Physics 227 Summer Quarter 2009

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Required Text: **Mathematical Methods in the Physical Sciences** by Boas

Grading: Two take-home exams

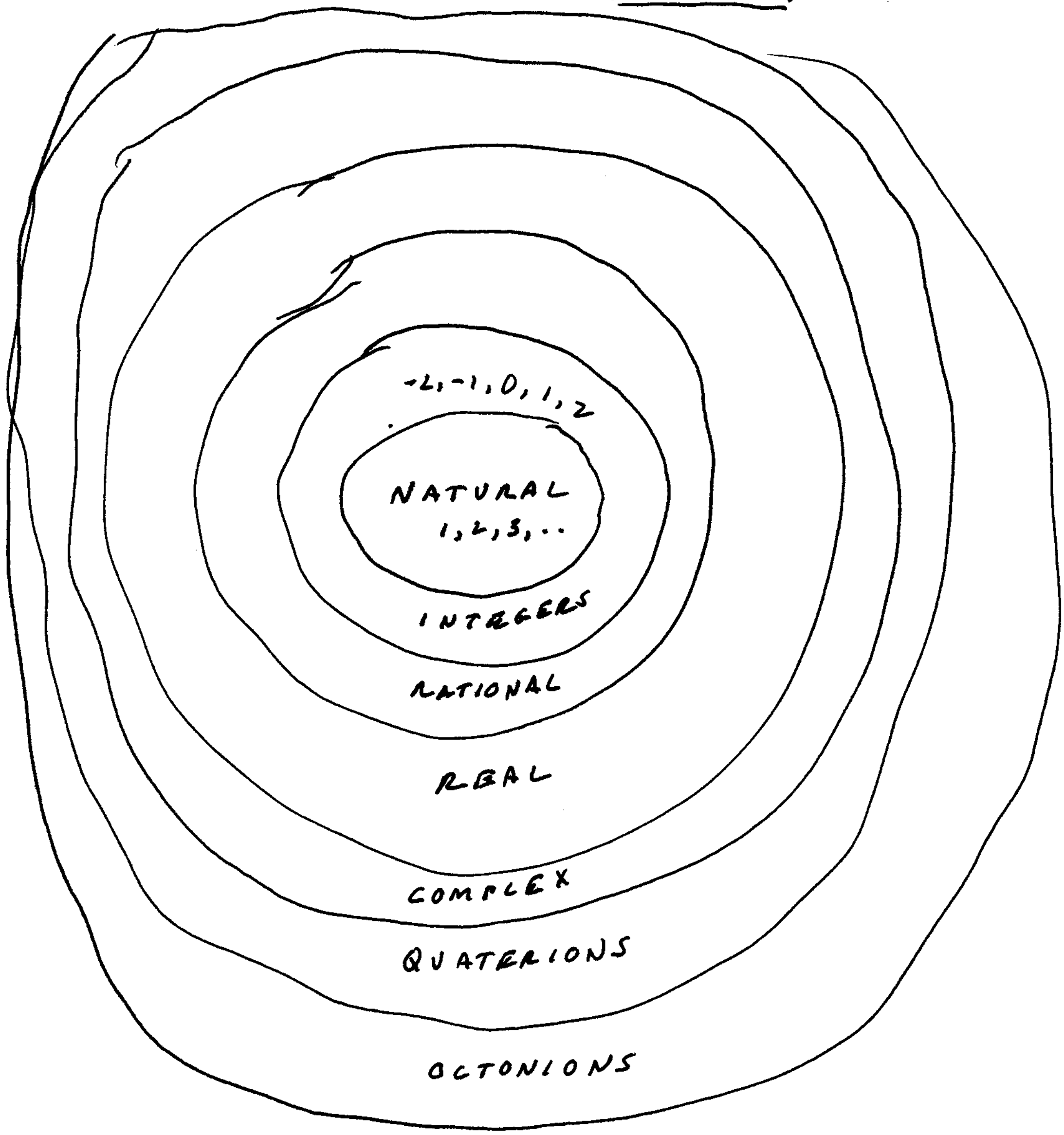
Week of	Topic	Chapter	
June 22	Complex Numbers	2	
	Vector Analysis	6	
	Linear Algebra	3, 10	
	Fourier and Other Series	7	
	Fourier and Other Transforms	15	
Exam	**Exam Posted**	**Exam Due**	**Exam will cover**
Exam 1	July 6	July 20	Weeks 1 to 3
Exam 2	July 27	August 24	Weeks 4 to 8

LECTURE 1 6-22-09

COURSE MECHANICS

COMPLEX NUMBERS

WHAT IS A NUMBER? $\boxed{\text{NAMES}}$



$\boxed{\text{DIMENSIONS}}$

1 REAL
2 COMPLEX
4 QUATERNIONS
8 OCTONIANS

PROPERTIES

ORDERED $a > b$

COMMUTATIVE $ab = ba$

ASSOCIATIVE $(ab)c = a(bc)$

MULTIPLICATIVE INVERSES $ab = 1$ $a = x$ $b = \frac{1}{x}$

NON-ASSOC
NON-COMM
$x_L x = e = x_R x$

	1 REAL	2 COMPLEX	4 QUATERNIONIC	8 OCTONIONS
MI	✓	✓	✓	✓
ASSOC	✓	✓	✓	
COMM	✓	✓		
ORDERED	✓			

WHY NO $d > 8$?

USED UP ALL OF THE PROPERTIES!

OTHER EXOTIC NUMBERS

HYPERCOMPLEX NUMBERS HYPERREAL

TRANSFINITE

SUPERREAL

HYPERREAL

SURREAL

COMPUTABLE

NON-COMPUTABLE

p-adic left hand expansion

polytopic

PHYSICISTS STOP AT COMPLEX

WHY?

QUATERNIONS <=> PAULI MATRICES

$1, i, j, k$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

DO NOT COMMUTE!

NUMBERS <=> GROUPS

PHYSICISTS USE GROUPS

IDEA: GROUPS <=> PHYSICAL SYMMETRIES

REAL NUMBERS = RATIONAL + IRRATIONAL

GEOMETRY

LINE

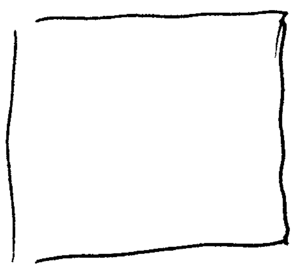
COMPLEX NUMBERS = PAIR OF REAL NUMBERS

$$(a, b)$$

$$a + bi$$

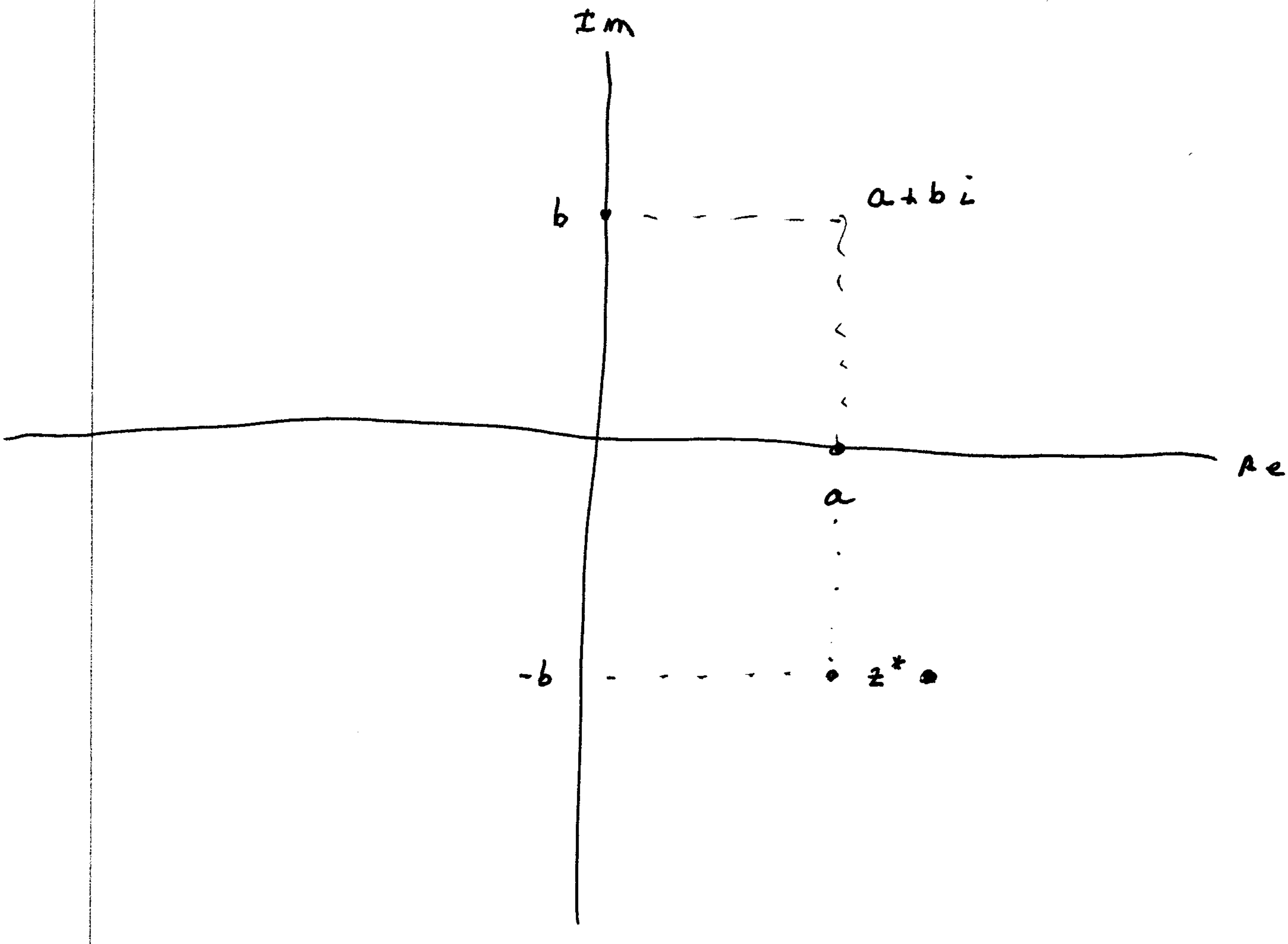
$$re^{i\varphi}$$

GEOMETRY



PLANE

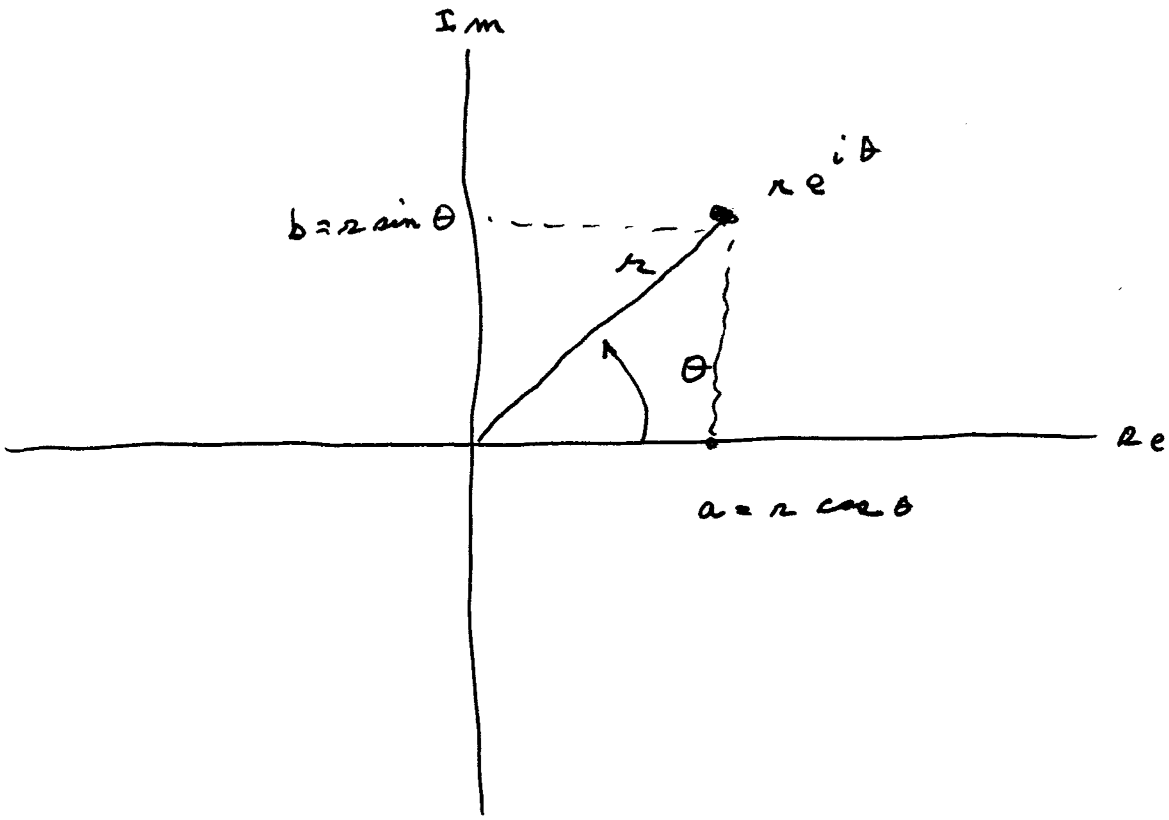
CARTESIAN ~~REPRESENTA~~



$z = a + bi$

$z^* = a - bi$

$|z| = \sqrt{a^2 + b^2} = \sqrt{z z^*}$



$$z = re^{i\theta}$$

$$z^* = re^{-i\theta}$$

ADDITION $\alpha = a + bi$

$\beta = c + di$

$$\alpha + \beta = (a+c) + (b+d)\,i$$

ADD COMPONENTS JUST LIKE ADDING VECTORS

MULTIPLICATION

$$\alpha\beta = (a+bi)(c+di)$$

$$= ac + (ad)i + (bc)i + \underbrace{bd(i)^2}_{-bd}$$

$$= (ac - bd) + (bc + ad)\,i$$

$$\alpha = re^{i\theta}$$

$$\beta = se^{i\varphi}$$

$$\alpha\beta = rse^{i(\theta+\varphi)}$$

POWERS AND ROOTS

$(re^{i\theta})^n = r^n e^{in\theta}$

~~POWERS AND ROOTS~~

$n < 1$ ROOTS integers $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$

ROOTS

$n \geq 1$ POWERS

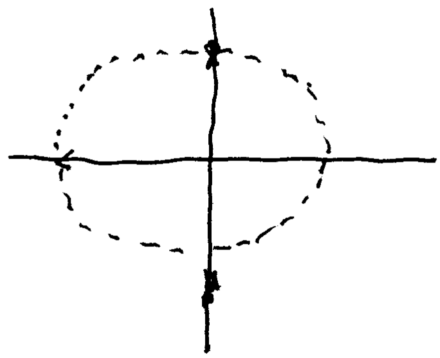
$$\sqrt{\alpha} = ?$$

$$\sqrt{a+bi} = ?$$

$$\sqrt{re^{i\theta}} = \sqrt{r}\, e^{i\theta/2}$$

$$\sqrt[3]{re^{i\theta}} = \sqrt{r}\, e^{i\theta/3}$$

WHEN $r = 1$, ROOTS OF 1



SQUARE ROOTS

$e^{i\theta} \longrightarrow e^{i\theta/2}$

$e^{i\pi} \quad e^{i\pi/2}$

$e^{-i\pi} \quad e^{-i\pi/2}$

$$e^{i\pi/2}\, e^{i\pi/2} = e^{i\pi} = -1$$

$$e^{-i\pi/2}\, e^{-i\pi/2} = e^{-i\pi} = -1$$

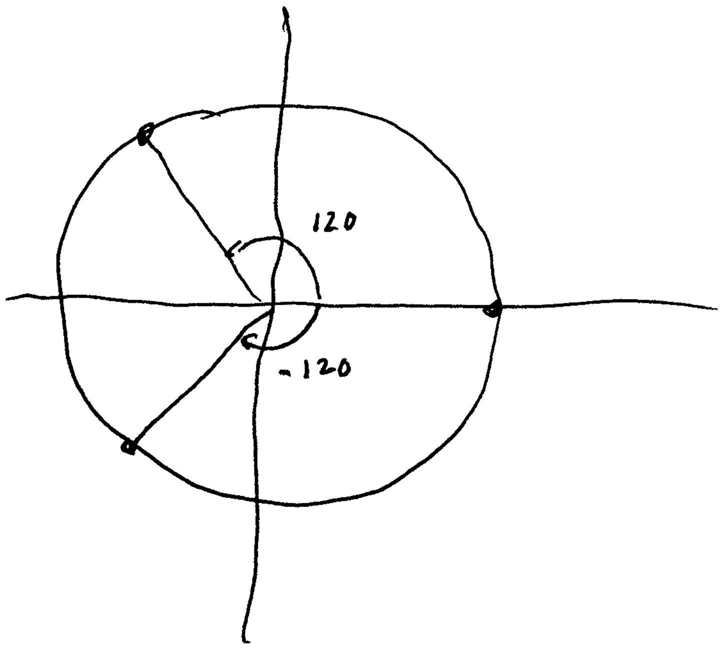
CUBE ROOTS

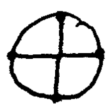
$$re^{i\theta} \longrightarrow \sqrt[3]{r}\,e^{i\theta/3}$$

$\cancel{\sqrt[3]{27}}$

$$\frac{0}{3} \qquad \frac{360}{3} \qquad \frac{720}{3} \qquad \frac{1080}{3}$$

$$0 \qquad 120 \qquad 240 \qquad 360$$





⋮

TRIG IDENTITIES

$$e^{i\varphi_1} e^{i\varphi_2} = e^{i(\varphi_1 + \varphi_2)}$$

$$\cos\varphi_1 + i\sin\varphi_1 + \cos\varphi_2 + i\sin\varphi_2 = \cos(\varphi_1+\varphi_2) + i\sin(\varphi_1+\varphi_2)$$

$$\cos\varphi_1 + \cos\varphi_2 = \cos(\varphi_1+\varphi_2)$$

$$\sin\varphi_1 + \sin\varphi_2 = \sin(\varphi_1+\varphi_2)$$

$c = \cos\varphi_2$

$d = \sin\varphi_2$

$a = \cos\varphi_1$

$b = \sin\varphi_1$

$$(a+bi)(c+di) = (ac-bd) + i\,(ad+bc)$$

$$e^{i\varphi_1} \quad e^{i\varphi_2}$$

$$e^{i(\varphi_1+\varphi_2)}$$

$$\cos\varphi_1 \cos\varphi_2 - \sin\varphi_1 \sin\varphi_2$$

$$\cancel{\sin\varphi_1 \sin\varphi_2}$$

$$+ i\,(\cos\varphi_1 \sin\varphi_2 + \sin\varphi_1 \cos\varphi_2)$$

$$\cancel{\cos\varphi_1 + i\sin\varphi_2}$$

$$\cos(\varphi_1+\varphi_2) + i\sin(\varphi_1+\varphi_2)$$

$$\cos(\varphi_1+\varphi_2) = \cos\varphi_1 \cos\varphi_2 - \sin\varphi_1 \sin\varphi_2$$

$$\sin(\varphi_1+\varphi_2) = \cos\varphi_1 \sin\varphi_2 + \sin\varphi_1 \cos\varphi_2$$