LECTURE 19 7-25-09

$$|\psi(0)\rangle = \frac{1}{\sqrt{2}}|E_1\rangle + \frac{1}{\sqrt{2}}|E_2\rangle$$

$\langle x|$

$$\psi(x,0) = \frac{1}{\sqrt{2}}\psi_1(x) + \frac{1}{\sqrt{2}}\psi_2(x)$$

$$|\psi(t)\rangle = \frac{1}{\sqrt{2}}|E_1\rangle e^{-iE_1t/\hbar} + \frac{1}{\sqrt{2}}|E_2\rangle e^{-iE_2t/\hbar}$$

$\langle x|$

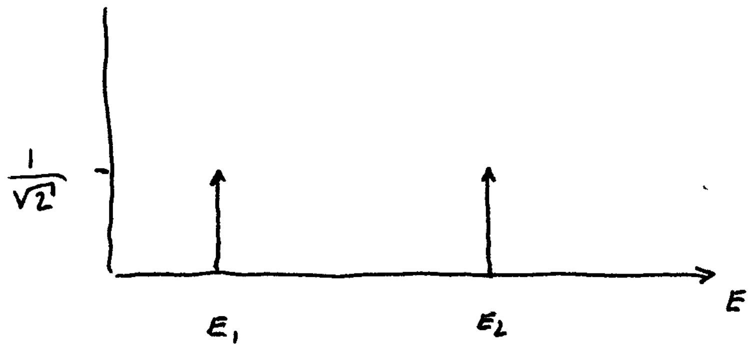
$$\psi(x,t) = \frac{1}{\sqrt{2}}\psi_1(x)e^{-iE_1t/\hbar} + \frac{1}{\sqrt{2}}\psi_2(x)e^{-iE_2t/\hbar}$$

IF YOU MEASURE ....

ENERGY
POSSIBLE
RESULTS : EIGENVALUES OF THE HAMILTONIAN

EIGENENERGIES

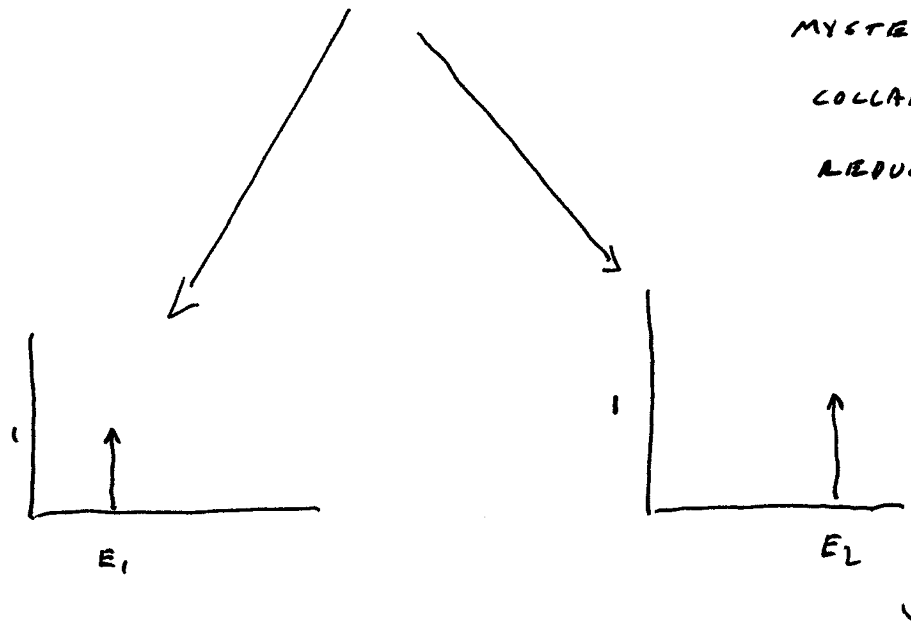
PROBABILITIES : HOW MUCH OF THAT EIGENSTATE IS IN $|\psi\rangle$



MYSTERIOUS QUANTUM JUMP

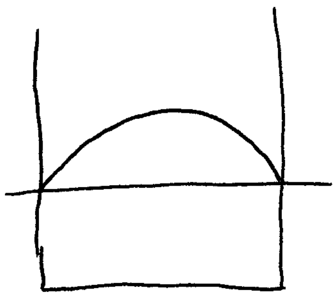
COLLAPSE OF THE WAVEFCN

REDUCTION OF THE STATE VECTOR



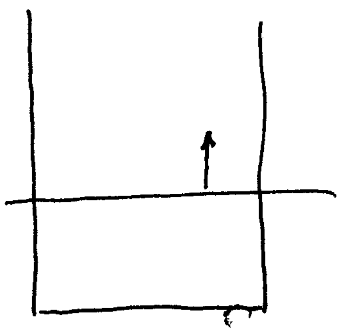
MYSTERIOUS BECAUSE :

IF YOU MEASURE POSITION



BEFORE

$\psi_1(x)$ eigenfunction of energy

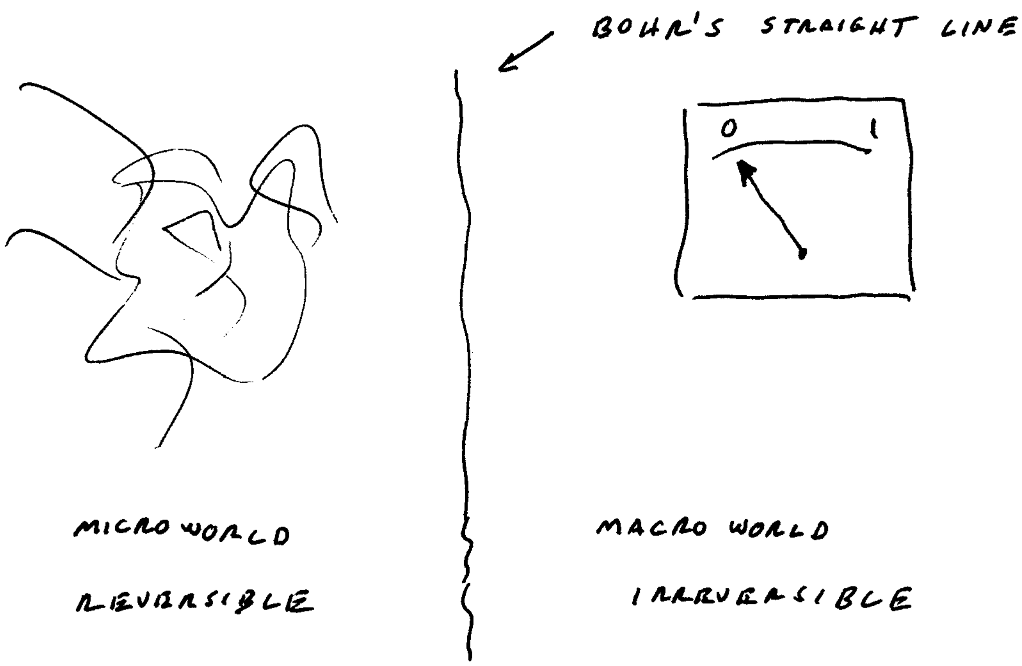


AFTER

eigenfunction of position

NRQM: This is instantaneous

RQM: NOT INSTANTANEOUS



MEASURE ENERGY $\Rightarrow$ WHAT ARE POSSIBLE RESULTS?

WHAT ARE THE ASSOCIATED PROBS?

COMMON SENSE

$$|\psi(0)\rangle = \frac{1}{\sqrt{2}}|E_1\rangle + \frac{1}{\sqrt{2}}|E_2\rangle$$

EQUAL MIXTURE OF E_1 AND E_2

EQUAL PROB OF E_1 AND E_2

TOTAL PROB = 1

$$\text{PROB}(E_1) = \frac{1}{2}$$

$$\text{PROB}(E_2) = \frac{1}{2}$$

$$\left(\frac{1}{\sqrt{2}}\right)^2 = \frac{1}{2}$$

$$\text{PROB} = |a_j|^2 \qquad \text{if } \sum_j |a_j|^2 = 1$$

FORMAL METHOD

MEASURE ENERGY

POSSIBILITIES : ev's OF H

$$E_1, E_2, E_3, \ldots$$

PROB OF EACH ONE : 50% E_1 50% E_2

$$\text{PROB}(E_1) = \left| \langle E_1 | \Psi(t) \rangle \right|^2$$

$$= \left| \left(\langle E_1 | \left(\frac{1}{\sqrt{2}} |E_1\rangle e^{-iE_1 t/\hbar} + \frac{1}{\sqrt{2}} |E_2\rangle e^{-iE_2 t/\hbar} \right) \right) \right|^2$$

$$= \left| \frac{1}{\sqrt{2}} e^{-iE_1 t/\hbar} \right|^2 = \frac{1}{2}$$

$$\text{PROB}(E_1) = \left| \int_0^a \psi_1^*(x) \left(\frac{1}{\sqrt{2}} \psi_1(x) e^{-iE_1 t/\hbar} + \frac{1}{\sqrt{2}} \psi_2(x) e^{-iE_2 t/\hbar} \right) dx \right|^2$$

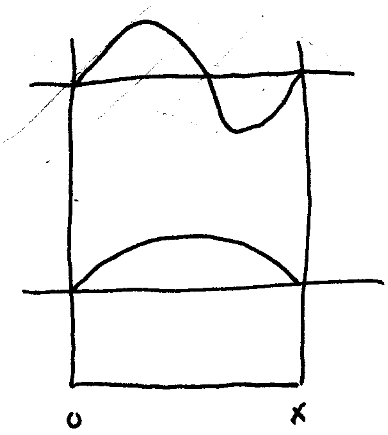
$$= \left| \frac{1}{\sqrt{2}} \frac{2}{a} \int_0^a \sin\left(\frac{\pi}{a} x\right) \sin\left(\frac{\pi}{a} x\right) e^{-iE_1 t/\hbar} \, dx \right.$$

$$\left. + \frac{1}{\sqrt{2}} \frac{2}{a} \int_0^a \sin\left(\frac{\pi}{a} x\right) \sin\left(\frac{2\pi}{a} x\right) e^{-iE_2 t/\hbar} \, dx \right|^2$$

$$= \left| \frac{1}{\sqrt{2}} \right|^2 = \frac{1}{2} \Longrightarrow 50\%$$

WORKS FOR ANY $f(x) = \Psi(x,0)$

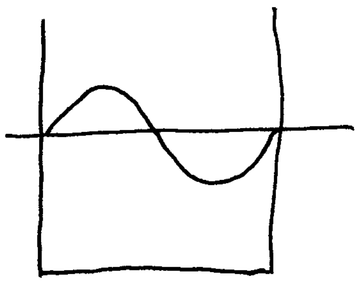
MEASURE POSITION ?



FIRST STATIONARY STATE $\psi_1(x)$

$$\text{PROB}(x)\,dx = P(x)\,dx = |\psi_1(x)|^2\,dx$$

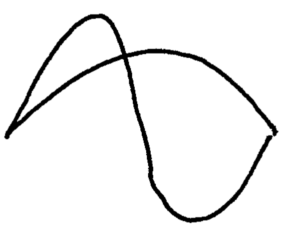
$P(x)$ is stationary



SECOND STATIONARY STATE $\psi_2(x)$

$$P(x)\,dx = |\psi_2(x)|^2\,dx$$

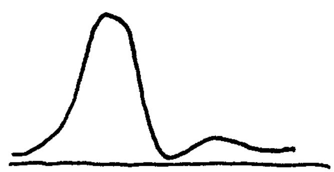
$P(x)$ is stationary



SUPERPOSITION STATE

$$\psi(x,0) = \left(\psi_1(x) + \psi_2(x)\right)\frac{1}{\sqrt{2}}$$

NOW $P(x)$ is not stationary!



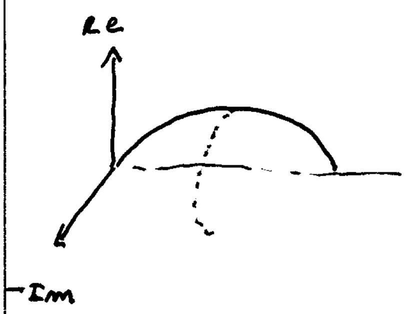
$\psi(x,0)$

$$P(x,0) = |\psi(x,0)|^2$$

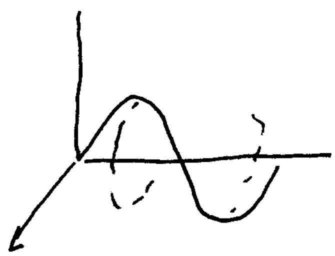
AS TIME GOES ON

$\times \quad e^{-iE_1t/\hbar}$

$\times \quad e^{-iE_2/\hbar}$



$\omega_1 = \frac{E_1}{\hbar}$



$\omega_2 = \frac{E_2}{\hbar} = 4\,\omega_1$

SIDE VIEW