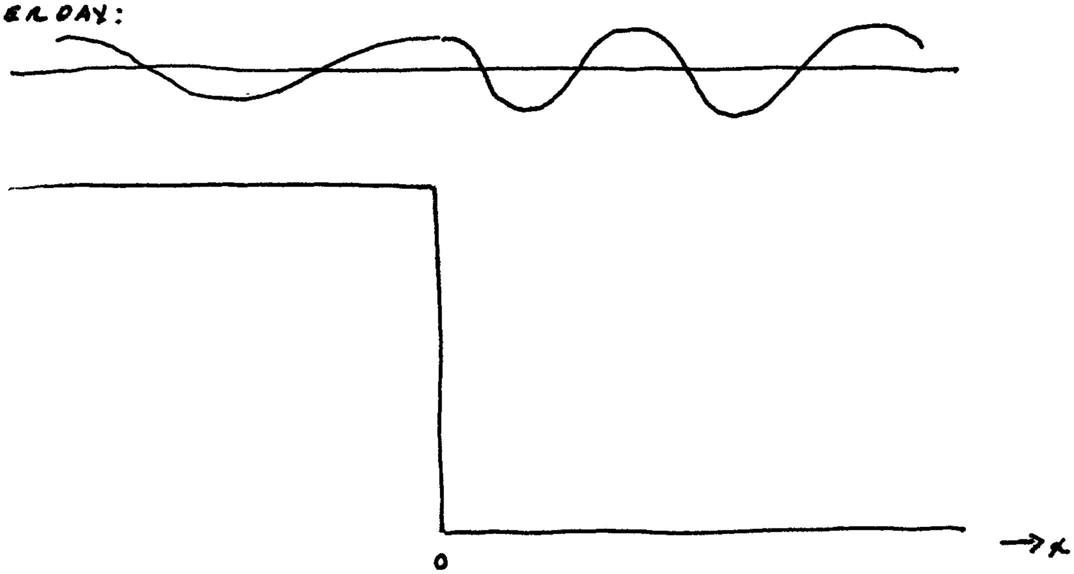
LECTURE 14 7-16-09

YESTERDAY:



$$\Psi_1(x) = Ae^{ik_1x} + Be^{-ik_1x} \qquad \Psi_2(x) = Ce^{ik_2x}$$

$$k_1 = \sqrt{2m(E - V_0)/\hbar^2} \qquad k_2 = \sqrt{2mE/\hbar^2}$$

LOWER SPATIAL FREQUENCY

HIGH SPATIAL FREQUENCY

"

SHORT SPATIAL WAVE LENGTH

B.C.'s

Ψ CONTINUOUS $\qquad \Psi_1(x=0) = \Psi_2(x=0)$

$\frac{d\Psi}{dx}$ CONTINUOUS $\qquad \left.\frac{d\Psi_1}{dx}\right|_{x=0} = \left.\frac{d\Psi_2}{dx}\right|_{x=0}$

WAVEFCN CONTINUOUS

$$\psi_1(0) = \psi_2(0)$$

$$A e^{ik_1(0)} + B e^{-ik_1(0)} = C e^{ik_2(0)}$$

$$\boxed{A + B = C}$$

DERIVATIVE CONTINUOUS
SLOPE CONTINUOUS

$$\left.\frac{d\psi_1}{dx}\right|_{x=0} = \left.\frac{d\psi_2}{dx}\right|_{x=0}$$

$$\frac{d}{dx}\left(A e^{ik_1 x} + B e^{-ik_1 x} = C e^{ik_2 x} \right)$$

$$ik_1 A e^{ik_1 x} - ik_1 B e^{-ik_1 x} = ik_2 C e^{ik_2 x}$$

$$e^{ik(0)} = 1$$

$$\boxed{k_1(A - B) = k_2 C}$$

FINITE POTENTIAL => ψ and ψ' CONTINUOUS

$$\psi_1(0) = \psi_2(0)$$

$$A e^{ik_1(0)} + B e^{-ik_1(0)} = C e^{ik_2(0)}$$

$$\boxed{A + B = C}$$

$$\psi_1'(0) = \psi_2'(0)$$

$$ik_1 A e^{0} - ik_1 B e^{0} = ik_2 C e^{0}$$

$$\boxed{k_1(A - B) = k_2 C}$$

=> SOLVE IN TERMS OF A

$$k_1(A - B) = k_2 C = k_2(A + B)$$

$$(k_1 - k_2) A = (k_1 + k_2) B$$

$$\boxed{B = \frac{k_1 - k_2}{k_1 + k_2} A} \qquad \boxed{C = \frac{2k_1}{k_1 + k_2} A}$$

$$B = r A \qquad C = t A$$

r = reflection amplitude

REFLECTION ~~COEFF~~ PROB COEFF $R = |r|^2$ $\quad$ PROB = $|AMP|^2$

$$R = \frac{\text{intensity reflected}}{\text{incident intensity}} = \frac{|B|^2}{|A|^2}$$

$$\boxed{R = \left(\frac{k_1 - k_2}{k_1 + k_2}\right)^2} \quad \frac{|A|^2}{|A|^2}$$

~~TRANSMISSION~~ TRANSMISSION COEFF

t = transmission amplitude

$$T + R = 1 \implies T = 1 - R$$

$$T = 1 - \left(\frac{k_1 - k_2}{k_1 + k_2}\right)^2$$

$$= \left(\frac{k_1 + k_2}{k_1 + k_2}\right)^2 - \left(\frac{k_1 - k_2}{k_1 + k_2}\right)^2$$

$$\boxed{T = \frac{4 k_1 k_2}{(k_1 + k_2)^2}}$$

N.B., IF YOU CALCULATE $\frac{|C|^2}{|A|^2}$ YOU DO NOT

GET THE RIGHT ANSWER $(1-R)$!

BECAUSE

FLUX = (INTENSITY) (VELOCITY)

OKAY FOR R because A and B have the same $(E-V)$

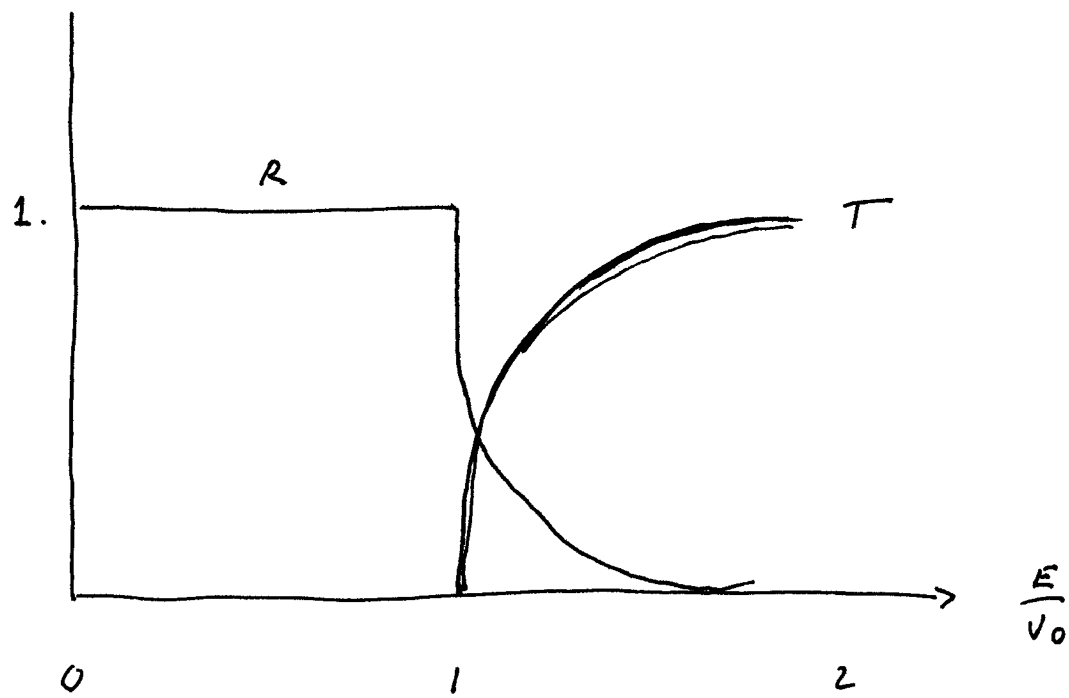
$$R = \frac{v_1 \, |B|^2}{v_1 \, |A|^2} = \frac{|B|^2}{|A|^2}$$

BUT NOT OKAY FOR T unless you include velocity

$$T = \frac{v_2 \, |C|^2}{v_1 \, |A|^2} = \frac{\frac{\hbar k_2}{m} \, |C|^2}{\frac{\hbar k_1}{m} \, |A|^2} = \frac{k_2}{k_1}\left(\frac{2k_1}{k_1+k_2}\right)^2$$

$$v_2 = \frac{p_2}{m} = \frac{\hbar k_2}{m}$$

$$T = \frac{4 k_1 k_2}{(k_1+k_2)^2}$$



$$R = 1 - T = \left[\frac{1 - \sqrt{1 + (V_0/E)}}{1 + \sqrt{1 + (V_0/E)}} \right]^2$$

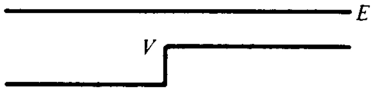
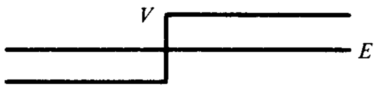
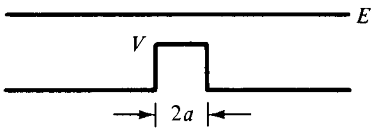
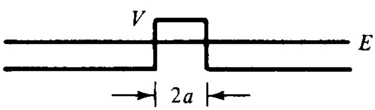
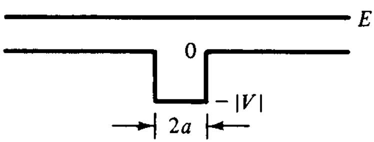
ELECTRONS $r = \dfrac{k_1 - k_2}{k_1 + k_2}$ $R = \left(\dfrac{k_1 - k_2}{k_1 + k_2} \right)^2$ $R = |r|^2$

LIGHT $r = \dfrac{n_1 - n_2}{n_1 + n_2}$

EM WAVES $r = \dfrac{Z_1 - Z_2}{Z_1 + Z_2}$ 377 OHMS

SOUND

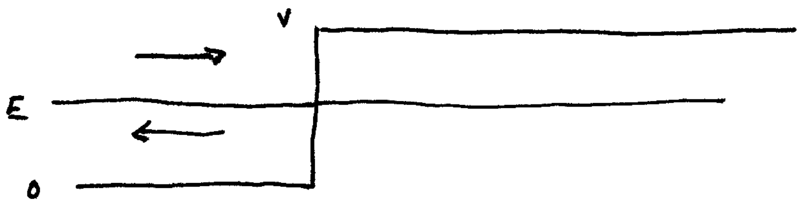
TABLE 7.2 Transmission coefficients for three elementary potential barriers

	$T = \dfrac{4k_2/k_1}{[1 + (k_2/k_1)]^2}$ $\left(\dfrac{k_2}{k_1}\right)^2 = 1 - \dfrac{V}{E}$
	$T = 0, \quad R = 1$
	$\dfrac{1}{T} = 1 + \dfrac{1}{4}\dfrac{V^2}{E(E - V)} \sin^2 (2k_2 a)$ $\dfrac{\hbar^2 k_2{}^2}{2m} = E - V$
	$\dfrac{1}{T} = 1 + \dfrac{1}{4}\dfrac{V^2}{E(V - E)} \sinh^2 (2\kappa a)$ $\dfrac{\hbar^2 \kappa^2}{2m} = V - E$
	$\dfrac{1}{T} = 1 + \dfrac{1}{4}\dfrac{V^2}{E(E + \lvert V \rvert)} \sin^2 (2k_2 a)$ $\dfrac{\hbar^2 k_2{}^2}{2m} = E - V = E + \lvert V \rvert$

The transmission coefficients corresponding to the one-dimensional potential configurations considered above are summarized in Table 7.2.

THE FIVE SIMPLE CASES

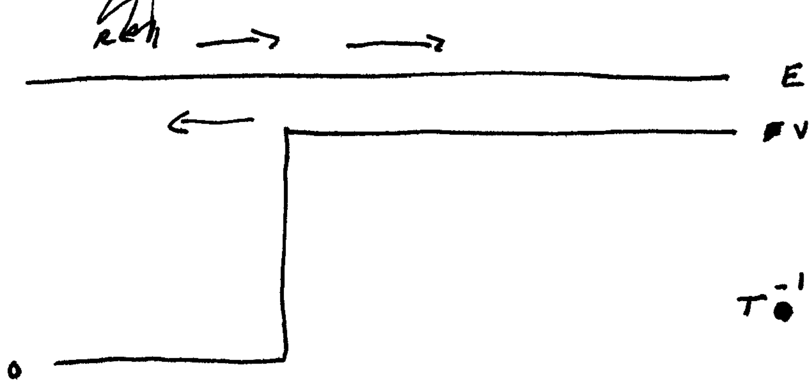
(1)



T=0

R=1

(2)

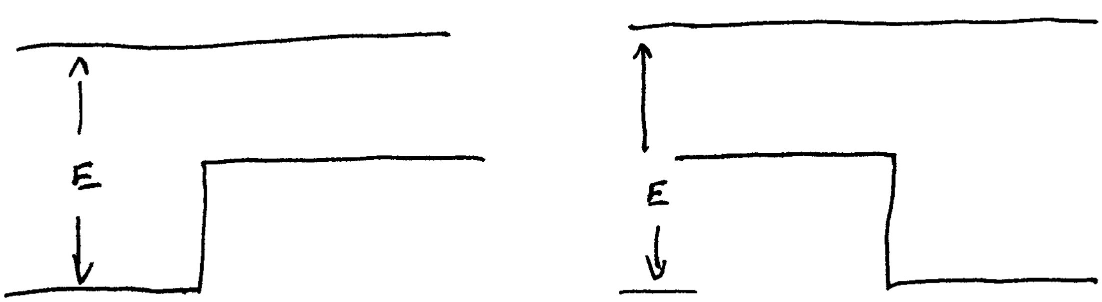


$$T^{-1} = \frac{(k_1 + k_2)^2}{4 k_1 k_2}$$

$$T = \frac{4 k_2 / k_1}{\left[1 + \left(\frac{k_2}{k_1}\right)\right]^2} = \frac{4 \; k_1 k_2}{(k_1 + k_2)^2}$$

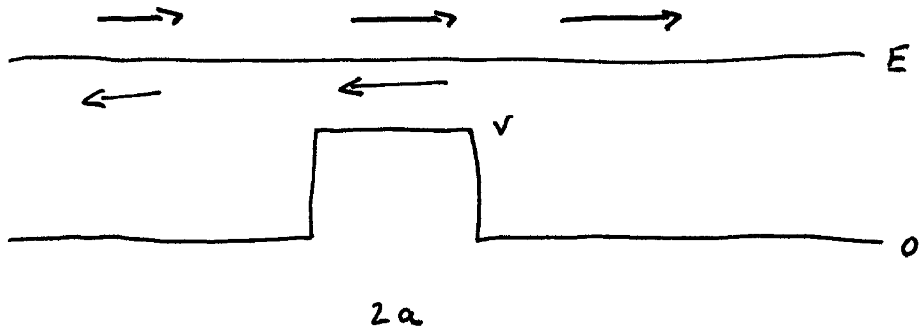
$$\left(\frac{k_2}{k_1}\right)^2 = \left(1 - \frac{V}{E}\right)$$

LIBOFF



$$R_{up} = R_{DOWN}$$

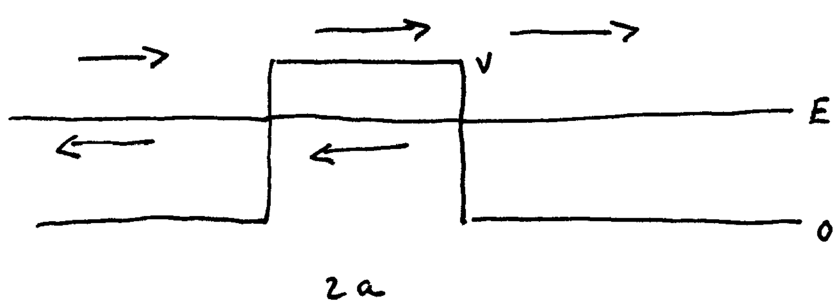
(3)



$$T^{-1} = 1 + \frac{1}{4}\frac{V^2}{E(E-V)}\sin^2(2k_2 a)$$

$$\frac{\hbar^2 k_2^2}{2m} = E - V$$

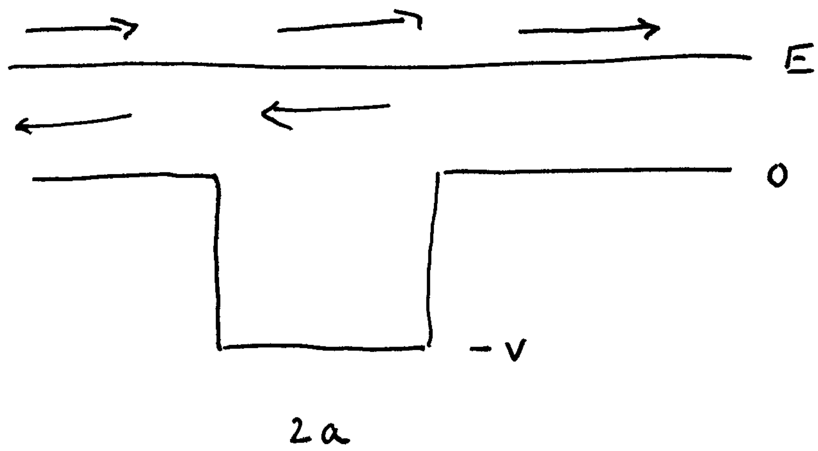
(4)



$$T^{-1} = 1 + \frac{1}{4}\frac{V^2}{E(V-E)}\sinh^2(2\varkappa a)$$

$$\frac{\hbar^2 \varkappa^2}{2m} = V - E$$

(5)



$$T^{-1} = 1 + \frac{1}{4}\,\frac{V^2}{E(E+|V|)}\,\sin^2(2k_2 a)$$

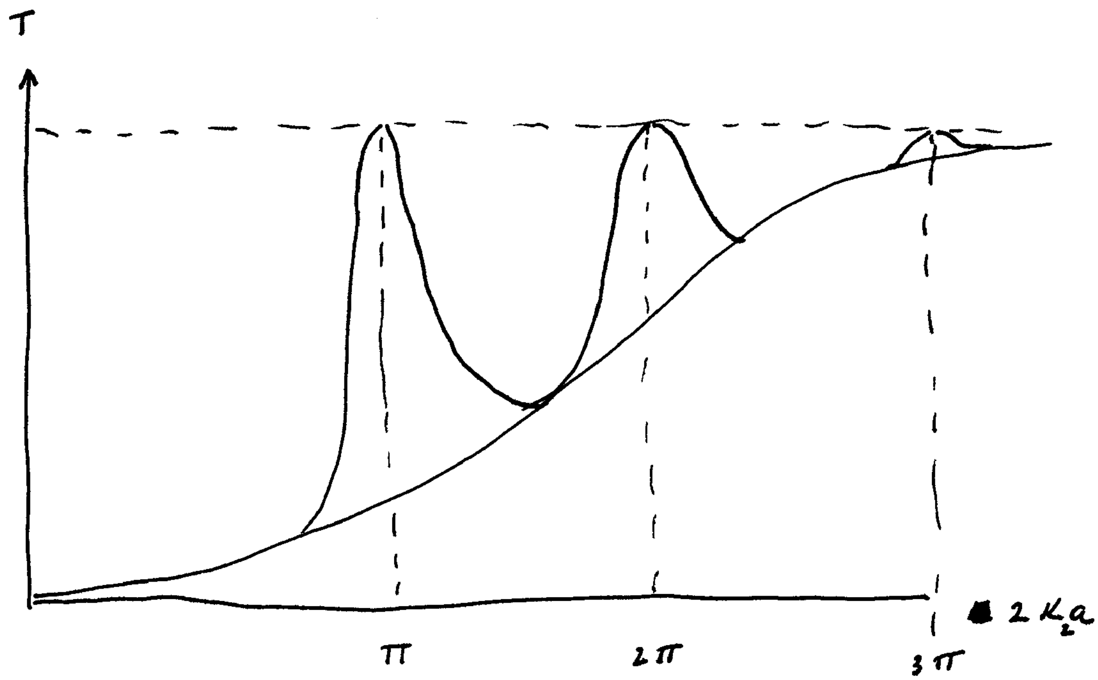
$$\frac{\hbar^2 k_2^2}{2m} = E + |V|$$

RESONANCE EFFECTS

when $\sin(2K_2a) = 0$, $T \to 1$

integral number of half wavelengths inside the well

electrons thru rare gases — RAMSAUER EFFECT



SEMICONDUCTOR EXAMPLE