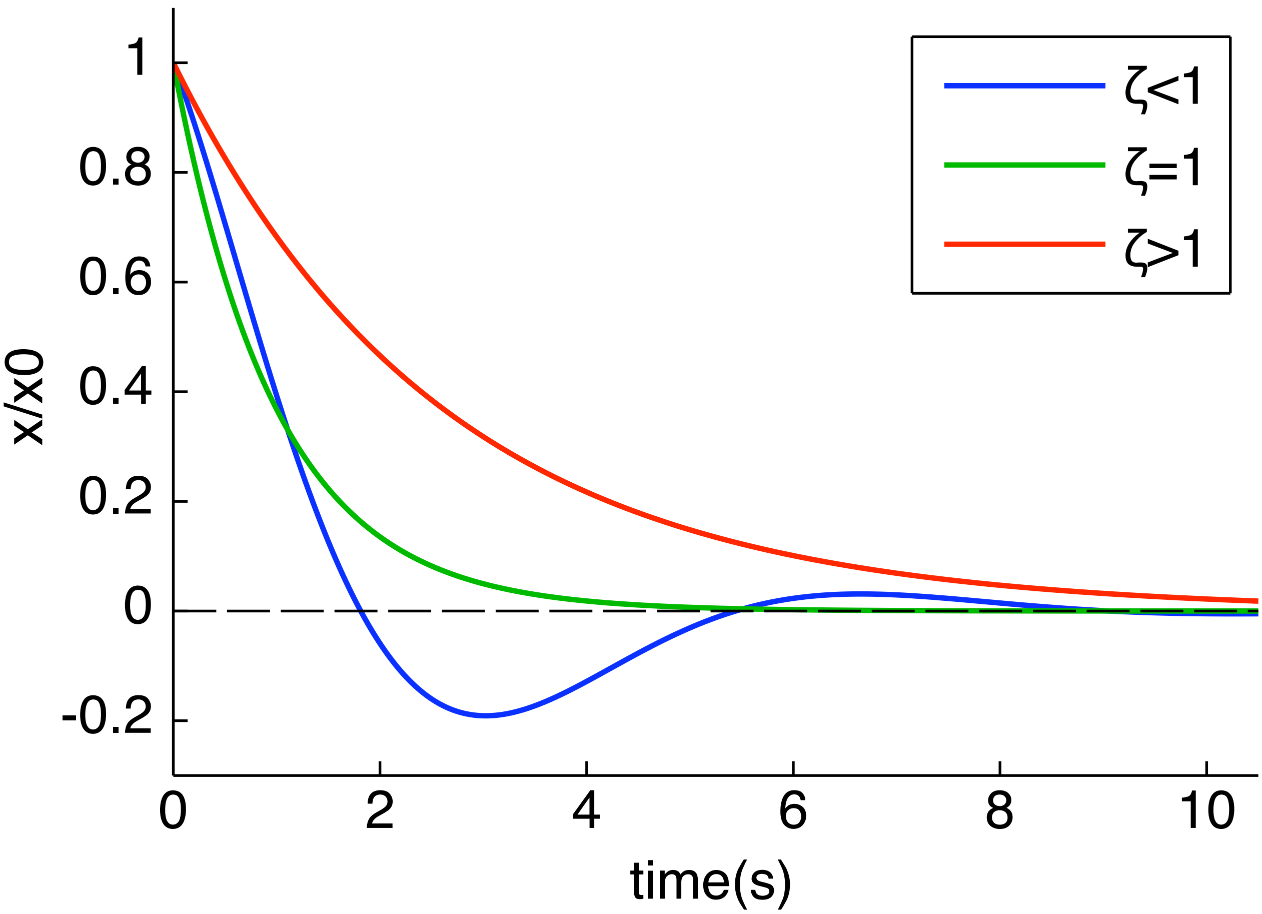
$$\omega = \sqrt{\omega_0^2 - \gamma^2}$$

$\gamma = 0$ NO DAMPING $\omega = \omega_0$

$\gamma = \omega_0$ CRITICALLY DAMPED $\omega = 0$

$\gamma > \omega_0$ OVER DAMPED ω imaginary


x
A
$Ae^{-bt/2m}$
$Ae^{-bt/2m}\cos\omega' t\ (\delta = 0)$
0
T
t
$-Ae^{-bt/2m}$
-A



ζ<1
ζ=1
ζ>1
1
0.8
0.6
0.4
0.2
0
-0.2
x/x0
0
2
4
6
8
10
time(s)

$F_T = m a$

$F_T = -\kappa x - \gamma v$

$$\left[\frac{d^2 \hat{x}}{dt^2} + \gamma \frac{d\hat{x}}{dt} + \kappa \hat{x}\right] e^{i\omega t} = \frac{\hat{F}}{m} e^{i\omega t}$$

$$(i\omega)^2 \hat{x} + \gamma (i\omega) \hat{x} + \kappa \hat{x} = \frac{\hat{F}}{m}$$

$$\hat{x} = \frac{\hat{F}}{m(\omega_0^2 - \omega^2 + i\gamma\omega)}$$

$\hat{x} = R \hat{F}$

R for response

χ for susceptibility

response = (suscep) (force)

$$R = \frac{1}{m(\omega_0^2 - \omega^2 + i\gamma\omega)}$$

R is complex

$Re(R)$

$Im(R)$

or

$r e^{i\varphi} = R$

$$\hat{F} = F_0 e^{i\Delta}$$

$$\hat{x} e^{i\omega t} = \rho e^{i\varphi} F_0 e^{i\Delta} = \rho F_0 e^{i(\theta+\Delta)} e^{i\omega t} = \rho F_0 e^{i(\omega t + \Delta + \theta)}$$

$$x = Re\,\hat{x} = \rho F_0 \cos(\omega t + \Delta + \theta)$$

(time dependent phase) (phase of affect drive) phase due to response

need ρ

$$\rho = \frac{1}{a + bi}$$

WHAT IS THE REAL PART? IMAGINARY PART?

$$\rho^2 = \frac{1}{a+bi}\left(\frac{a-bi}{a-bi}\right)$$

$$= \frac{a-bi}{a^2+b^2}$$

$$Re[\rho] = \frac{a}{a^2+b^2} \qquad Im[\rho] = \frac{-b}{a^2+b^2}$$

$$\rho^2 = \rho \rho^*$$

$$= \frac{1}{m(\omega^2 - \omega_0^2 + i\gamma\omega)} \quad \frac{1}{m(\omega^2 - \omega_0^2 - i\gamma\omega)}$$

$$= \frac{1}{m^2\left[(\omega^2 - \omega_0^2)^2 + \gamma^2\omega^2\right]}$$

$$|\rho| = \sqrt{z\, z^*}$$

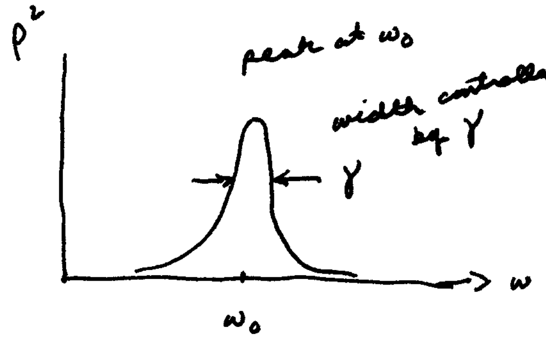
ANGLE

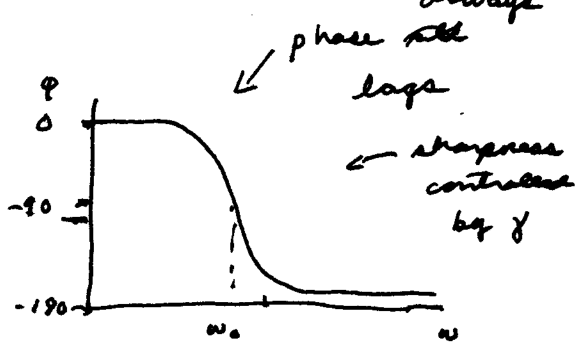
$$R = \rho e^{i\varphi}$$

$$\frac{1}{R} = \frac{1}{\rho} e^{-\varphi}$$

$$= m(\omega_0^2 - \omega^2 + i\gamma\omega)$$

$$\tan(\varphi) = \frac{Im}{Re} = \frac{-\gamma\omega}{\omega_0^2 - \omega^2}$$





$\rho^2 \sim$ square of amp $\quad \infty \rightarrow \frac{1}{\gamma^2\omega^2}$

$\sim$ energy in oscillator

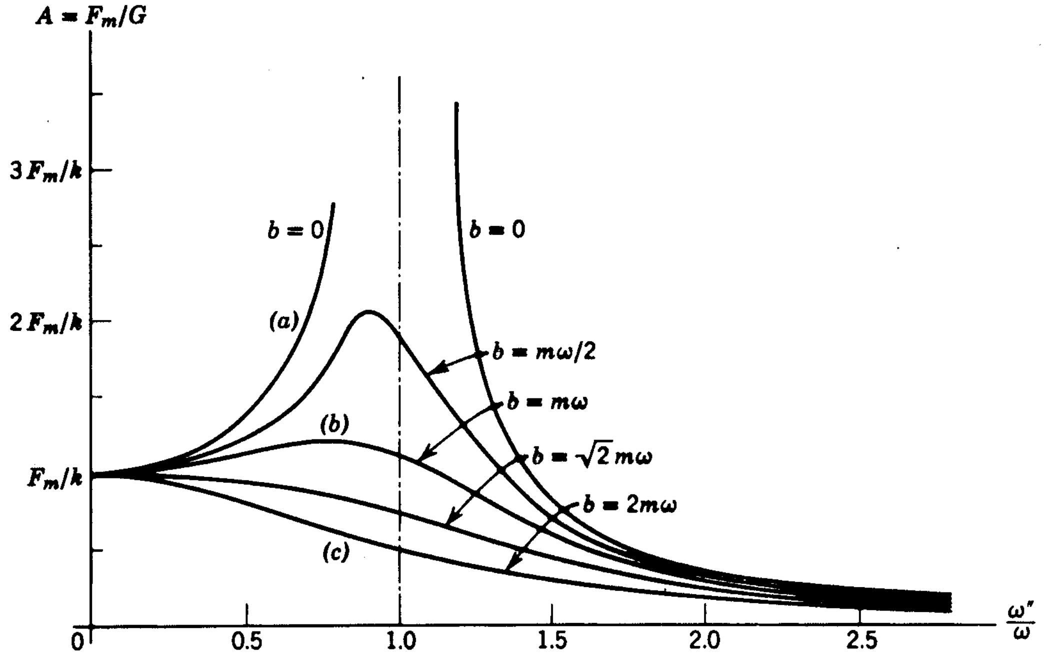
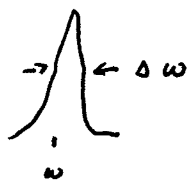


Fig. 15–20 The amplitude of a driven damped simple harmonic oscillator is plotted versus the ratio of the driving frequency ω'' to the undamped natural frequency ω. Curves for five different degrees of damping are shown; curve (*a*) shows no damping and curve (*c*) high damping. We notice that the resonant peak moves nearer and nearer the vertical line at $\omega''/\omega = 1$ as b becomes smaller and smaller.

THE Q

$$Q = \frac{\omega}{\Delta\omega}$$



$$Q = \frac{\omega_0}{\gamma}$$

MOSSBAUER EFFECT $\quad Q \sim 10^{13}$!

$E = 14{,}000$ ev

$\Delta E \sim 1$ meV !

phonons 10 meV