LECTURE 17 7-22-09

COURSE IS 50% OVER!

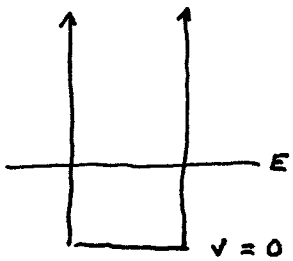
QM

EM

TISE

$$\left[\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + V(x)\right]\psi = E\psi$$

QM

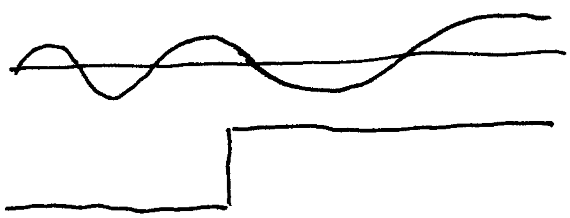


$V \rightarrow \infty \quad \psi \rightarrow 0$

$E > V \quad \psi \sim \sin kx$ or $\cos kx$

$$\psi_n(x) = \sqrt{\frac{2}{a}}\,\sin\!\left(\frac{n\pi}{a}x\right)$$

QUANTIZED DUE TO B.C.'s



$V(x)$

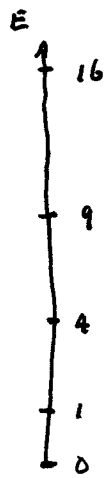
STEADY STATE $\Rightarrow$ TISE

NO QUANTIZATION

BOUND STATES

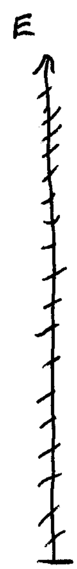
ENERGY QUANTIZED

DISCRETE SPECTRUM



SCATTERING STATES
CONTINUUM STATES

CONTINUOUS SPECTRUM



TDSE $\qquad H\ \Psi(x,t) = i\hbar \frac{d}{dt} \psi(x,t)$ $\qquad$ ALWAYS TRUE

TISE $\qquad H\ \psi_n(x) = E_n\ \psi_n(x)$

HAMILTONIAN OPERATOR

ENERGY EIGENVALUE

STATIONARY STATES
ENERGY EIGENSTATES
ENERGY EIGENFCNS

ANALOGY

$$\begin{pmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = 2 \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

H OP $\qquad$ e$\vec{v}$ OF H

ev of H associated with this e$\vec{v}$

STRATEGY:

WANT TO SOLVE TDSE

FIRST SOLVE TISE

THEN IT IS EASY TO WRITE DOWN SOLN TO TDSE

SOLVING TISE : FIND ev's and e$\vec{v}$'s
ev's " ef's

DIAGONALIZE HAMILTONIAN

ANALOGY BETWEEN LINEAR ALGEBRA AND DIFF EQNS

$$H \rightarrow \begin{pmatrix} 3 & & \\ & 2 & \\ & & 1 \end{pmatrix}$$

$$3 \quad \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \qquad\qquad 2 \quad \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \qquad\qquad 1 \quad \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

ev_1 $\quad e\vec{v}_1$ $\qquad\qquad ev_2$ $\quad e\vec{v}_2$ $\qquad\qquad ev_3$ $\quad e\vec{v}_3$

E_1 $\qquad\qquad\qquad E_2$ $\qquad\qquad\qquad E_3$

IF AT $t=0$

$$\text{STATE} \rightarrow \begin{pmatrix} a \\ b \\ c \end{pmatrix}$$

THEN AT LATER TIMES

TIME-DEPENDENT PHASE FACTORS

$$\text{STATE} \rightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} = a \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} e^{-iE_1 t/\hbar} + b \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} e^{-iE_2 t/\hbar}$$

$$+ c \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} e^{-iE_3 t/\hbar}$$

$$H = -\frac{\hbar^2}{2m} \frac{d^2}{dx^2}$$

e v

E_1 E_2 E_3

e f

$\sin(k_1 x)$ $\sin(k_2 x)$ $\sin(k_3 x)$

$\sin\left(\frac{\pi}{a} x\right)$ $\sin\left(\frac{2\pi}{a} x\right)$ $\sin\left(\frac{3\pi}{a} x\right)$

$$E_n = \frac{\hbar^2 k_n^2}{2m} = \frac{\hbar^2}{2m} \left(\frac{n\pi}{a}\right)^2$$

$$E_n = \left(\frac{\hbar^2 \pi^2}{2ma^2}\right) n^2$$

$$\begin{pmatrix} a \\ b \\ c \end{pmatrix} = a \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + b \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + c \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix} = a \qquad d_1 = \int_0^a \psi_1^*(x)\, \Psi(x,0)\, dx$$

MULTIPLY POINT-BY-POINT
THEN SUM = INTEGRATE

$$\begin{pmatrix} 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix} = b \qquad d_2 = \int_0^a \psi_2^*(x)\, \Psi(x,0)\, dx$$

$$\begin{pmatrix} 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix} = c \qquad d_3 = \int_0^a \psi_3^*(x)\, \Psi(x,0)\, dx$$

$$\vdots$$

MULTIPLY TERM-BY-TERM
AND THEN SUM

$$\langle E_3 \mid \Psi(0) \rangle$$

$$|\psi(0)\rangle = \frac{1}{\sqrt{2}}|E_1\rangle + \frac{1}{\sqrt{2}}|E_2\rangle$$

$$\begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ 0 \end{pmatrix}$$

$$|\psi(t)\rangle = \frac{1}{\sqrt{2}}|E_1\rangle e^{-iE_1t/\hbar} + \frac{1}{\sqrt{2}}|E_2\rangle e^{-iE_2t/\hbar}$$

$$\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} e^{-iE_1t/\hbar} + \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} e^{-iE_2t/\hbar}$$

$$= \frac{1}{\sqrt{2}}\begin{pmatrix} e^{-iE_1t/\hbar} \\ e^{-iE_2t/\hbar} \\ 0 \end{pmatrix}$$

$$|\psi(0)\rangle = \frac{1}{\sqrt{2}}|E_1\rangle + \frac{1}{\sqrt{2}}|E_2\rangle$$

$\langle x|$

$$\langle x|\psi(0)\rangle = \frac{1}{\sqrt{2}}\langle x|E_1\rangle + \frac{1}{\sqrt{2}}\langle x|E_2\rangle$$

$$\psi(x,0) = \frac{1}{\sqrt{2}}\psi_1(x) + \frac{1}{\sqrt{2}}\psi_2(x)$$

50 : 50 SUPERPOSITION OF GROUND STATE AND FIRST EXCITED STATE

$$\psi(x,t) = \frac{1}{\sqrt{2}}\psi_1(x)\,e^{-iE_1t/\hbar} + \frac{1}{\sqrt{2}}\psi_2(x)\,e^{-iE_2t/\hbar}$$

$$= \frac{1}{\sqrt{2}}\sqrt{\frac{2}{a}}\,\sin\left(\frac{\pi}{a}x\right)e^{-iE_1t/\hbar}$$

$$+ \frac{1}{\sqrt{2}}\,\sin\left(\frac{2\pi}{a}x\right)e^{-iE_2t/\hbar}$$

CLASSICAL HAMILTONIAN

$$H = \frac{p^2}{2m} + V(x)$$

TOTAL ENERGY

KINETIC ENERGY

POTENTIAL

QM

$$H_{op} = \frac{p_{op}^2}{2m} + V(x_{op})$$

need a basis x-space position space

$$p_{op} \rightarrow i\hbar \frac{d}{dx}$$

$$x_{op} \rightarrow x$$

$$\left[\frac{-\hbar^2}{2m} \frac{d^2}{dx^2} + V(x) \right] \psi(x,t) = i\hbar \frac{d}{dt} \psi(x,t) \qquad \text{TDSE}$$

$$\left[\qquad \qquad \qquad \right] \psi_1(x) = E_1 \psi_1(x) \qquad \text{TISE}$$