**Lecture 16** **July 20, 2009**

**Is there a problem or isn't there?**
**Double slit calculations**
**Local Realistic Theories and Einstein Locality**
**The Einstein-Podolsky-Rosen Experiment**
**Feynman's Path Integral Formulation of QM**

# Is there a problem, or isn't there?

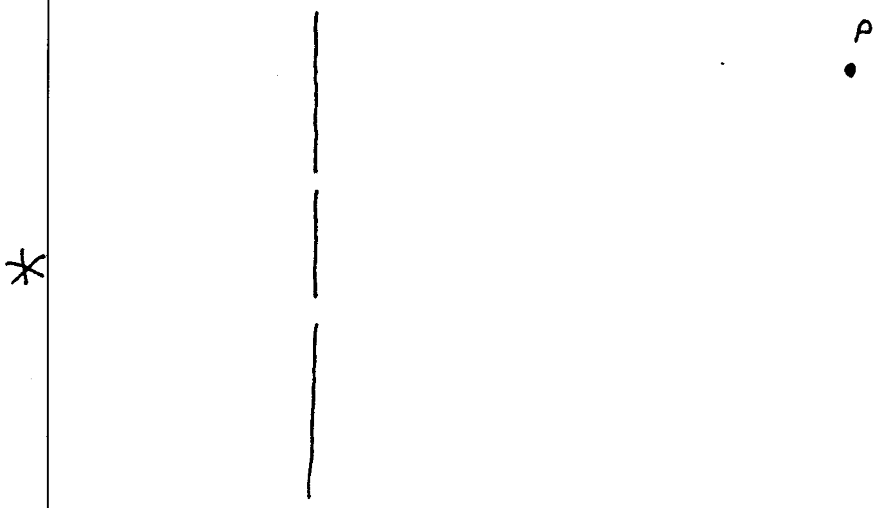
This thing is completely characteristic of all of the particles of nature, and of a universal character, so if you want to hear about the character of physical law it is essential to talk about this particular aspect... It will be difficult. But the difficulty is psychological and exists in the perpetual torment that results from saying to yourself, "But how can it be like that" which is a reflection of an uncontrolled but utterly vain desire to see it in terms of something familiar. I will not describe it in terms of something familiar; I will simply describe it. There was a time when the newspapers said that only twelve men understood the theory of relativity. I do not believe there ever was such a time. There might have been a time when only one man did, because he was the only one who caught on, before he wrote his paper. But after people read the paper a lot of people understood the theory of relativity in some way or another, certainly more than twelve. On the other hand, I think I can safely say that nobody understands quantum mechanics. So do not take the lecture too seriously, feeling that you have to understand in terms of some model what I am going to describe, but just relax and enjoy it. I am going to tell you what nature behaves like. If you will simply admit that she maybe does behave like this, you will find her a delightful, entrancing thing. Do not keep saying to yourself, if you can possibly avoid it, "But how can it be like that?" because you will get "down the drain" into a blind alley from which nobody has yet escaped. Nobody knows how it can be like that.

**Feynman 1964**

We have always had a great deal of difficulty understanding the world view that quantum mechanics represents. At least I do, because I'm an old enough man that I haven't got to the point that this stuff is obvious to me. Okay, I still get nervous about it... You know how it always is, every new idea, it takes a generation or two until it is obvious that there's no real problem. I cannot define the real problem, therefore I suspect there's no real problem, but I'm not sure there's no real problem.

**Feynman 1982**

# EXAMPLE 1



GIVEN:

$|\psi_1(P)| = 2$

$|\psi_2(P)| = 6$

100 e/s WITH ONLY 1 OPEN

(a) HOW MANY e/s AT P WITH ONLY 2 OPEN?

$|\psi_1|^2 = (2)^2 = 4 \quad \Rightarrow \quad 4\,(25\ e/s) = 100\ e/s$

$|\psi_2|^2 = (6)^2 = 36 \quad\quad 36\,(25\ e/s) = 900\ e/s$

~~average~~ sum = 1000

(b) HOW MANY e/s AT P IF 1 and 2 INTERFERE CONSTRUCTIVELY?

$$\psi_1 + \psi_2 = 8$$

$$|\psi_1 + \psi_2|^2 = 8^2 = 64$$

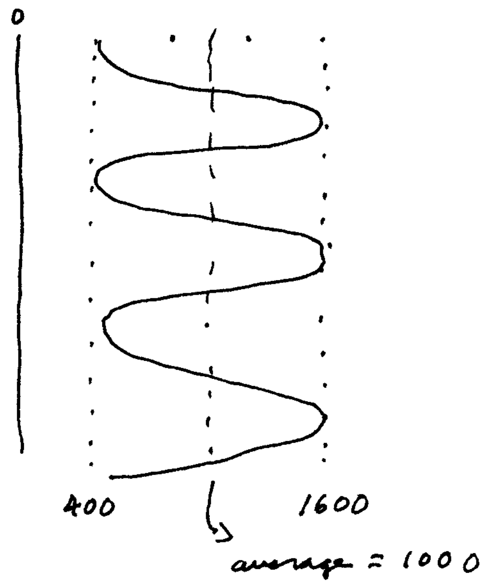
$$64\ (25\ e/s) = 1600\ e/s$$

(c) HOW MANY e/s AT P IF 1 AND 2 INTERFERE DESTRUCTIVELY?

$$|\psi_1 + \psi_2| = 4$$

$$|\psi_1 + \psi_2|^2 = 4^2 = 16$$

$$16\ (25\ e/s) = 400\ e/s$$



EXAMPLE 2

EQUAL SLITS

$|\psi_1|^2 =$ 100 e/s

$|\psi_2|^2 =$ 100 e/s

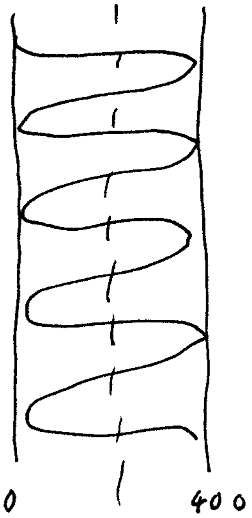
SUM = 200 e/s

CONSTRUCTIVE

$|\psi_1 + \psi_2|^2 = |20|^2 =$ 400 e/s

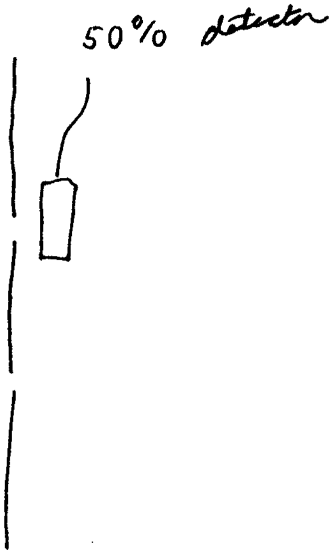
DESTRUCTIVE

$|\psi_1 + \psi_2|^2 = |0|^2 =$ 0 e/s



average = 200

EXAMPLE 3



1 50 e/s DETECTED $\sqrt{50} \approx 7.07$

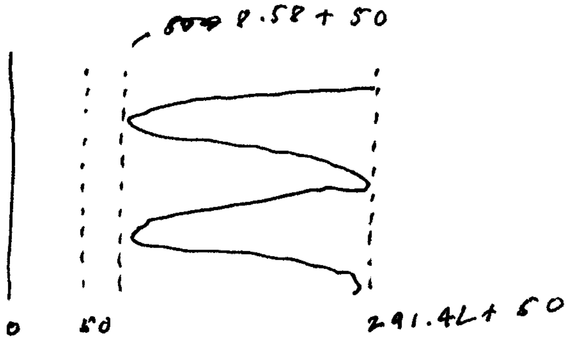
50 e/s NOT DETECTED

2 100 e/s NOT DETECTED $\sqrt{100} = 10$

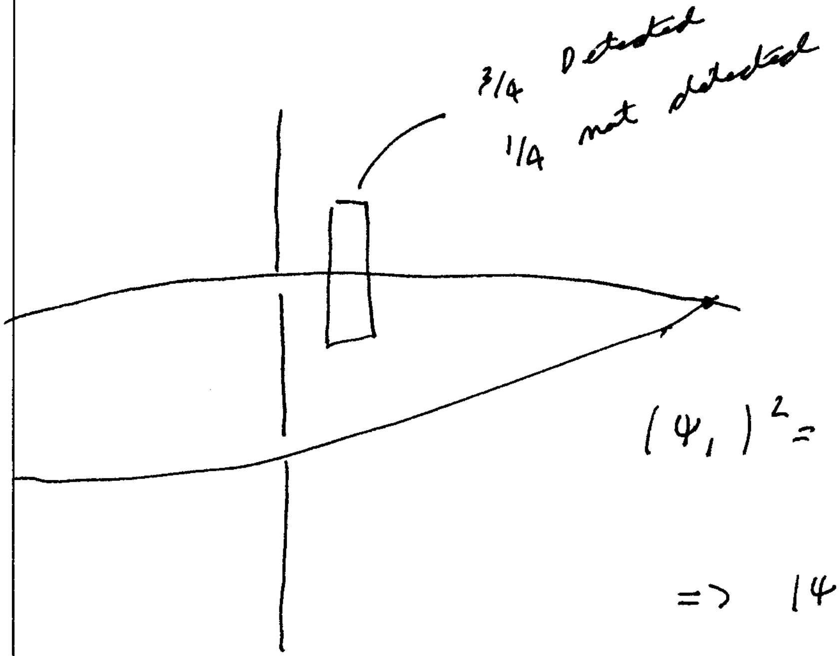
P (NOT DETECTED) = $|7.07 + 10|^2 = 291.42$

$|-7.07 + 10| = 8.58$

P (DETECTED) = 50



EXAMPLE 4



$$(\psi_1)^2 = \frac{1}{4}|\psi_2|^2$$

$$\Rightarrow |\psi_1| = \frac{1}{4}|\psi_2|$$

$$|\psi_1^0|^2 = \frac{3}{4}|\psi_2|^2$$

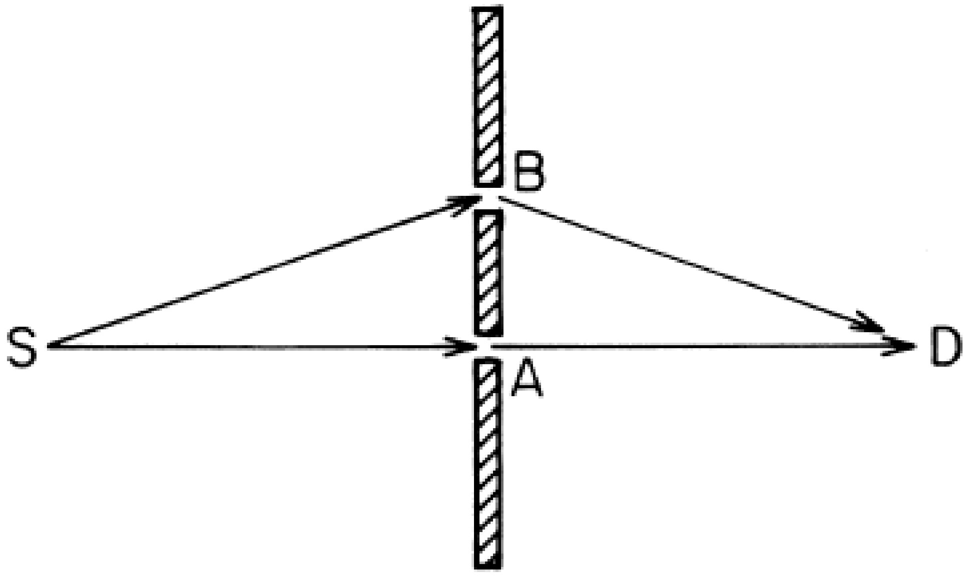
$$|\psi_1^0| = \sqrt{\frac{3}{4}}\,|\psi_2|$$

$$I = \underbrace{(\psi_1+\psi_2)^2}_{\text{DON'T KNOW WHICH SLIT}} + \underbrace{(\psi_1^0)^2}_{\text{DO KNOW WHICH SLIT}}$$

$$\frac{I_{MAX}}{I_{MIN}} = \frac{|\frac{1}{2}\psi_2+\psi_2|^2 + \frac{3}{4}\psi_2^2}{|\frac{1}{2}\psi_2-\psi_2|^2 + \frac{3}{4}\psi_2^2} = \frac{3}{1}$$

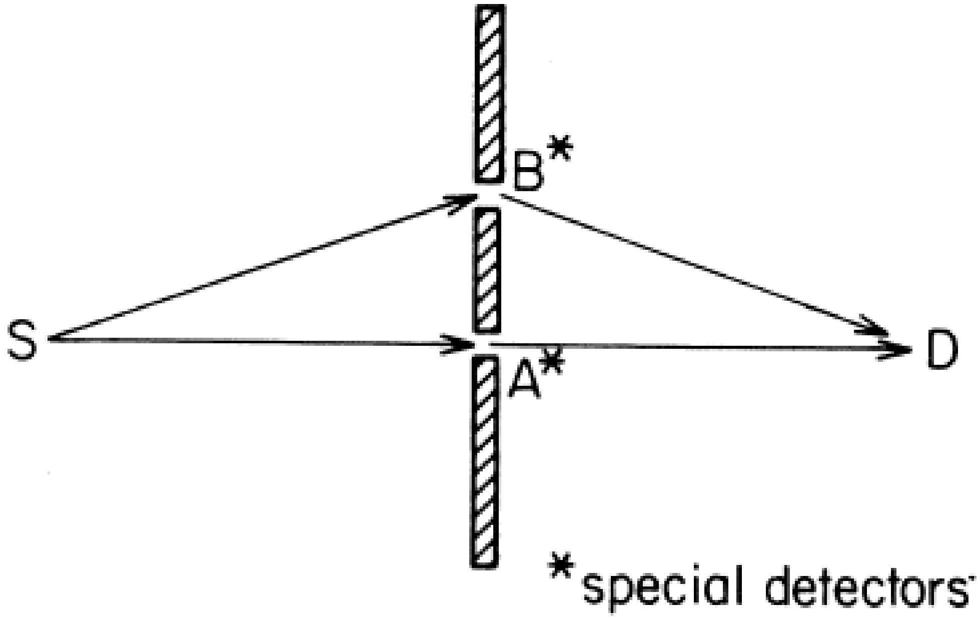
**If you do not detect them**

**Interference? Yes!!!**



**If you do detect them**

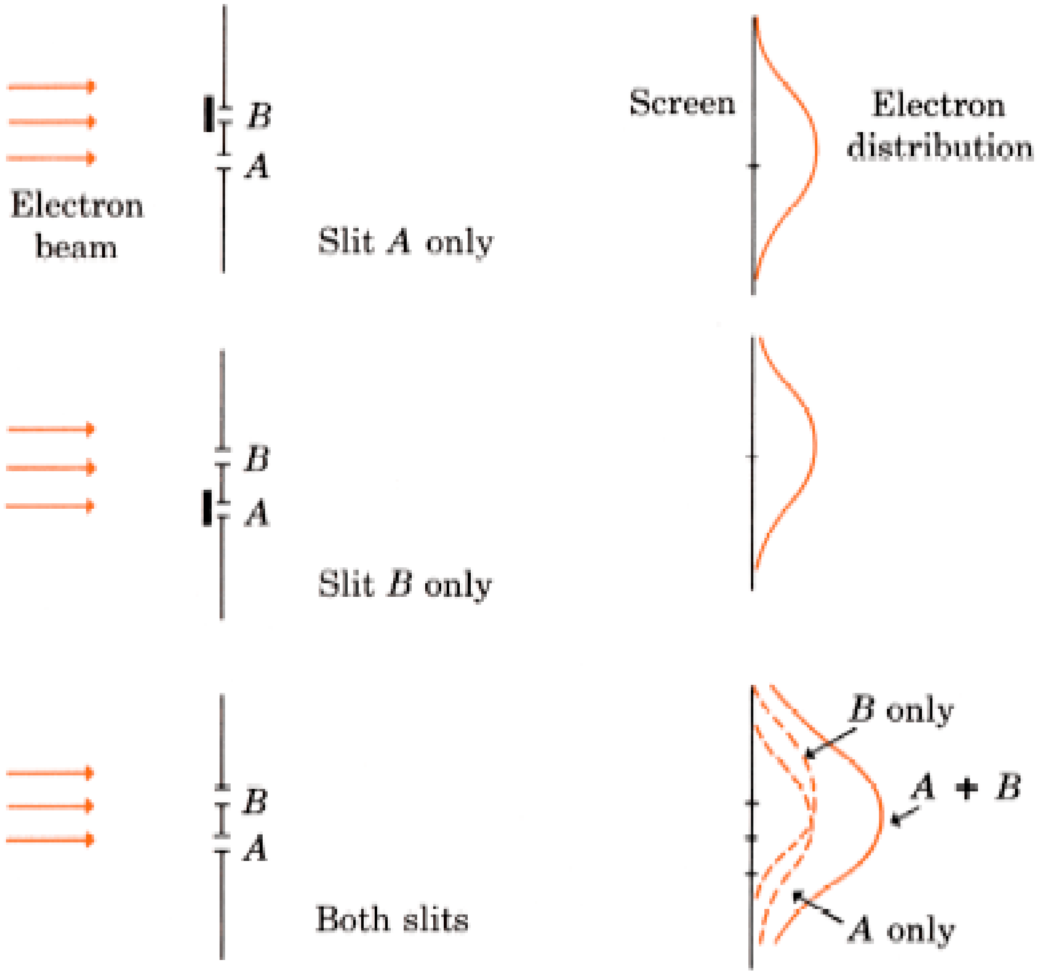
**Interference? No!!!**



**What if you detect some of them?**

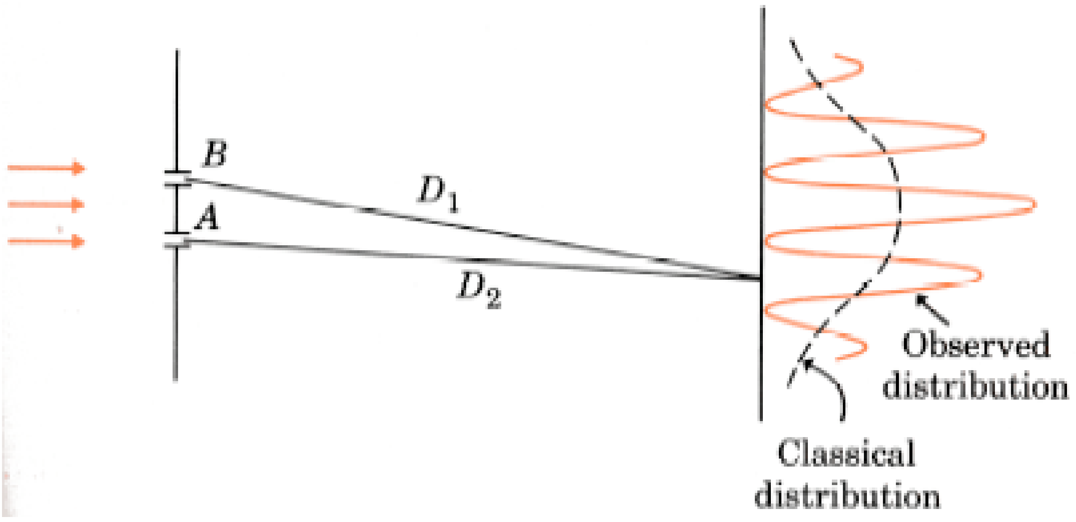
# Classical Particles

# Interference? No!!!



# Classical Waves and Photons

# Interference? Yes!!!



# EXAMPLE 1

## EQUAL SLITS

With Slit 1 open 100 photons/second => $A_1=10$

With Slit 2 open 100 photons/second => $A_2=10$

## HOW MANY WHEN 1 AND 2 CONSTRUCTIVE?

$A = A_1+A_2 = 20$

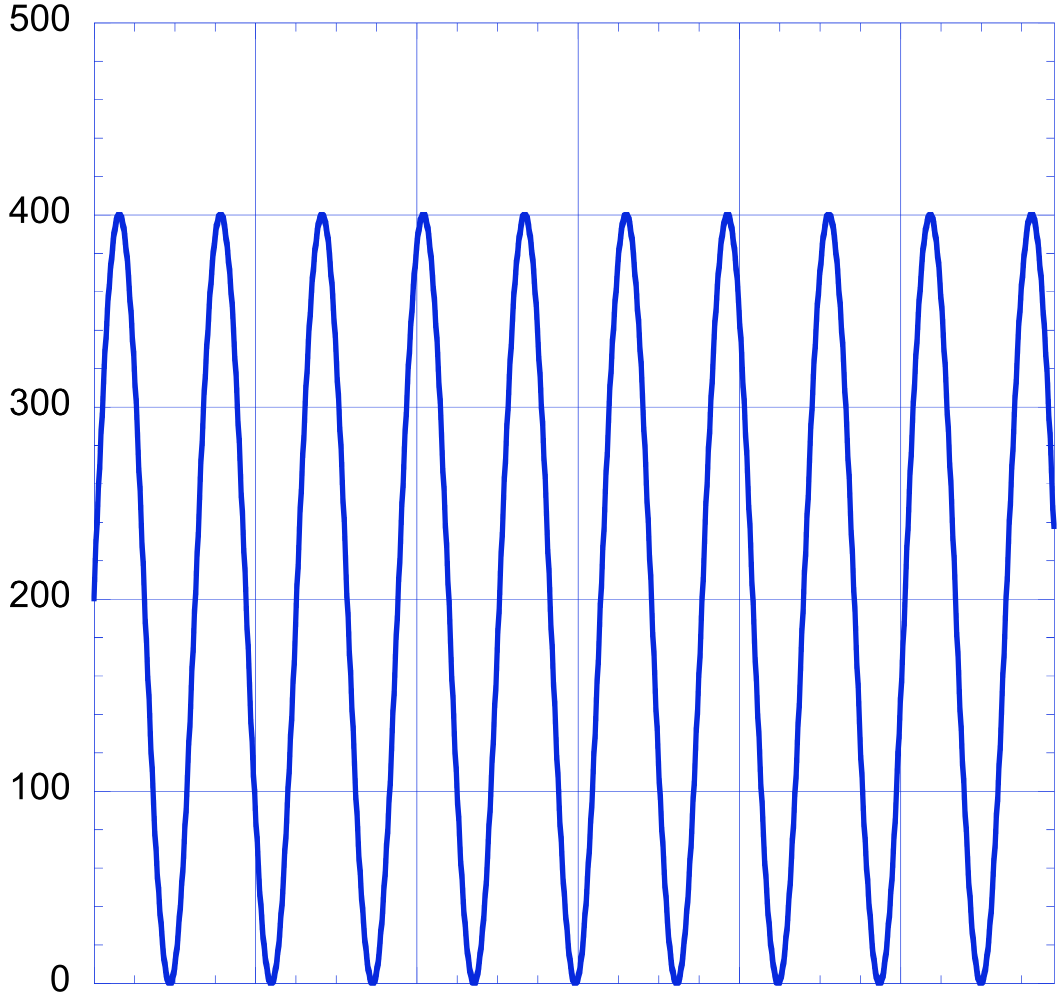
$A^2 = 400$ photons/second

## HOW MANY WHEN 1 AND 2 DESTRUCTIVE?

$A = A_1+A_2 = 0$

$A^2 = 0$ photons/second

constructive => 400 photons/sec
destructive => 0 photons/sec
with only slit 1 open => 100
with only slit 2 open => 100
with both open no interference = 200 photons/sec
200 with both open = average of interference 0.5 (400 + 0)



# EXAMPLE 2

$A_1=2$

$A_2=6$

With slit 1 open 100 photons/second

## HOW MANY WITH ONLY SLIT 2 OPEN?

$(A_1)^2 = 4$

4*(25 photons/sec) = 100 photons/second

$(A_2)^2 = 36$

36*(25 photons/sec) = 900 photons/second

## HOW MANY WHEN 1 AND 2 CONSTRUCTIVE?

$A = A_1 + A_2 = 8$

$A^2 = 64$

64*(25 photons/sec) = 1600 photons/second

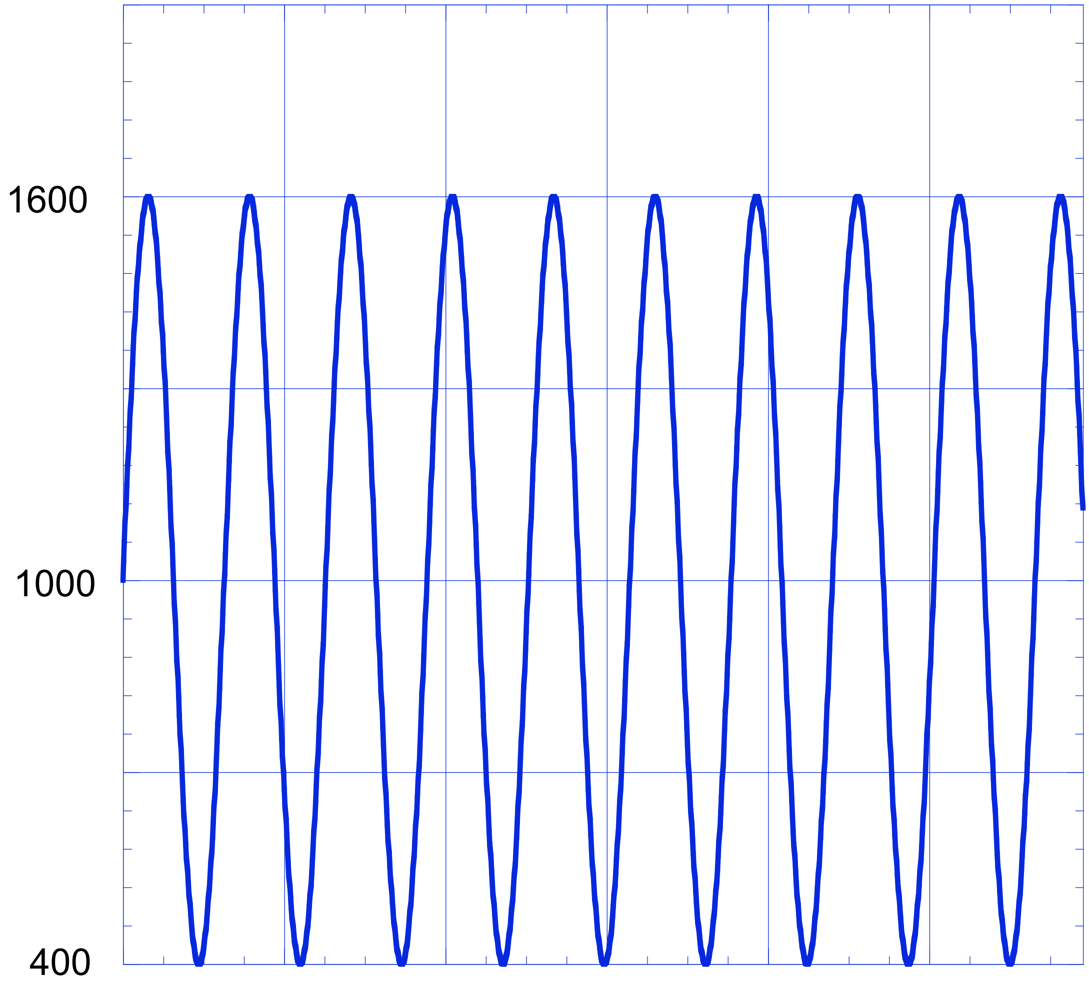
## HOW MANY WHEN 1 AND 2 DESTRUCTIVE?

$A = A_2 - A_1 = 4$

$A^2 = 16$

16*(25 photons/sec) = 400 photons/second

constructive => 1600 photons/sec
destructive => 400 photons/sec
with only slit 1 open => 100 photons/sec
with only slit 2 open => 900 photons/sec
with both open no interference = 200 photons/sec
1000 with both open = average of interference 0.5 (1600 + 400)



# EXAMPLE 3

put a partial detector behind slit 1

for a total of 100 photons/second thru slit 1

50 photons/second are not detected => $A_1 = 7.07$

50 photons/second are detected => $A_{1d} = 7.07$

no partial detector behind slit 2 => $A_2 = 10$

100 photons/second not detected thru slit 2

## FOR THE PHOTONS NOT DETECTED:

**HOW MANY CONSTRUCTIVE?**

$A = A_1 + A_2 = 7.07 + 10$

$A^2 = 291.42$ photons/second

**HOW MANY DESTRUCTIVE?**

$A = A_1 + A_2 = 10 - 7.07$

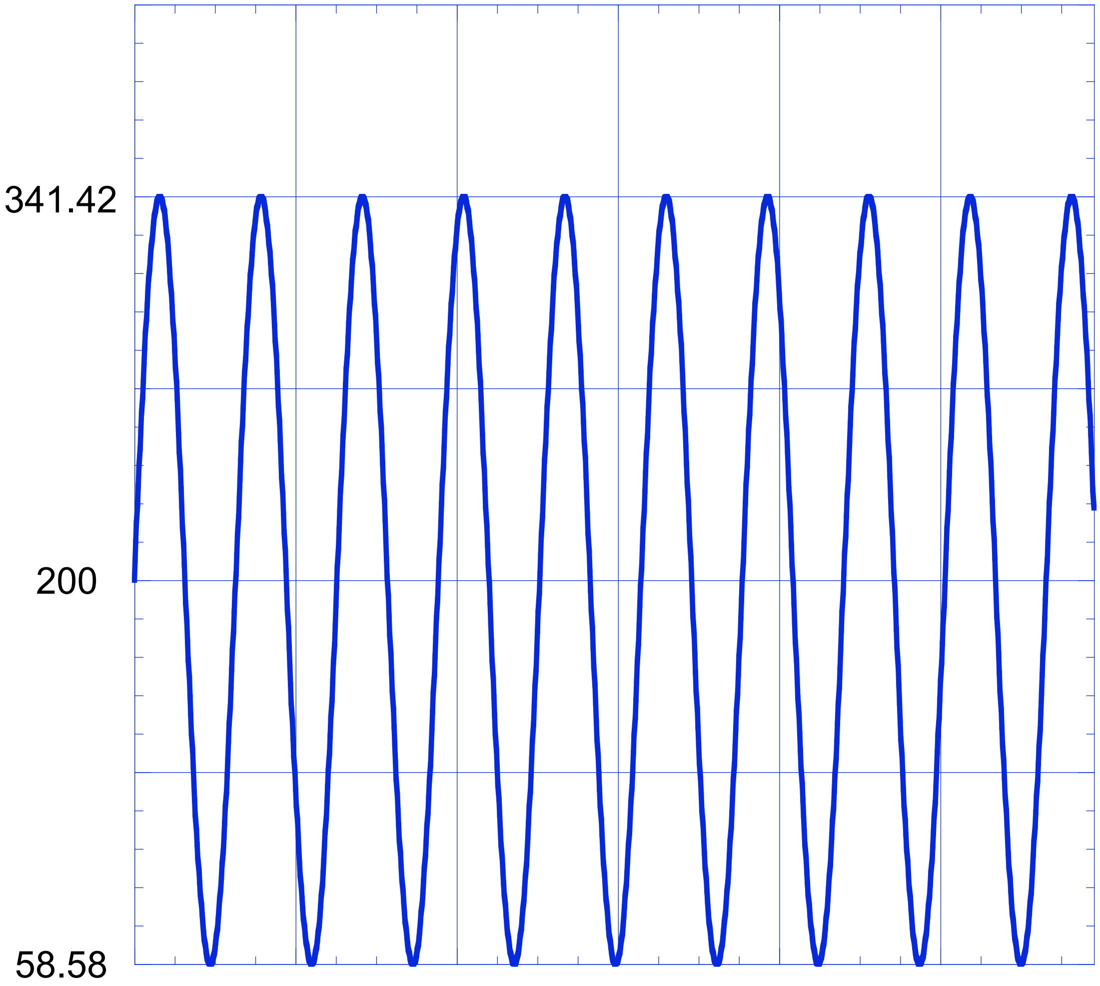
$A^2 = 8.58$ photons/second

## FOR THE PHOTONS DETECTED:

**NO INTERFERENCE**

$(A_{1d})^2 = 50$ photons/second

with only slit 1 open => 100 photons/sec
of which 50 photons/sec are detected
and of which 50 photons/sec are not detected
with only slit 2 open => 100 photons/sec
not detected constructive => 291.42 photons/sec
not detected destructive => 8.58 photons/sec



# EXAMPLE 4

equal slits

put a partial detector behind slit 1

3/4 of the photons/second are not detected $A_1$

1/4 of the photons/second are detected $A_{1d}$

$(A_{1d})^2 = 1/4\ (A_2)^2 \Rightarrow A_{1d} = 1/2\ A_2$

$(A_1)^2 = 3/4\ (A_2)^2 \Rightarrow A_1 = (3/4)^{0.5}\ A_2$

The total intensity is proportional to

$I \sim (A_1 + A_2)^2 + (A_{1d})^2$

The Intensity Contrast = $I_{max}/I_{min}$ is given by

$((1/2)*A_2 + A_2)^2 + 3/4\ (A_2)^2$ divided by

$((1/2)*A_2 - A_2)^2 + 3/4\ (A_2)^2$

**So, the Intensity Contrast = 3/1**

# Quantum Entanglement

## EPR: The Einstein Podolsky Rosen Paradox

**"If S1 and S2 are two systems that have interacted in the past, but are now arbitrarily distant, the real factual situation of S1 does not depend on what is done with S2 which is spatially separated from the former."**

**EPR is based on three premises:**

**1) REALISM**
**Observed regularities are caused by physical reality independent of human observers.**

**2) INDUCTIVE INFERENCE**
**Consistent observations produce legitimate conclusions.**

**3) EINSTEIN SEPARABILITY (LOCALITY)**
**No influence can propagate faster than light.**

**LOCAL REALISTIC THEORIES (LRT's) obey premises 1, 2, and 3**

**LRT and QM disagree**

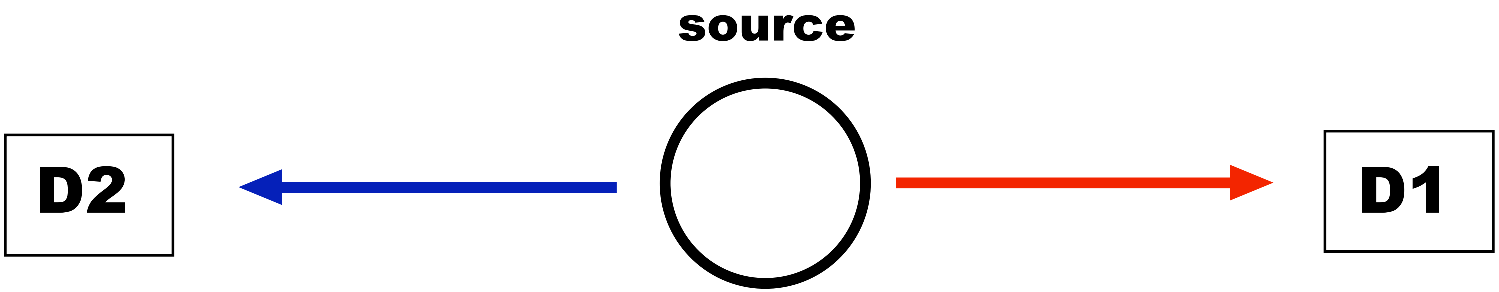
**QM agrees with experiments**

**=> We must give up 1, 2, or 3 !!!**

**1) The universe exists without us**
**2) The scientific method**
**3) Locality**

**So, we give up locality.**

# The EPR Paradox



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D2

D1

Measure X2
=> know X1

Measure P1
=> know P2

Measure both => violate

$\Delta x \Delta p \geq$ hbar/2

of lanthanum is 7/2, hence the nuclear magnetic moment as determined by this analysis is 2.5 nuclear magnetons. This is in fair agreement with the value 2.8 nuclear magnetons determined from La III hyperfine structures by the writer and N. S. Grace.[9]

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[9] M. F. Crawford and N. S. Grace, Phys. Rev. **47**, 536 (1935).

This investigation was carried out under the supervision of Professor G. Breit, and I wish to thank him for the invaluable advice and assistance so freely given. I also take this opportunity to acknowledge the award of a Fellowship by the Royal Society of Canada, and to thank the University of Wisconsin and the Department of Physics for the privilege of working here.

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MAY 15, 1935 PHYSICAL REVIEW VOLUME 47

# Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?

A. Einstein, B. Podolsky and N. Rosen, *Institute for Advanced Study, Princeton, New Jersey*

(Received March 25, 1935)

In a complete theory there is an element corresponding to each element of reality. A sufficient condition for the reality of a physical quantity is the possibility of predicting it with certainty, without disturbing the system. In quantum mechanics in the case of two physical quantities described by non-commuting operators, the knowledge of one precludes the knowledge of the other. Then either (1) the description of reality given by the wave function in quantum mechanics is not complete or (2) these two quantities cannot have simultaneous reality. Consideration of the problem of making predictions concerning a system on the basis of measurements made on another system that had previously interacted with it leads to the result that if (1) is false then (2) is also false. One is thus led to conclude that the description of reality as given by a wave function is not complete.

## 1.

Any serious consideration of a physical theory must take into account the distinction between the objective reality, which is independent of any theory, and the physical concepts with which the theory operates. These concepts are intended to correspond with the objective reality, and by means of these concepts we picture this reality to ourselves.

In attempting to judge the success of a physical theory, we may ask ourselves two questions: (1) "Is the theory correct?" and (2) "Is the description given by the theory complete?" It is only in the case in which positive answers may be given to both of these questions, that the concepts of the theory may be said to be satisfactory. The correctness of the theory is judged by the degree of agreement between the conclusions of the theory and human experience. This experience, which alone enables us to make inferences about reality, in physics takes the form of experiment and measurement. It is the second question that we wish to consider here, as applied to quantum mechanics.

Whatever the meaning assigned to the term *complete*, the following requirement for a complete theory seems to be a necessary one: *every element of the physical reality must have a counterpart in the physical theory*. We shall call this the condition of completeness. The second question is thus easily answered, as soon as we are able to decide what are the elements of the physical reality.

The elements of the physical reality cannot be determined by *a priori* philosophical considerations, but must be found by an appeal to results of experiments and measurements. A comprehensive definition of reality is, however, unnecessary for our purpose. We shall be satisfied with the following criterion, which we regard as reasonable. *If, without in any way disturbing a system, we can predict with certainty (i.e., with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity.* It seems to us that this criterion, while far from exhausting all possible ways of recognizing a physical reality, at least provides us with one

# I.9 BOHR'S REPLY

## COMMENTARY OF ROSENFELD (1967)

This onslaught came down upon us as a bolt from the blue. Its effect on Bohr was remarkable. We were then in the midst of groping attempts at exploring the implications of the fluctuations of charge and current distributions, which presented us with riddles of a kind we had not met in electrodynamics. A new worry could not come at a less propitious time. Yet, as soon as Bohr had heard my report of Einstein's argument, everything else was abandoned: we had to clear up such a misunderstanding at once. We should reply by taking up the same example and showing the right way to speak about it. In great excitement, Bohr immediately started dictating to me the outline of such a reply. Very soon, however, he became hesitant: "No, this won't do, we must try all over again . . . we must make it quite clear . . . ." So it went on for a while, with growing wonder at the unexpected subtlety of the argument. Now and then, he would turn to me: "What *can* they mean? Do *you* understand it?" There would follow some inconclusive exegesis. Clearly, we were farther from the mark than we first thought. Eventually, he broke off with the familiar remark that he "must sleep on it." The next morning he at once took up the dictation again, and I was struck by a change in the tone of the sentences: there was no trace in them of the previous day's sharp expressions of dissent. As I pointed out to him that he seemed to take a milder view of the case, he smiled: "That's a sign," he said, "that we are beginning to understand the problem." And indeed, the real work now began in earnest: day after day, week after week, the whole argument was patiently scrutinized with the help of simpler and more transparent examples. Einstein's problem was reshaped and its solution reformulated with such precision and clarity that the weakness in the critics' reasoning became evident, and their whole argumentation, for all its false brilliance, fell to pieces. "They do it 'smartly,'" Bohr commented, "but what counts is to do it right."

The refutation of Einstein's criticism does not add any new element to the conception of complementarity, but it is of great importance in laying bare a very deep-lying opposition between Bohr's general philosophical attitude and the still widespread habits of thought belonging to a glorious but irrevocably bygone stage in the evolution of science. Physical concepts, Einstein used to say, are "free creations of the mind." In the case under debate, the "criterion of reality" he proposed has very much this character, and it turns out to yield a striking illustration of the pitfalls to which one may be exposed by such arbitrary constructions of concepts. In spite of its apparent clarity, the criterion in question contains in fact a very essential ambiguity, hidden in the seemingly harmless restriction "without disturbing the system." To disclose this ambiguity, however, it is necessary to renounce any pretension to impose upon nature our own preconceived notion of what "elements of reality" ought to be, and humbly take guidance, as Bohr exhorts us to do, in what we can learn from nature herself.

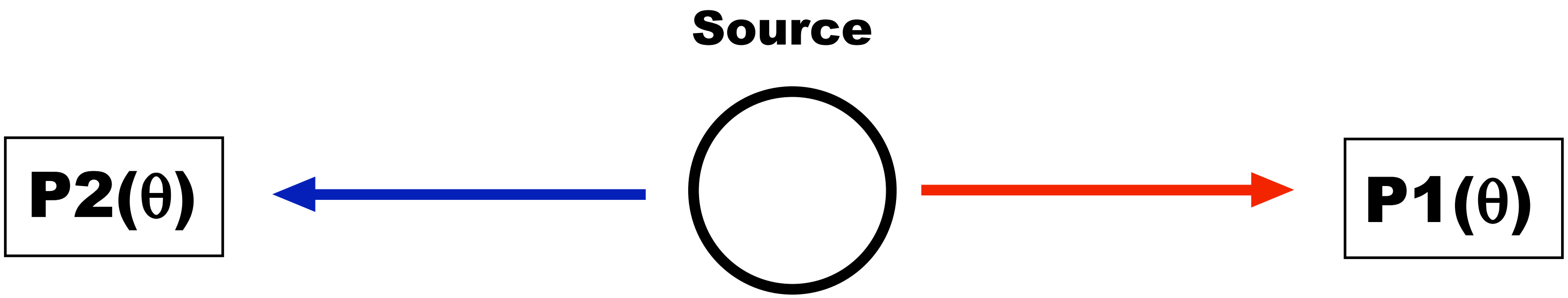
……

When one realizes the fundamental nature of the issue at stake, it becomes easier to understand the state of exaltation in which Bohr accomplished this work. The writing of his reply, its typing, polishing, retyping and sending off to print did not take more than six weeks—an astonishing speed when one knows how slow his usual pace was. It was impressive to watch him thus at the height of his powers, in utmost concentration and unrelenting effort to attain clarity through painstaking scrutiny of every detail—true as ever to his favourite Schiller aphorism "Nur die Fülle führt zur Klarheit." He was particularly well served on this occasion by his uncommon ability to go into the opponent's views, dissect his arguments and turn them to the advantage of the truth. In this, however, he always proceeded with complete openmindedness, and only rejoiced in victory if in winning it he had also deepened his own insight into the problem.

The contest about the completeness of the quantal description of physical phenomena was the last clash between the two giants. The confrontation of their diverging conceptions of the nature of scientific knowledge had now reached the limits set by confining it to the problems of the physical world. That there was no hope of carrying it further was soon made clear by Einstein himself, who commented on Bohr's position that it was logically possible, but "so very contrary to my scientific instinct that I cannot forego the search for a more complete conception." Bohr was very unhappy about this deadlock, for he admired Einstein precisely for the way in which he had laid stress on the epistemological aspects of classical physics and, at an early stage, of quantum theory also. In fact, Einstein's approach to these problems had been so closely similar to his own, and such a source of inspiration to him, that he found Einstein's later lack of understanding doubly disheartening. On the other hand, he had good reason to look back with satisfaction on a controversy which had put to such severe test his own conception of the complementarity of physical phenomena, and even, in this last dispute about an alleged "criterion of reality," the underlying general ideas he had formed of the most fundamental aspects of human knowledge and man's position in the universe.

# The Bohm-EPR Experiment

## Measure the polarization/spin



**Source**

**P2(θ)**

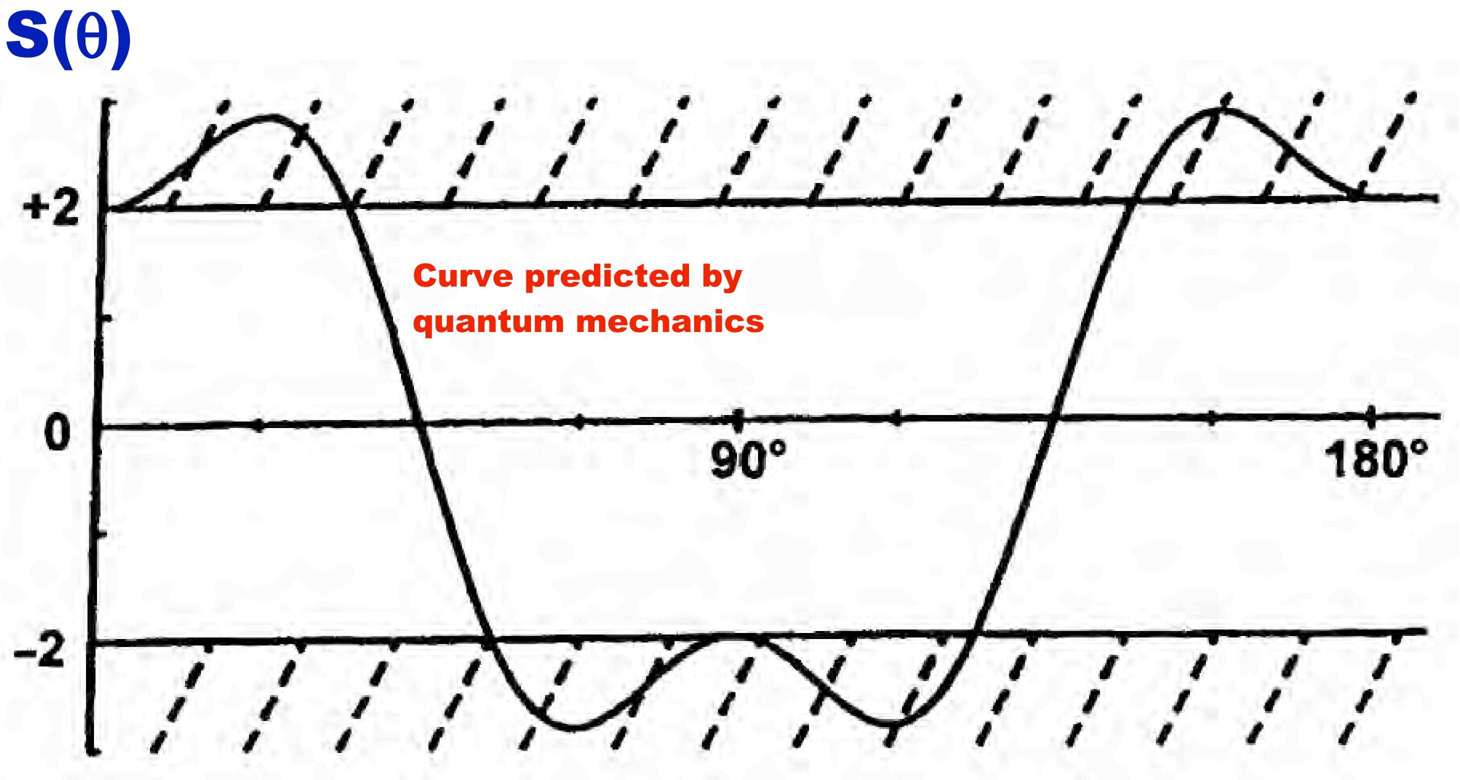
**P1(θ)**

**Measure P1**
**=> know P2**
**if you**
**measure it**
**along the**
**same axis**

**zero total momentum and**
**zero total angular momentum**

**Measure P1**
**=> know P2**
**if you**
**measure it**
**along the**
**same axis**

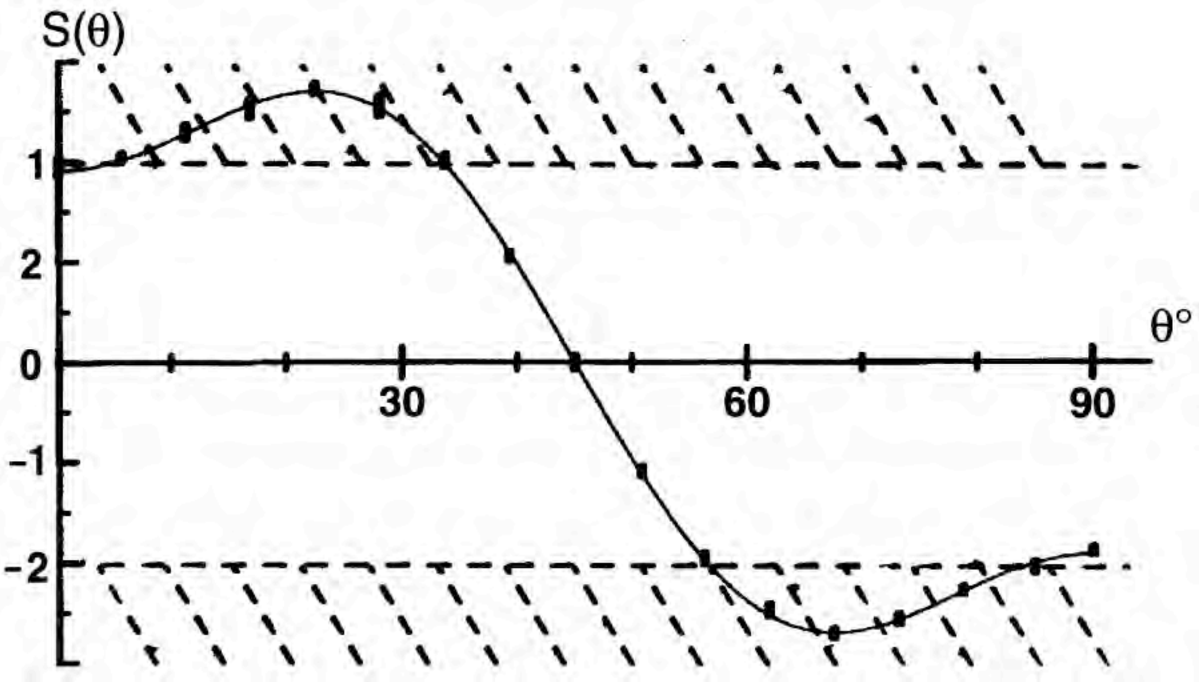
# Bell’s Prediction



Curve predicted by quantum mechanics

**Bell’s Theorem => all local realistic theories stay inside the interval [-2, +2]**

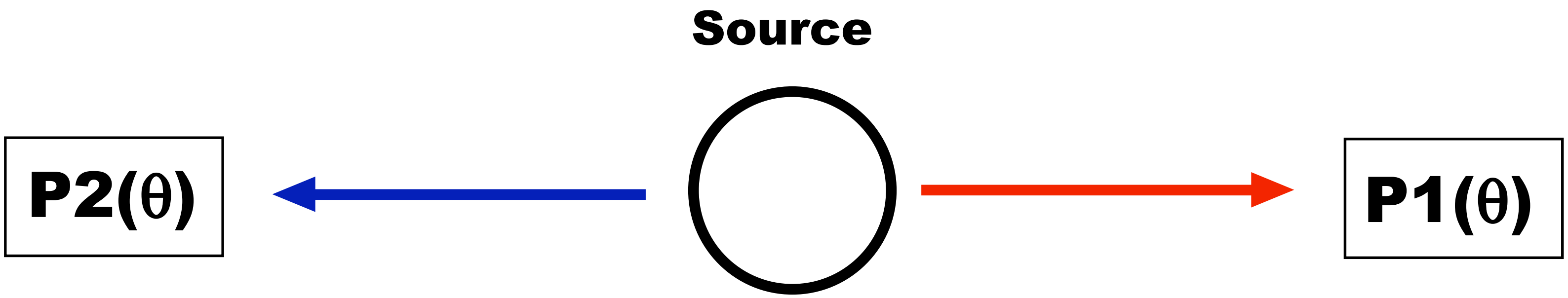
# Aspect’s Results



**Local realism is violated by 22 sigma !!!**

$$\frac{N_{++}(\vec{a},\vec{b}) + N_{--}(\vec{a},\vec{b}) - N_{+-}(\vec{a},\vec{b}) - N_{-+}(\vec{a},\vec{b})}{N_{++}(\vec{a},\vec{b}) + N_{--}(\vec{a},\vec{b}) + N_{+-}(\vec{a},\vec{b}) + N_{-+}(\vec{a},\vec{b})}$$

# My Personal Resolution of
# The Bohm-EPR Experiment



Source

P2(θ)

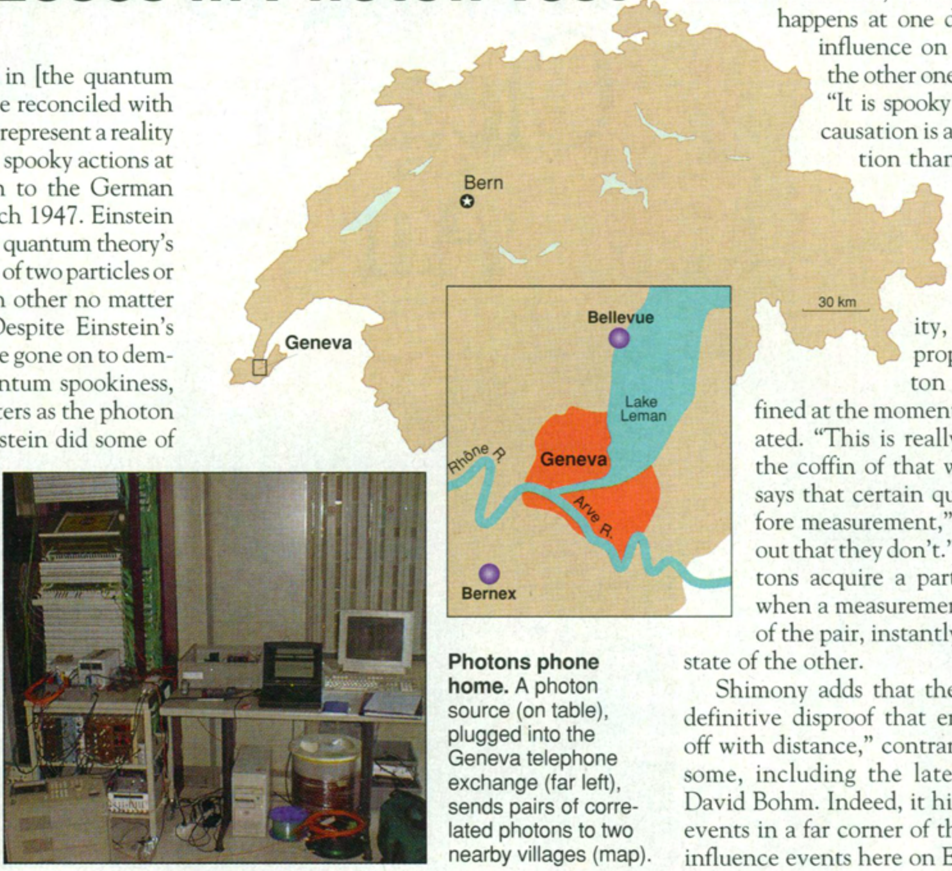
P1(θ)

**Both particles are in both detectors, so measuring in either detector collapses the state vector for both particles.**

QUANTUM MECHANICS

# Quantum Spookiness Wins, Einstein Loses in Photon Test

"I cannot seriously believe in [the quantum theory] because it cannot be reconciled with the idea that physics should represent a reality in time and space, free from spooky actions at a distance," wrote Einstein to the German physicist Max Born in March 1947. Einstein was particularly bothered by quantum theory's oddball claim that the states of two particles or photons can influence each other no matter how far apart they are. Despite Einstein's misgivings, researchers have gone on to demonstrate the reality of quantum spookiness, and now—just 140 kilometers as the photon flies from Bern, where Einstein did some of his greatest research—a Swiss group has provided the best demonstration yet of quantum "action at a distance."



**Photons phone home.** A photon source (on table), plugged into the Geneva telephone exchange (far left), sends pairs of correlated photons to two nearby villages (map).

The new result, announced earlier this month at a quantum computation workshop in Turin, Italy, and hailed as "very important" by Boston University theorist Abner Shimony, shows that links between quantum entities persist over distances of up to several kilometers. Some theorists have speculated that these correlations would weaken with distance, says another quantum mechanics expert, John Rarity of the Defence Evaluation and Research Agency in Malvern, United Kingdom. But in the Swiss result, "we've now got to 10 kilometers' separation, and quantum mechanics is apparently still holding."

As early as 1926, the Austrian physicist Erwin Schrödinger, a father of quantum theory, pointed out that the theory allows a single, pure quantum state—a particular polarization, for example—to be spread across two objects, such as a pair of simultaneously created photons. In the lingo of quantum mechanics, the photons are "entangled," and they remain entangled even when they fly apart. Then quantum theory predicts that a measurement on one photon will influence the outcome of a measurement on its distant twin. This is the action at a distance that Einstein detested, as it appears to be at odds with the prohibition of faster-than-light effects in his theory of special relativity. But short-range laboratory experiments, notably those of Alain Aspect and his colleagues in Paris in 1982, have backed the quantum claim.

With a little help from Swiss Telecom, Nicolas Gisin and his group at the University of Geneva have now demonstrated quantum action at a distance on a large scale by turning the countryside around Geneva into a giant quantum laboratory. Gisin's team created pairs of entangled photons, using a specially constructed, suitcase-sized generator in central Geneva, and sent them through fiber-optic lines to the two small villages of Bellevue and Bernex, 10.9 kilometers apart, where the streams of photons were analyzed and counted.

The total energy of each entangled pair is fixed, but the energy of each photon in a pair can vary within a narrow range. An analyzer is effectively an energy filter, offering each photon a random choice of either being counted, or of being lost from the experiment. Each photon makes its choice depending on its energy and the setting of the analyzer, explains Wolfgang Tittel, one of Gisin's colleagues. When the photon counts were relayed to Geneva via a second fiber-optic system and compared, they turned out to be correlated. Each photon in a pair knows what its distant partner does, and does the same thing.

"Even if you change a [setting] only on one end, it has an influence on what happens on the other end," Gisin says. "There is indeed spooky action at a distance, in the sense that what happens at one detector has some influence on what happens at the other one." Adds Shimony, "It is spooky in the sense that causation is a more subtle relation than we had ever realized. I think Einstein loses on this point."

The implication, says Rarity, is that certain properties of the photon twins aren't defined at the moment the pairs are created. "This is really another nail in the coffin of that world view which says that certain quantities exist before measurement," he says. "It turns out that they don't." Instead, the photons acquire a particular state only when a measurement is made on one of the pair, instantly determining the state of the other.

Shimony adds that the result is "pretty definitive disproof that entanglement falls off with distance," contrary to proposals by some, including the late British theorist David Bohm. Indeed, it hints that quantum events in a far corner of the universe might influence events here on Earth.

Gisin points to more down-to-earth implications for telecommunications, implications presumably not lost on Swiss Telecom: "If these correlations hold over very long distances … then they could be exploited for a variety of applications, especially quantum cryptography." Contrary to Einstein's fears, quantum correlations can't be exploited to transfer information faster than light. "You cannot control what will be transmitted; therefore, you can't send an SOS message, or any other … message, by means of quantum correlation," says Shimony. But these correlations could in principle create two perfect copies of random digits in two places. These could serve as the key to some code.

Not only would the transmission be error free; it might also be uncrackable. "Any eavesdropper who tried to eavesdrop these quantum channels would break the correlation," explains Gisin; the two parties could detect the intrusion by comparing parts of the received signals, which should be identical. Quantum spookiness might be just the thing to foil a spook.

**–Andrew Watson**

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*Andrew Watson is a science writer in Norwich, U.K.*

From the newspaper:

# It’s physics, Jim, but not as we know it

Scientists have set a new record in sending information through thin air using the revolutionary technology of quantum teleportation---although Mr. Spock may have to wait a little longer for a Scotty to beam him up with it.

“Einstein said that if quantum mechanics is right, then the world is crazy … Well, Einstein was right. The world is crazy.”

--Greenberger

The world isn’t crazy.
The world is non-local.
We are crazy to insist that it be local.

The job of a scientist is to listen carefully to nature, not to tell nature how to behave.

I am not happy with all the analyses that go with just classical theory, because nature is not classical, dammit, and if you want to make a simulation of nature you’d better make it quantum mechanical and by golly it is a wonderful problem.

--Feynman

**Bohr:** You showed that space and time depend on the observer.  So why can't quantum mechanics show that reality---at least dynamics---depends on observation?

**Einstein:** A good joke should not be told twice.

# NR Path Integrals

## Two formulations of classical mechanics

**Hamiltonian formulation**

$H = T + V$

=> Schrodinger equation formulation of QM

**Lagrangian formulation**

$L = T - V$

=> Path integral formulation of QM

## Ten good things about the path integral formulation

## One bad thing about the path integral formulation

then you can start with the Lagrangian and then create a Hamiltonian and work out the quantum mechanics, more or lessuniquely. But this thing (1) involves the key variables, positions, at two different times and therefore, it was not obvious what to do to make the quantum-mechanical analogue.

I tried - I would struggle in various ways. One of them was this; if I had harmonic oscillators interacting with a delay in time, I could work out what the normal modes were and guess that the quantum theory of the normal modes was the same as for simple oscillators and kind of work my way back in terms of the original variables. I succeeded in doing that, but I hoped then to generalize to other than a harmonic oscillator, but I learned to my regret something, which many people have learned. The harmonic oscillator is too simple; very often you can work out what it should do in quantum theory without getting much of a clue as to how to generalize your results to other systems.

So that didn't help me very much, but when I was struggling with this problem, I went to a beer party in the Nassau Tavern in Princeton. There was a gentleman, newly arrived from Europe (Herbert Jehle) who came and sat next to me. Europeans are much more serious than we are in America because they think that a good place to discuss intellectual matters is a beer party. So, he sat by me and asked, « what are you doing » and so on, and I said, « I'm drinking beer. » Then I realized that he wanted to know what work I was doing and I told him I was struggling with this problem, and I simply turned to him and said, ((listen, do you know any way of doing quantum mechanics, starting withaction - where the action integral comes into the quantum mechanics? » « No », he said, « but Dirac has a paper in which the Lagrangian, at least, comes into quantum mechanics. I will show it to you tomorrow. »

Next day we went to the Princeton Library, they have little rooms on the side to discuss things, and he showed me this paper. What Dirac said was the following : There is in quantum mechanics a very important quantity which carries the wave function from one time to another, besides the differential equation but equivalent to it, a kind of a kernal, which we might call $K(x', x)$, which carries the wave function $\psi$ *(x)* known at time *t*, to the wave function y *(x')* at time, *t* +ε. Dirac points out that this function *K* was *analogous* to the quantity in classical mechanics that you would calculate if you took the exponential of $i\varepsilon$, multiplied by the Lagrangian $L(\dot{x}, x)$ imagining that these two positions *x,x'* corresponded *t* and t +ε. In other words,

$$K(x', x) \text{ is analogous to } e^{i\varepsilon L(\frac{x'-x}{\varepsilon}, x)/\hbar}$$

Professor Jehle showed me this, I read it, he explained it to me, and I said, « what does he mean, they are analogous; what does that mean, *analogous?* What is the use of that? » He said, « you Americans ! You always want to find a use for everything! » I said, that I thought that Dirac must mean that they were equal. « No », he explained, « he doesn't mean they are equal. » « Well », I said, « let's see what happens if we make them equal. »

So I simply put them equal, taking the simplest example where the Lagrangian is $^1/_2 Mx^2 - V(x)$ but soon found I had to put a constant of proportionality A in, suitably adjusted. When I substituted $Ae^{i\varepsilon L/\hbar}$ for $K$ to get

$$\psi(x', t+\varepsilon) = \int A \exp\left[\frac{i\varepsilon}{\hbar} L\left(\frac{x'-x}{\varepsilon}, x\right)\right] \psi(x, t)\, dx \qquad (3)$$

and just calculated things out by Taylor series expansion, out came the Schrödinger equation. So, I turned to Professor Jehle, not really understanding, and said, « well, you see Professor Dirac meant that they were proportional. » Professor Jehle's eyes were bugging out - he had taken out a little notebook and was rapidly copying it down from the blackboard, and said, « no, no,this is an important discovery. You Americans are always trying to find out how something can be used. That's a good way to discover things! » So, I thought I was finding out what Dirac meant, but, as a matter of fact, had made the discovery that what Dirac thought was analogous, was, in fact, equal. I had then, at least, the connection between the Lagrangian and quantum mechanics, but still with wave functions and infinitesimal times.

It must have been a day or so later when I was lying in bed thinking about these things, that I imagined what would happen if I wanted to calculate the wave function at a finite interval later.

I would put one of these factors $e^{i\varepsilon L}$ in here, and that would give me the wave functions the next moment, $t+\varepsilon$ and then I could substitute that back into (3) to get another factor of $e^{i\varepsilon L}$ and give me the wave function the next moment, $t + 2\varepsilon$, and so on and so on. In that way I found myself thinking of a large number of integrals, one after the other in sequence. In the integrand was the product of the exponentials, which, of course, was the exponential of the sum of terms like εL. Now, $L$ is the Lagrangian and ε is like the time interval dt, so that if you took a sum of such terms, that's exactly like an integral. That's like Riemann's formula for the integral $\int L dt$, you just take the value at each point and add them together. We are to take the limit as ε - 0, of course. Therefore, the connection between the wave function of one instant and the wave function of another instant a finite time later could be obtained by an infinite number of integrals, (because ε goes to zero, of course) of ex-

### RICHARD P. FEYNMAN

# The development of the space-time view of quantum electrodynamics

*Nobel Lecture, December 11, 1965*

We have a habit in writing articles published in scientific journals to make the work as finished as possible, to cover all the tracks, to not worry about the blind alleys or to describe how you had the wrong idea first, and so on. So there isn't any place to publish, in a dignified manner, what you actually did in order to get to do the work, although, there has been in these days, some interest in this kind of thing. Since winning the prize is a personal thing, I thought I could be excused in this particular situation, if I were to talk personally about my relationship to quantum electrodynamics, rather than to discuss the subject itself in a refined and finished fashion. Furthermore, since there are three people who have won the prize in physics, if they are all going to be talking about quantum electrodynamics itself, one might become bored with the subject. So, what I would like to tell you about today are the sequence of events, really the sequence of ideas, which occurred, and by which I finally came out the other end with an unsolved problem for which I ultimately received a prize.

I realize that a truly scientific paper would be of greater value, but such a paper I could publish in regular journals. So, I shall use this Nobel Lecture as an opportunity to do something of less value, but which I cannot do elsewhere. I ask your indulgence in another manner. I shall include details of anecdotes which are of no value either scientifically, nor for understanding the development of ideas. They are included only to make the lecture more entertaining.

I worked on this problem about eight years until the final publication in 1947. The beginning of the thing was at the Massachusetts Institute of Technology, when I was an undergraduate student reading about the known physics, learning slowly about all these things that people were worrying about, and realizing ultimately that the fundamental problem of the day was that the quantum theory of electricity and magnetism was not completely satisfactory. This I gathered from books like those of Heitler and Dirac. I was inspired by the remarks in these books; not by the parts in which everything was proved and demonstrated carefully and calculated, because I couldn't

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As a by-product of this same view, I received a telephone call one day at the graduate college at Princeton from Professor Wheeler, in which he said, "Feynman, I know why all electrons have the same charge and the same mass" "Why?" "Because, they are all the same electron!" And, then he explained on the telephone, "suppose that the world lines which we were ordinarily considering before in time and space - instead of only going up in time were a tremendous knot, and then, when we cut through the knot, by the plane corresponding to a fixed time, we would see many, many world lines and that would represent many electrons, except for one thing. If in one section this is an ordinary electron world line, in the section in which it reversed itself and is coming back from the future we have the wrong sign to the proper time - to the proper four velocities - and that's equivalent to changing the sign of the charge, and, therefore, that part of a path would act like a positron." "But, Professor", I said, "there aren't as many positrons as electrons." "Well, maybe they are hidden in the protons or something", he said. I did not take the idea that all the electrons were the same one from him as seriously as I took the observation that positrons could simply be represented as electrons going from the future to the past in a back section of their world lines. That, I stole!

action principle in classical mechanics. I was learning from these discussions with Feynman that the integrated action of classical theory, in a sense more precise than ever before appreciated, is—apart from a universal factor, $\hbar=1.054\times10^{-27}$ g $\mathrm{cm}^2$/sec—only another name for the phase of the probability amplitude associated with the classical history.

Visiting Einstein one day, I could not resist telling him about Feynman's new way to express quantum theory. "Feynman has found a beautiful picture to understand the probability amplitude for a dynamical system to go from one specified configuration at one time to another specified configuration at a later time. He treats on a footing of absolute equality every conceivable history that leads from the initial state to the final one, no matter how crazy the motion in between. The contributions of these histories differ not at all in amplitude, only in phase. And the phase is nothing but the classical action integral, apart from the Dirac factor, $\hbar$. This prescription reproduces all of standard quantum theory. How could one ever want a simpler way to see what quantum theory is all about! Doesn't this marvelous discovery make you willing to accept quantum theory, Professor Einstein?" He replied in a serious voice, "I still cannot believe that God plays dice. But maybe," he smiled, "I have earned the right to make my mistakes."

Undeterred I persisted, and still do, in regarding Feynman's PhD thesis as marking a moment when quantum theory for the first time became simpler than classical theory. I began my upcoming graduate course in classical mechanics with Feynman's idea that the microscopic point particle makes its way from *A* to *B*, not by a unique history, but by pursuing every conceivable history with democratically equal probability amplitude. Only out of Huygens's principle, only out of the concept of constructive and destructive interference between these contributions—and this only in an approximation—could one understand the existence of the classical history. Feynman sat there and took the course notes, of which I still have a mimeographed copy. On many a puzzling point he helped us both to find new light by discussions in class and out.

## Any Career for the Kid from Far Rockaway?

While Richard was working on his thesis, his father, Melville Arthur Feynman, sales manager for a medium-sized uniform company, made a brief call on me in my office one day. How important he had been in

## Chapter I.2

# Path Integral Formulation of Quantum Physics

### The professor's nightmare: a wise guy in the class

As I noted in the preface, I know perfectly well that you are eager to dive into quantum field theory, but first we have to review the path integral formalism of quantum mechanics. This formalism is not universally taught in introductory courses on quantum mechanics, but even if you have been exposed to it, this chapter will serve as a useful review. The reason I start with the path integral formalism is that it offers a particularly convenient way of going from quantum mechanics to quantum field theory. I will first give a heuristic discussion, to be followed by a more formal mathematical treatment.

Perhaps the best way to introduce the path integral formalism is by telling a story, certainly apocryphal as many physics stories are. Long ago, in a quantum mechanics class, the professor droned on and on about the double-slit experiment, giving the standard treatment. A particle emitted from a source $S$ (Fig. I.2.1) at time $t = 0$ passes through one or the other of two holes, $A_1$ and $A_2$, drilled in a screen and is detected at time $t = T$ by a detector located at $O$. The amplitude for detection is given by a fundamental postulate of quantum mechanics, the superposition principle, as the sum of the amplitude for the particle to propagate from the source $S$ through the hole $A_1$ and then onward to the point $O$ and the amplitude for the particle to propagate from the source $S$ through the hole $A_2$ and then onward to the point $O$.

Suddenly, a very bright student, let us call him Feynman, asked, "Professor, what if we drill a third hole in the screen?" The professor replied, "Clearly, the amplitude for the particle to be detected at the point $O$ is now given by the sum of three amplitudes, the amplitude for the particle to propagate from the source $S$ through the hole $A_1$ and then onward to the point $O$, the amplitude for the particle to propagate from the source $S$ through the hole $A_2$ and then onward to the point $O$, and the amplitude for the particle to propagate from the source $S$ through the hole $A_3$ and then onward to the point $O$."

The professor was just about ready to continue when Feynman interjected again, "What if I drill a fourth and a fifth hole in the screen?" Now the professor is visibly

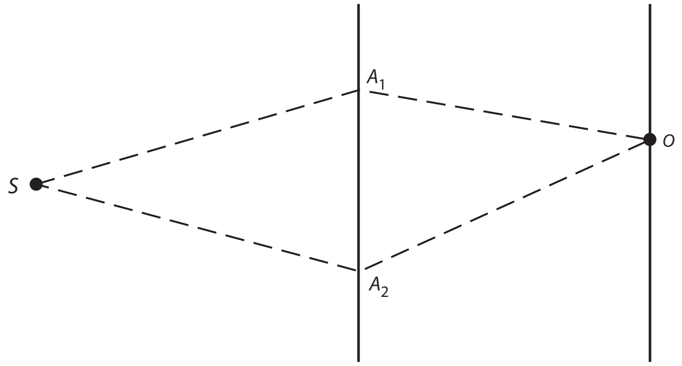

**Figure I.2.1**

losing his patience: "All right, wise guy, I think it is obvious to the whole class that we just sum over all the holes."

To make what the professor said precise, denote the amplitude for the particle to propagate from the source $S$ through the hole $A_i$ and then onward to the point $O$ as $\mathcal{A}(S \to A_i \to O)$. Then the amplitude for the particle to be detected at the point $O$ is

$$\mathcal{A}(\text{detected at } O) = \sum_i \mathcal{A}(S \to A_i \to O) \tag{1}$$

But Feynman persisted, "What if we now add another screen (Fig. I.2.2) with some holes drilled in it?" The professor was really losing his patience: "Look, can't you see that you just take the amplitude to go from the source $S$ to the hole $A_i$ in the first screen, then to the hole $B_j$ in the second screen, then to the detector at $O$, and then sum over all $i$ and $j$?"

Feynman continued to pester, "What if I put in a third screen, a fourth screen, eh? What if I put in a screen and drill an infinite number of holes in it so that the

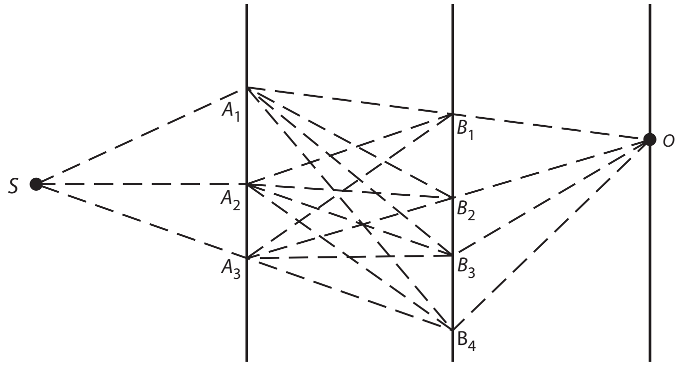

**Figure I.2.2**

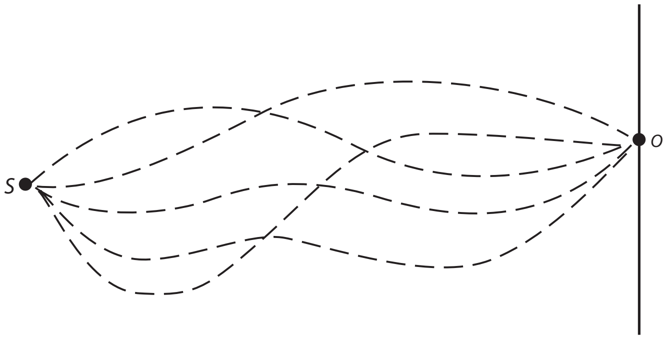

**Figure I.2.3**

screen is no longer there?" The professor sighed, "Let's move on; there is a lot of material to cover in this course."

But dear reader, surely you see what that wise guy Feynman was driving at. I especially enjoy his observation that if you put in a screen and drill an infinite number of holes in it, then that screen is not really there. Very Zen! What Feynman showed is that even if there were just empty space between the source and the detector, the amplitude for the particle to propagate from the source to the detector is the sum of the amplitudes for the particle to go through each one of the holes in each one of the (nonexistent) screens. In other words, we have to sum over the amplitude for the particle to propagate from the source to the detector following all possible paths between the source and the detector (Fig. I.2.3).

$$\mathcal{A}(\text{particle to go from } S \text{ to } O \text{ in time } T) =$$
$$\sum_{(\text{paths})} \mathcal{A}\big(\text{particle to go from } S \text{ to } O \text{ in time } T \text{ following a particular path}\big) \quad (2)$$

Now the mathematically rigorous will surely get anxious over how $\sum_{(\text{paths})}$ is to be defined. Feynman followed Newton and Leibniz: Take a path (Fig. I.2.4), approximate it by straight line segments, and let the segments go to zero. You can see that this is just like filling up a space with screens spaced infinitesimally close to each other, with an infinite number of holes drilled in each screen.

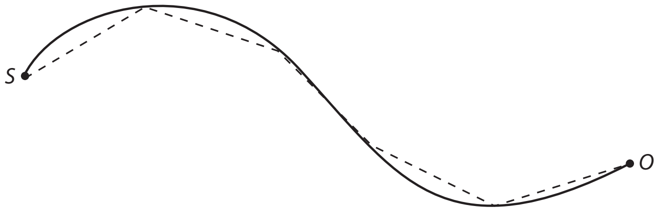

**Figure I.2.4**

# NR Path Integrals

## Two formulations of classical mechanics

**Hamiltonian formulation**

**H = T + V**

**=> Schrodinger equation formulation of QM**

**Lagrangian formulation**

**L = T - V**

**=> Path integral formulation of QM**

## Ten good things about the path integral formulation

## One bad thing about the path integral formulation

# Michio Kaku lists seven advantages of the path integral formulation of quantum mechanics:

1. The path integral formalism yields a simple, covariant quantization of complicated systems with constraints, such as gauge theories. While calculations with the canonical approach are often prohibitively tedious, the path integral method yields the results rather simply, vastly reducing the amount of work.

2. The path integral formalism allows one to go easily back and forth between the other formalisms, such as the canonical or the various covariant approaches. In the path integral approach, these various formulations are nothing but different choices of gauge.

3. The path integral formalism is based intuitively on the fundamental principles of quantum mechanics. Quantization prescriptions, which may seem rather arbitrary in the operator formalism, have a simple physical interpretation in the path integral formalism.

4. The path integral formalism can be used to calculate nonperturbative as well as perturbative results.

5. The path integral formalism is based on c-number fields, rather than q-number operators. Hence, the formalism is much easier to manipulate.

6. At present, there are a few complex systems with constraints that can only be quantized in the path integral formalism.

7. Renormalization theory is much easier to express in terms of path integrals.

M. Kaku QFT: A modern introduction (1993)

## Path integral gives us insight into the extremely nonlocal nature of quantum mechanics.

**So, why not teach the path integral method from the very beginning?**

**Path integral is much more difficult than Schrodinger equation for simple NRQM problems, viz., hydrogen atom and spin.**

**On the other hand, easier or comparable to the canonical method for relativistic problems.**